Cubic Sp-Line Interpolation for Forth Order Polynomial Function

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ABSTRACT
When any higher order polynomial function performed as a time signal then the resultant graph may be distorted. Function generator cannot generate ten power pera unit frequency this signals also needs some slant average value. This demerit has been removed under in cubic sp-line. Due to the third order polynomial function has performed with its average approximation.

In this paper we generate the signal apply interpolation formula on it and make it smooth and accurate. Finally we compare without and with cubic sp-line interpolation graphs. This algorithm performed in MatLab.

Keywords
Interpolation, Cubic Sp-Line Node Description, Tri-Diagonal System, Decomposition, Forward Substitution, Backward Substitution

1. INTRODUCTION
Any signal is nothing but a combination or pure integral addition of equation. When equation has "n" no. of values and it will generate the signal then this generated signal have some noise and distortion due to large no of time with small variations. This term indicates the degree of that equation means if degree is high distortion also increases. Interpolation removes this demerit because interpolation is nothing but an estimate a value of (a function or a series) between two known values.

2. CUBIC SP-LINE ALGORITHM
Expectation in Cubic Sp-Line is when the quality of interpolation to increase with increasing degree of the polynomial used but this is not true.

For various function F the corresponding interpolation polynomials may tend to oscillate more and more between nodes as "n" increases. Hence, we must be prepared for possible numerical instability which is representing in fig.1.

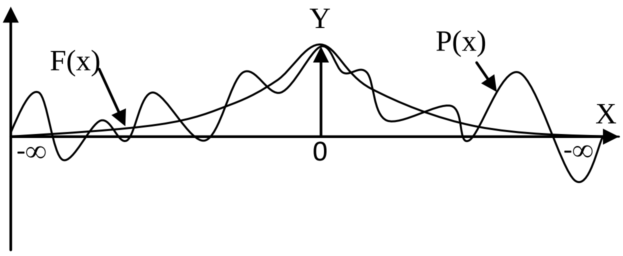

Fig.1 Representation of Polynomial As a Signal

Hence, it has proved that for equidistance nodes the maximum errors even approaches infinity as "n" tens to infinity.

Such oscillations are avoided by the method of Sp-Line. The name is borrowed from a draft man's Sp-Line, which is an elastic rod, band to conform to the points. The mathematical idea is this, instead of single high degree polynomial over an interval K. Given by **a≤ x ≤b**.

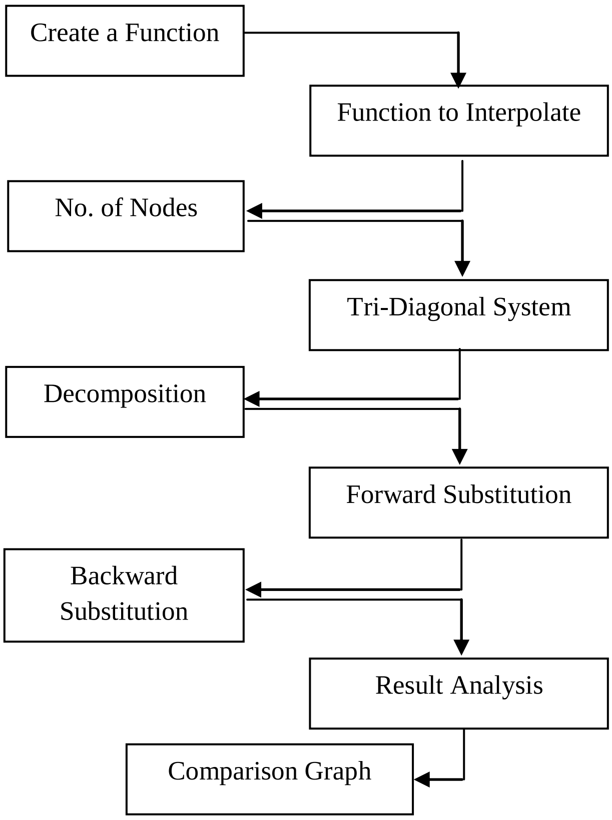

Fig. 2 Cubic Sp-Line Algorithm

3. ALGORITHM DESCRIPTION
Higher order polynomial function is applied to the MatLab with respective commands. This polynomial function generates by "rand" command in MatLab.

Further process described by given below.

A. Function To Interpolate

Interpolation means addition of n+ 1 subinterval in the given polynomial function.

$$I(x) = \sum_{k=0}^{n+1} F(k)$$

...By KBS and YKS

Where, I(x) represent interpolation of that function.

Using interpolation signal makes smoother due to interval division process. That is why; cubic sp-line interpolation is a piece wise incessant curve transitory through each of the principles.

Interpolation is divided into two parts:

If second derivation of piece wise polynomial function is zero at starting and ending points then it is called "natural interpolation". And if second derivation of piece wise polynomial function has some value at starting and ending points (in terms of signals) then it is called "clamped interpolation" [2].

B. *Number of Nodes*

Number of nodes represents the general overview of the given polynomial signal. The total number of nodes is even where the centre node is called peak node and remaining nodes are called general nodes of the given polynomial signal.

If any signal like sine wave where the peak node is in 0V D.C. line then divide this frequency into two parts and find peak node and general nodes for each signal then add those signals.

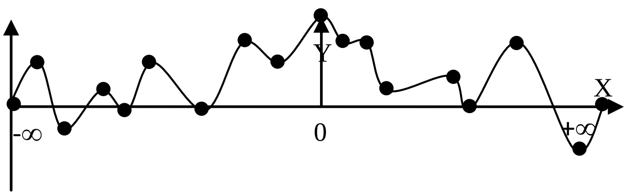

Fig.3 No. of Nodes

Total no. of nodes is even but accepts peak node general nodes are always odd [1].

C. *Tri-Diagonal System*

Tri-Diagonal system means the area covered in subinterval signals in terms of 0 and 1 in which only the diagonal elements are present other elements are zero. That is why; it also used in 2D image processing. It creates the matrix of given polynomial signal. But matrix represents other elements also like upper and lower elements. This element indicates where the signal turned and also represents the amount of noise.

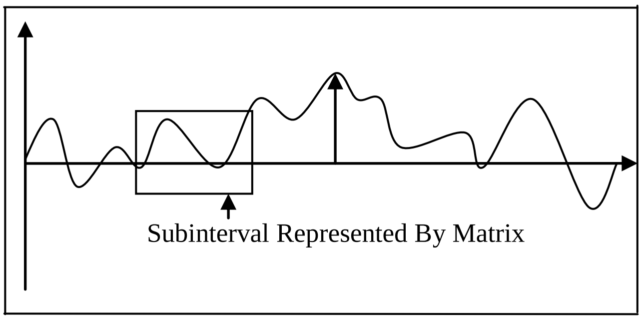

Fig.4 Tri-Diagonal System

D. *Decomposition*

Decomposition means average value of tri-diagonal system with respective area in cubic Sp-Line.

Mathematically, Decomposition is represented by;

$$D(x) = \sum_{m=\min value}^{\max value} \sqrt{I(x)_{m-1}{}^{2}}$$

...By KBS

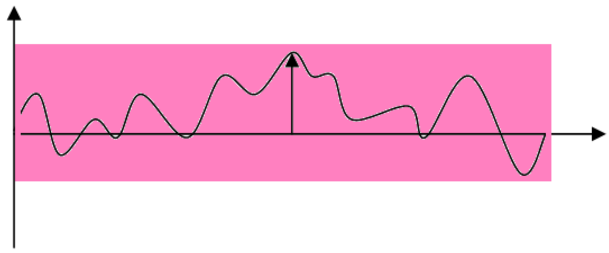

Fig. 5 After Decomposition Area of that signal in terms of matrix

E. *Forward Substitution*

Forward substitution removes the upper elements for the given matrix. But it represents lower elements and diagonal elements. This remove decides noise or sharp curve has been removed at upper elements. It also indicate when maximum peak in the polynomial function comes it will automatically zero. This process is given below.

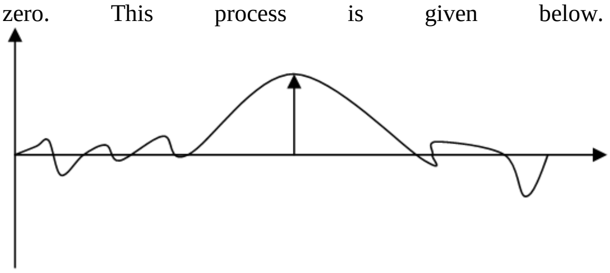

Fig.6 After Forward Communication

F. *Backward Substitution*

Backward substitution removes the lower elements for the given matrix. But it represents upper elements and diagonal elements. This remove decides noise or sharp curve has been removed at lower elements. It also indicate when minimum peak in the polynomial function comes it will automatically zero. This process is given below. At the end diagonal elements are present in the matrix in piecewise polynomial function. This matrix pivots are nothing but an average value of the respected polynomial signal where upper triangular matrix and lower triangular matrix elementary is zero in fourth order cubic Sp-Line.

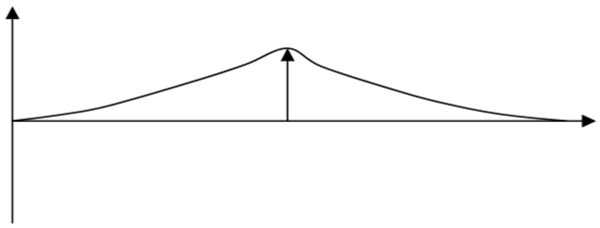

Fig.7 After Backward Substitution

4. RESULT ANALYSIS

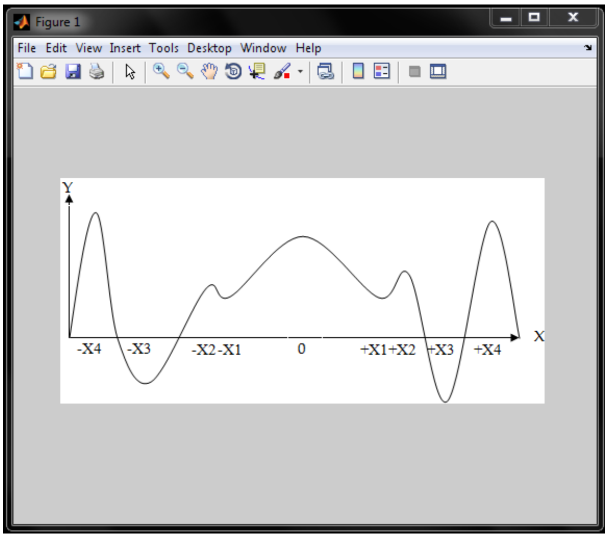

Fig.8 Without implementation of Cubic Sp-Line

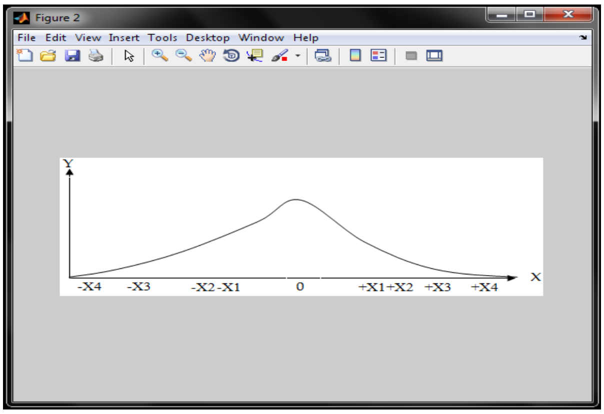

Fig.9 With Implementation of Cubic Sp-Line

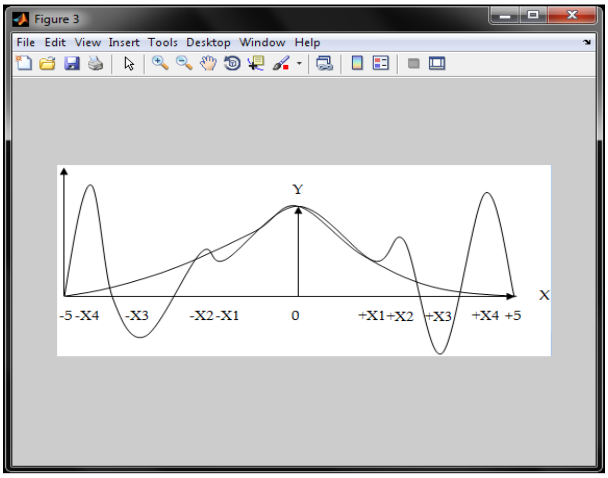

Fig. 10 Final Polynomial Function

5. APPLICATIONS

Due to this structure it is also used in multipurpose signal analysis. It is also used in image brightness where bi-linear transformation can be used on each 3x3 window.

4^{th} order Cubic Sp-Line can be used to smooth out the sudden transients in digital signal i.e. it consumes lower bandwidth reducing harmonics.

6. CONCLUSION

Implementation is very easy by Cubic Sp-Line. And they produces easy curve in higher order polynomial function. In cubic Sp-Line code it built less distortion at the infinite terms as well as sharp peak input graph.

7. FUTURE WORK

Our code is only used for 4^{th} order derivation. But if request is responsive to the smoothness of derivatives in higher term then our code is not the best choice.

8. REFERENCE

[1] Carl De Boor, “Engineering Mathematics”, Pearson Publication, 2nd Edition, Year-2006, pp.848-849 and pp.861-866.

[2] Samuel D. Conte, “Elementary Numerical Analysis An Algorithmic Approach”, Tata McGraw Hill, 3rd Edition, Year-2005, pp.251-293.