Local Adaptive Bilateral Filter with Variation for Deblurring

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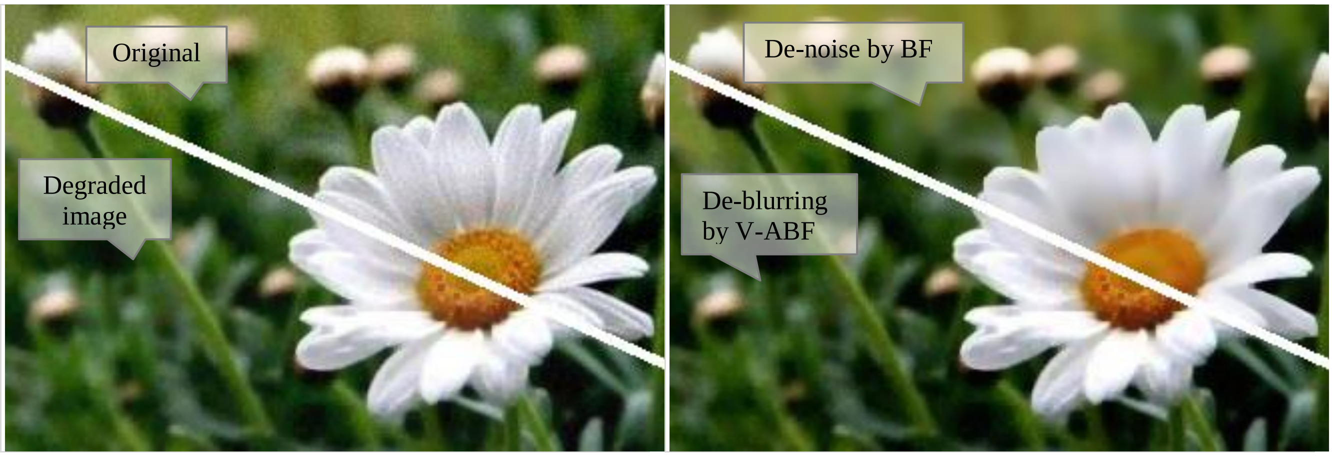


Fig. 1. BF and V-ABF Reduce degree of low resolution and improve major edges. Deblurring example by V-ABF has sharp edges.

ABSTRACT
In this study, alternative application of bilateral filter for image deblurring and enhancement is discovered. The concept of Total variation model is added to BF. Based on analyzing force distribution rules of variance, standard deviation is managed to distinguish degree of degrade. The optimization solution of total variation is gained by tracking minimum change channels and keep maximum edges. Experimentation proves that the new V-ABF can solve the deblurring problem where original BF is solution for de-noising.

General Terms
Pattern Recognition

Keywords
Variance adaptive bilateral filter, de-blurring, restoration, smoothing, sharpening, filtering, total variance

1. INTRODUCTION
Image restoration, that is, the computation of the degree of blur and noise in a given image sequence and restore original image, is a well-known problem in image processing and has received significant attention in recent years. Total Variation (TV) is one example of a widely used approach to model image degrade. Numerous applications have been developed to by this model e.g., [8,9,10,11,12]. Solution of the model is optimization of keeping edges and limitation of change.

While Bilateral Filter (BF) with duo Gaussian filters has been addressed in image de-noising as simple and effective filter, new developments of BF application were found e.g., [13,15,16].

This paper addresses to BF with reference of TV model for new development. Considering BF from global and local views, it is quite noticeable that relative change of Gaussian BF options creates dynamic range of vision effect. The longer the Gaussian standard deviations, the blur effect found more clearly.

To improve the accuracy of the blur/edge estimation on an image suffered from degrading it would be helpful to use TV model. If the estimation of blur and noise is known, the noise and blur can be removed correctly. In fact, this estimation may be is covered under variation map, so variance can give control degree on restoration BF process for blur/noise image (cf. fig.2). The proposed framework is an adaptive bilateral filter, which includes explicit modeling of the restoration process as well image regularization terms, and is solved via efficient TV model.

The input to model is an simple image for enhancement or degraded image with blur/noise for restoration, while the output are the corresponding enhanced/restored image, map of variance (fig.3c), local and global standard Gaussian deviation (fig.3b) and a comparison of BF and V-ABF output (fig.3d). As demonstrated in this paper, this join of estimation of variance, deviation regulation, and channel selection by minimization of change, are essential techniques when apply BF and TV concept.

Before proceeding with the explicit description of the proposed framework, the color image in Fig. 1 is illustrated. It is a natural scene, very challenging and appropriate to demonstrate the advantage of the approach. In this figure, daisy become more flat after BF, but if get more clear edge by V-ABF filter.

2. OUTLINE OF PAPER
The organization of this paper is as follows. Section 3 consists of notations and background materials. The algorithms are

discussed in Section 4. Section 5 presents implementation proposed algorithm. Discussions on V-ATV with BF and TV are in section 6. Future work and a concluding remark are given in Section 7, 8.

3. CONTRIBUTIONS AND RELATED WORK

Bilateral Filtering for grey and color images was introduced by Tomasi and Manduchi [6]. Bilateral filter makes images smooth while preserving edges, by means of a nonlinear combination of nearby image values. It combines grey levels or colors based on both their geometric closeness and their photometric similarity, and prefers near values to distant values in both domain and range.

Many applications continue to expand the capability of the basic method in various aspects: Tone Mapping [8], Tone Management [9], Virtual Video Exposure [10] Flash / No-Flash [11,12].

Total variation minimizing models from Rudin, Osher, and Fatemi [1], have become popular and successful for image restoration. New developments were developed basing on Total variation: In-painting in Transformed Domains [13]. Super-resolution [15], Diffusion Tensors Images [16].

In this paper, an adaptive version of Bilateral Filter with concept of Total Variation model is developed: find optimal solution balancing between maximum variation and minimum of changes. Result is V-ABF that give enhancement effect: ignore a part of details and keep major edges. The algorithm of V-ABF is based on Lanman's implementation of BF and bilateral image abstraction [18].

4. FILTERING

4.1 Total Variation

Firstly total variation model from ROF [1] is recalled. Denote u as M-dimension vector function defined in space $\Omega \subset \Re^N$:

$$u:\Omega \to \Re^M, u(x) := (u_1(x),..u_M(x)), \qquad (1)$$

$$u_i:\Omega \to \Re, i = 1,.., M$$

∇ is divergent operator:

$$\nabla u := (\nabla u_1,...\nabla u_M) : \Omega \to \Re^{M\times N} \qquad (2)$$

Denote Euclidean scalar product by

$$< u, v > := \sum_{i=1}^{M} < u_i, v_i >$$

and the L_2 Euclidean norm by $\|\cdot\|$

$|\nabla u(x)|$ is L_2 Euclidean norm of u :

$$|\nabla u(x)| = \sqrt{\sum_i^M u_i^2}, u \in \Re^M \qquad (3)$$

Total variation (TV) then is defined by formula:

$$TV(u) = \int_\Omega |\nabla u(x)| dx \qquad (4)$$

Then v is a solution for input u to minimize energy function [1]:

$$E_\lambda(v) = \|v-u\|^2 - \lambda TV(v), \qquad (5)$$

Vectoral TV norm of u is given by

$$\int_\Omega |Du| = \sum_{i=1}^{M} TV(u_i) = \sum_{i=1}^{M} \int_\Omega |\nabla u_i(x)| dx \qquad (6)$$

From next section index i will be denoted on superscript (u^i) to leave subscript note for bilateral filter.

4.2 Bilateral filter

The Gaussian filter, is a smoothing filter but at the cost of less distinct edges. For $x \in \Omega$:

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad (7)$$

The Bilateral Filter [2] with global constants σ_s^∞ and σ_r^∞ smooth surfaces, just as the Gaussian Filter, while maintaining sharp edges in the image:

$$BF[u]_x = \frac{1}{W_x} \sum_{y\in S} G_{\sigma_s^\infty}(\| x-y \|) G_{\sigma_r^\infty}(| u(x)-u(y) |) u(y) \qquad (8)$$

Where σ_s^∞ and σ_s^∞ are unchanged for Ω, $| u(x)-u(y) |$ is density difference in locations at x and $y \in \Omega$. W_x is normalized weight at x :

$$W_x = \sum_{y\in S} G_{\sigma_s^\infty}(\| x-y \|) G_{\sigma_r^\infty}(| u(x)-u(y) |) \qquad (9)$$

For color image $u(x) := (u_1(x), u_2(x), u_3(x))$

$$BF[u]_x = \frac{1}{W_x} \sum_{y\in S} G_{\sigma_s^\infty}(\| x-y \|) G_{\sigma_r^\infty}(\| \mathbf{C}_x - \mathbf{C}_y \|) \mathbf{C}_y \qquad (10)$$

Where $\| \mathbf{C}_x - \mathbf{C}_y \|$ is difference of color.

4.3 Adaptive bilateral filter by variation for deblurring grey image

Now σ_s and σ_r from (8) are presented as function of $|\nabla u(x)|$, defined from (3):

$$\sigma_s^V(x) : \Omega \to [\max(0, \sigma_s^\infty - \varepsilon_s), \sigma_s^\infty + \varepsilon_s] \qquad (11)$$

$$\sigma_s^V(x) = f_s(|u(x)|) \qquad (12)$$

$$\sigma_r^V(x) : \Omega \to [\max(0, \sigma_r^\infty - \varepsilon_r), \sigma_r^\infty + \varepsilon_r] \qquad (13)$$

$$\sigma_r^V(x) = f_r(|u(x)|) \qquad (14)$$

where $\varepsilon_s > 0, \varepsilon_r > 0$.

In order to keep W_x (9) in ABF the same as in BF (8), histogram of $\sigma_s^V(u)$ must be the same as $|\nabla u(x)|$, so that:

$$(\sigma_s^V(x) - \sigma_s^\infty + \varepsilon_s) / 2\varepsilon s_s = (|\nabla u(x)| - n) / (m-n) \qquad (15)$$

where

$$m = \max_\Omega |\nabla u(x)|, n = \min_\Omega |\nabla u(x)|.$$

Consequently, (14) leads to equation:

$$\sigma_s^V(x) = 2\varepsilon_s * (|\nabla u(x)| - n) / (m-n) + \sigma_s^\infty - \varepsilon_s \qquad (16)$$

Definitions (17,18) for $\sigma_r^V(u)$ are similar to (15,16):

$$(\sigma_r^V(x) - \sigma_r^\infty + \varepsilon_r)/2\varepsilon_r = (|\nabla u(x)| - n)/(m-n) \qquad (17)$$

$$\sigma_r^V(x) = 2\varepsilon_r * (|\nabla u(x)| - n)/(m-n) + \sigma_r^\infty - \varepsilon_r \qquad (18)$$

Hence let σ_s be σ_s^∞ or $\sigma_s^V(u)$; and σ_r be σ_r^∞ or $\sigma_r^V(u)$, now basic adaptive bilateral filters for grey image are defined:

$$BBF(u_x) = BBF(u_x, \sigma_s, \sigma_r) = \frac{1}{W_x} \sum_{y \in S} G_{\sigma_s}(\| x - y \|) G_{\sigma_r}(|u_x - u_y|) u_y \qquad (19)$$

Table 1. BASIC TV BILATERAL FILTERS

$[\sigma_s, \sigma_r]$	σ_s^∞	$\sigma_s^V(u)$
σ_r^∞	$BBF^V(u, \sigma_s^\infty, \sigma_r^\infty)$ (20)	$BBF^V(u, \sigma_s^V, \sigma_r^\infty)$ (21)
$\sigma_r^V(u)$	$BBF^V(u, \sigma_s^\infty, \sigma_r^V)$ (22)	$BBF^V(u, \sigma_s^V, \sigma_r^V)$ (23)

The filters have four forms, see table 1. These basic filters create full BF combination of local and global options:

(20) is the same global BF predefined in (8);

(21) and (22) mixes global and local options;

(23) uses local options only.

Finally Adaptive Bilateral Filter by Variation (V-ABF) is:

$$ABF^V(u_x, \sigma_x) = BBF^V(u_x, \hat{\sigma}_x)$$

where

$$\hat{\sigma}_x = \arg\min_{\sigma \in [\sigma_s, \sigma_r]} |BBF^V(u_x, \sigma_x) - u_x| \qquad (24)$$

4.4 V-ABF and TV model

The *min* condition in (24) leads V-ABF to have minimum change $\|v-u\|^2$ - the first part of energy function (5) of TV model. According to conditions (15) and (17), the V-ABF filter has as big σ_s^V and σ_r^V at x as big variation $|\nabla u(x)|$ there. This leads to make $TV(v)$ get as big as possible – the second condition of energy function (5). So edges where variation is usually high will be kept well by the conditions.

The *arg* in (24) is a selection from four combinations of local and global options, where only local options with variation present support for maximum $TV(v)$ in (5). The selection has effect of using weight λ in the (5), that manages degree of two parts of $E(v)$. So V-ABF followed well the concept of TV model to get optimization solution for energy function.

4.5 Adaptive bilateral filter by variation for deblurring color image

Color image is presented by *3-D* function u:

$$u: \Omega \to \Re^3, u(x) := (u^1(x), u^2(x), u^3(x)), \qquad (25)$$

Where channel $u^i: \Omega \to \Re, i = 1,2,3$

Given initial global $\sigma_s^\infty, \sigma_r^\infty$, divergence ε_s and ε_r, basic bilateral filter by variation for each channel $u^i, i = 1,2,3$ is described:

$$BBF(u^i) = BBF(u^i, \sigma_s^i, \sigma_r^i) = \frac{1}{W_x} \sum_{y \in S} G_{\sigma_s^i}(\| x - y \|) G_{\sigma_r^i}(|u^i(x) - u^i(y)|) u^i(y) \qquad (26)$$

Finally, $BBF^V(u_x^i, \sigma_x^i) = BBF^V(u_x^i, \hat{\sigma}_x^i)$ (27)

where

$$\hat{\sigma}_x^i = \arg\min_{\sigma^i \in [\sigma_s^i, \sigma_r^i]} |BBF^V(u_x^i, \sigma_x^i) - u_x^i| \qquad (28)$$

So this V-ABF color version is different of (10): V-ABF take local variation $|\nabla u^i(x)|$ to control Gaussian options for each channel *i*.

5. IMPLEMENTATION

5.1 Deblurring grey image

ALGORITHM V-ABF
FOR DEBLURRING GREY IMAGE

Given: blurred grey image $u(x)$, initial global $\sigma_s^\infty, \sigma_r^\infty$, divergence ε_s and ε_r.

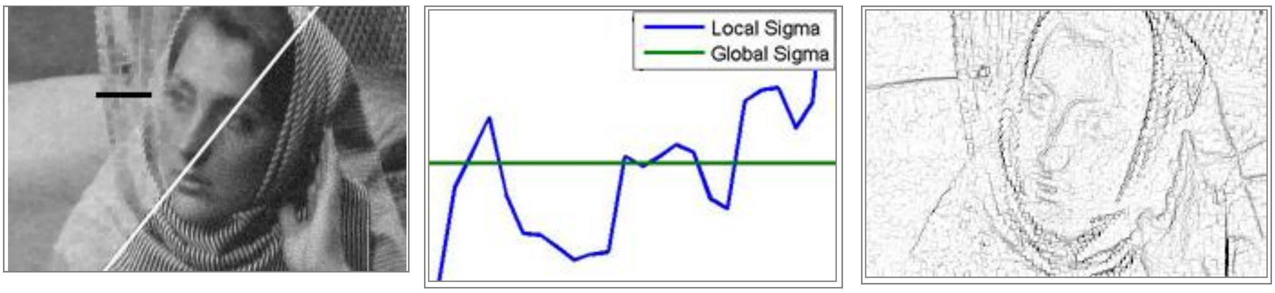
The deblurring V-ABF algorithm takes the following steps:

Step 1: Find local variation $|\nabla u(x)|$ by formula 3.

Step 2: Define local $\sigma_s^V(x)$ and $\sigma_r^V(x)$ following formula 16 and 18 accordingly.

Step 3: Calculate basic V-ABF values by formula 20-23. Then get final output $v(x) = ABF^V(u_x, \sigma_x)$ by formula 24.

The algorithm is presented by diagram in fig.4



a) b) c)

d) e) f)

Fig. 2. Effect of BF and V-ABF for "Barbara" image

a) Original image in right part, degraded image by command *imclose(I,true(3))* in left part

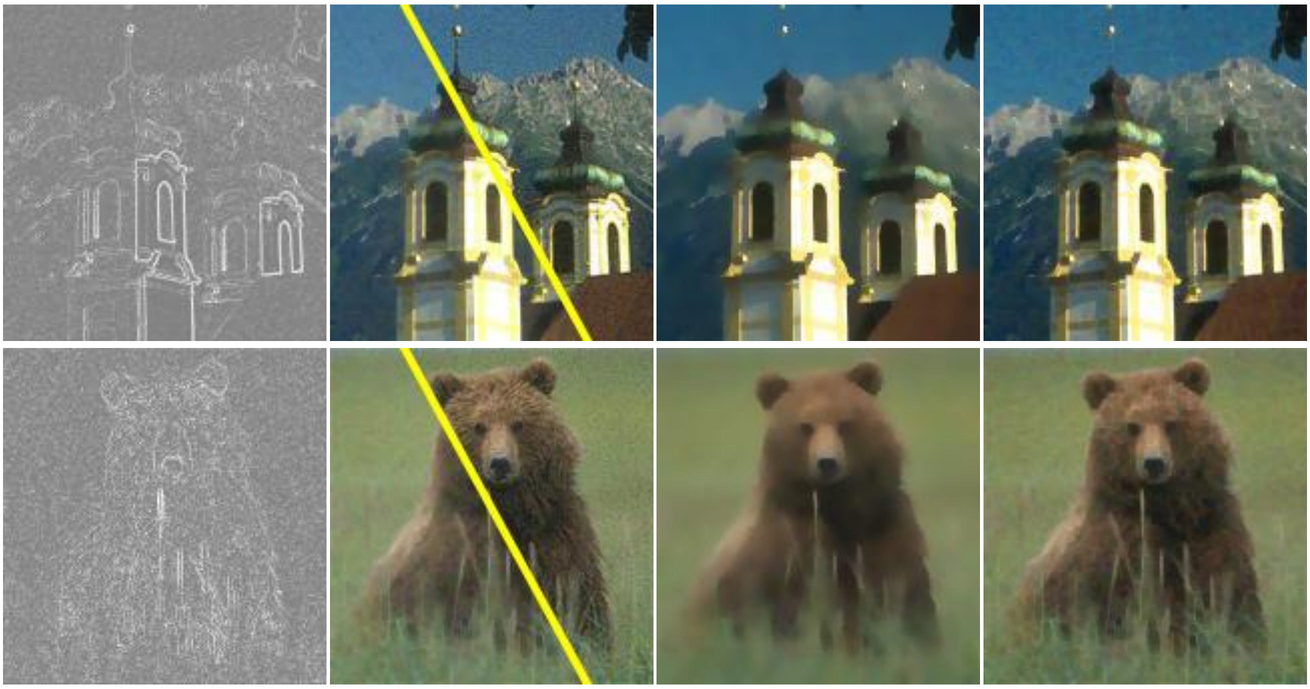
b) Global σ_r^{∞} in green and local σ_r^{V} in blue, according to black line in (a).

c) Variation signal $|\nabla u|$

d) Original signal in blue, BF signal in green and V-ABF signal in red, according to black line in (a)

e) Result by BF with $\sigma_s^{\infty} = 3, \sigma_r^{\infty} = 0.1$

f) Result by TV-ABF with local σ_s^{V} and σ_r^{V}



a) V signal of L channel (CieLab)

b) Right part: original color image.
Left part: degraded color image

c) De-noise by BF, $\sigma_s^{\infty} = 3, \sigma_r^{\infty} = 0.1$

d) Enhancement by V-ABF $\sigma_s^{V} \in [2.8, 3.2], \sigma_r^{V} \in [.95, 1.05]$

Fig. 3. Effect of BF and V-ABF for color image

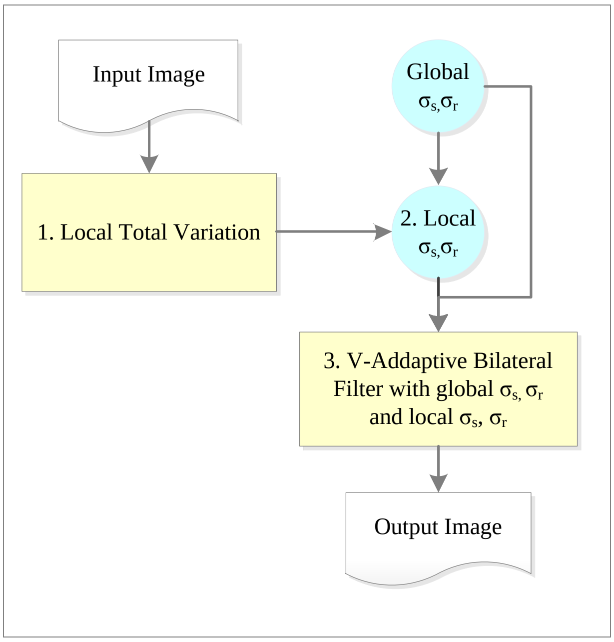


Fig. 4. Diagram of V-Adaptive Bilateral Filter with global and local sigma

Example of deblurring grey image

Barbara image is an example for V-ABF algorithm. Firstly, original image was blurred by command *imclose(I,true(3))* (fig.2.a). Local variation $|\nabla u(x)|$ of blurred image has visual signal, presented in Fig.3.c. Step 2 of the algorithm gives local $\sigma_s^V(x)$ and $\sigma_r^V(x)$. Fig.2.c show straight line of $\sigma_r^\infty(x) = 0.1$ and curve line of $\sigma_r^V(x)$ according to black line in fig.2.a.

Result signal of $ABF^V(u_x,\sigma_x)$ is red line in fig.2.d, being aligned with signal of blurred image (blue line) and result of bilateral filter (green line), according to black line in fig.2.a. The red line has more shift in zone of edge, see arrow in fig.2.d. Fig.2.f is final output of V-ABF, and fig.2.e - result of BF algorithm. Two images sound the same but if look carefully in details, then fig.2f gives better edges, for example, see arrows in the images fig 2e, 2f.

Deblurring color image

ALGORITHM V-ABF
FOR DEBLURRING COLOR IMAGE

Given: Blurred color image $u(x), u: \Omega \rightarrow \Re^3$, initial global σ_s^∞, σ_r^∞, divergence ε_s and ε_r.

The deblurring V-ABF algorithm takes the following three steps for each channel *i, i=1,2,3:*

Step 1: Find local variance $|\nabla u^i(x)|$ by formula 3.

Step 2: Define local $\sigma_s^{V,i}(x)$ and $\sigma_r^{V,i}(x)$ following formula 16 and 18 accordingly.

Step 3: Calculate basic V-ABF values by formula 25. Then get output $v^i(x) = ABF^{V,i}(u_x^i, \sigma_x^i)$ by formula 27-28.

When above task were completed, combine all channels i into final output image.

Example of deblurring color image

Fig.3 shows some examples. Color images firstly were blurred in fig.3.b. Variation of L channel (CieLab representation) of each blurred image is shown in fig.3.a. Final result of bilateral algorithm is in fig.3.c. and fig.3.d is result of V-ABF. The images in fig.3.c have more significant edges than images in fig.3.c.

6. DISCUSSION

V-ABF is an adaptive version of BF, which uses variation of TV model to customize locally Gaussian options. It gives effect of better keeping edges while blur small details.

The V-ABF algorithm is based on Lanman's implementation of BF [1] and bilateral image abstraction [17]. BF take one round to check all items of input image and then apply two Gaussian filters. So BF has $O(n)$ complexity. V-ABF take two rounds: the first round is for step 1 of the algorithm, the second round runs step 2 and 3. The last round checks local neighbours of each pixel to calculate local values. If k is the size of neighbour zone, complexity of V-ABF is $O(n+kn) = O((1+k)n) \cong O(kn)$.

In experimental implementation the V-ABF is found good enough with $\sigma_s^\infty = 3 \pm 0.5$ and $\sigma_r^\infty = 0.1 \pm 0.05$.

7. FUTURE WORK

V-ABF is tested for deblurring grey and color images. Performance for small and medium image size is acceptable but it take long for big images. Algorithm may need to be checked on optimization for better performance. Some other application like de-noising, edge detection, artistic drawing could be implemented by V-ABF.

8. CONCLUSION

This work presents a new efficient application of Bilateral Filter for deblurring and enhancement for color images. The proposed V-ABF algorithm constitutes adaptive standard deviation generation from variation, basing total variation model for image restoration. The algorithm has been tested on several image categories from the publicly available image database provided by the University of California Berkeley, and the results show that the algorithm is robust to various image scenarios at different scales.

Future research of V-ABF algorithm for de-noising, edge detection or artistic drawing could lead to wider range application of the algorithm.

9. ACKNOWLEDGMENT

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