Study and Performance in a MIMO System to Receiving

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ABSTRACT
Modern systems of Wireless Communications have limited capacity and processing techniques are required more sophisticated in order to improve system performance. One of the techniques in vogue is the application of MIMO (Multiple Input Multiple Output), known to improve the channel capacity and or the probability of bit error. The purpose of this paper is the comparison of two techniques for MIMO systems. The first technique using a series of antennas on the side of the terminal (and therefore much RF chain) is known to improve system performance and that there is a feedback loop containing the state of the channel or not. However, the use of these antennas on the terminal requires a lot of power because the remains of additional RF chains. The second one is for the terminal to choose the best antenna corresponding to the channel conditions. For the uplink, this requires a return to the channel state radio base station to mobile. The performance will always be suboptimal compared to using multiple channels in reception. But the terminal would need to maintain single RF chains which potentially save a lot of power.

Keywords
Minimum mean-squared error (MMSE), Maximum likelihood (ML), Bit error rate (BER), Successive interference cancellation (SIC), zero Forcing (ZF), Multiple-Input Multiple-Output (MIMO).

1. INTRODUCTION
Digital technology completes the range of applications offered by the analog transmissions. Simpler and more robust than analog media, media digital communication already allow many services one of the main reasons for this delay is the willingness of manufacturers to integrate the latest technologies to increase substantially the spectral efficiency of mobile systems. One of them, combining multiple antennas for transmission and reception, is the subject of this thesis. Such multiple antenna systems, MIMO systems or (for Multi -Input Multi-Output), have the potential to significantly increase the capacity of wireless transmissionand, if they exist for more than a decade, experienced a marked revival. Several architecturesas have been developed in parallel, from the spatial multiplexing space-time codes, and all offering incredible performance improvement of transmission systems. However, this multi-antenna technology does not work in all environments, and must comply with many constraints. It is increasingly growing and affluent innovations. This work therefore aims firstly to raise some specific spatiotemporal systems such restrictions, and secondly to optimize their performance.

It also gives the main characteristics and limitations of multi-antenna systems, before proposing the intended applications. Then we give the various diversity techniques that can combat fading, followed by a realistic modeling of the MIMO channel to become familiar with the spatial aspect brings an increase in the number of antennas, the capacity is one of the parameters most important, we present, therefore, the capacity of SISO and MISO systems MIMO. And proposes to involve the efferent equalizer types used for antenna selection at receiver and the most performance and we propose a simulation of multi-antenna systems which allows both the calculated capacity of MIMO systems and evaluated the antenna bit error rate according to the number ( TEB / Ne) with equalization.

2. MATHEMATICAL MODEL AND SIMULATION
2.1 Introduction
The simulated transmission system correspondsto the system number of structures for the receiver (2 x2) of the MIMO channel.
- WithSISO equalizer(ZF, MMSE, ML).
- 1-MIMO Equalizerzero forcing(ZF).
- 2-MIMO EqualizerMinimum MeanSquareError (MMSE).
- 3-MIMO equalizer with maximumlikelihood (ML), Maximum Likelihood(ML).
- 4-MIMO withzeroforcing equalizer the successive Interference Combination(ZF-SIC) withoptimal control.
- The 5-MIMO with MMSE equalizer with SIC and optimal control.

2.2 Sisowith equalizer(ZF, MMSE, ML)
Now we consider the casewhere there is an "input signal output signal" SISO system. For this reason, we will try to make the "RBA" bit error rate under the application of equalization (ZF, ML, MMSE) at the receiver. We assume that the channel is Rayleigh and use the BPSK channel.

![Fig 1: BER for SISO system with equalizer (ZF, ML, MMSE)](image)

Observations
As desired, the simulation results show good agreement with theory.

Simulation results show that ZF and MMSE equalizer and ML gives the same (bit error rate) "BER". Finally, I’m not sure whether we can call it as the MMSE equalization (ZF, ML) as there is no interference terms. We conclude that the application of equalization (ZF, ML) in a system “SISO” gives the same "BER". Since they are the same ERR, The problem is the increase in noise for bad subchannels. The diversity order is, however, MR-MT (which implies that there is no diversity if MR = MT).

2.3 MIMO with Zero Forcing Equalizer (ZF)

We now consider the case where there is a formation of a Multiple Input Multiple Output (MIMO) antenna system. For this reason, we will limit our discussion to 2 transmit and 2 receive antennas (which has a 2x2 MIMO channel). We assume that the channel is Rayleigh fading and multi-chain. We use BPSK modulation.

For a MIMO system (2, 2), the probable use of a device for two transmission antennas is as follows:

- Under normal transmission, we send symbols in the first time interval, the 2nd time interval, etc...
- However, as we now have two transmit antennas, we can group the symbols into sets of two, the first time interval, the second time interval, and so on.
- Note that since we are grouping two symbols and sending in a time interval, we need 2 time intervals to complete the transmission - data rate is doubled!
- This is the simple explanation of a probable MIMO transmission with two transmit antennas and two receiving antennas.

2.3.1 The application of (ZF) for MIMO (2, 2)

We will now try to understand mathematically how to extract the two symbols that interfere with other [5]. These signals received at the first receiving antenna and the following form:

\[ y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \]  

The signals received on these second receiving antenna and the following form:

\[ y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \]  

Or: \[ y_1, y_2 \] the symbols received at the first and second receiving antennas respectively.

h_{1,1} is the channel for the first antenna transmitting the worm first receiving antenna.

h_{1,2} is the channel for the first antenna transmitting the worm second receiving antenna.

h_{2,1} is the channel for the first antenna transmitting the worm 2nd receiving antenna.

h_{2,2} is the channel for the second antenna transmitting the worm 2nd receiving antenna.

\( y_1 \) and \( y_2 \) are two symbols transmitted.

\( n_1, n_2 \) is the noise on the two receiving antennas?

We assume that \( h_{1,1}, h_{1,2}, h_{2,1}, h_{2,2} \) known and the receiver also knows \( y_1, y_2 \) then the unknown is \( x_1, x_2 \) two equations and two unknowns may have solved? Yes. For further understanding, the above equation can be represented in matrix notation as follows:

\[ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = [H]^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \]  

2.4. MIMO with MMSE Equalizer

2.4.1 The application of (MMSE) for the MIMO system (2, 2)

As expected, the simulated results with the MIMO system (2x2) using the BPSK modulation in a Rayleigh channel shows that the results obtained in the matching as a system (1x1) to the BPSK channel the Rayleigh.
We will now try to understand mathematically how to extract the two symbols that interfered with others. The signals received at the first receiving antenna and the following form:

\[ y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1}h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1. \]  

(6)

The signals received on the second receiving antenna and the following form:

\[ y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1}h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2. \]  

(7)

Or: \( y_1, y_2 \) are the symbols received on the first and the second receiving antennas respectively.

- \( h_{1,1} \) is the channel for the first antenna transmitting the worm first receiving antenna.
- \( h_{1,2} \) is the channel for the first antenna transmitting the worm 2nd receiving antenna.
- \( h_{2,1} \) is the channel for the first antenna transmitting the worm 2nd receiving antenna.
- \( h_{2,2} \) is the channel for the second antenna transmitting the worm 2nd receiving antenna.
- \( x_1, x_2 \) are symbols transmitted.
- \( n_1, n_2 \) are the Pruits on the two receiving antennas.

We assume that \( h_{1,1}, h_{1,2}, h_{2,1}, h_{2,2} \) are known and the receiver also knows \( y_1, y_2 \). Then the unknown is \( x_1, x_2 \). Two equations and two unknowns may have solved? Yes.

For convenience, the above equation can be represented in matrix notation as follows:

\[ y = Hx + n. \]  

(8)

The Minimum Mean Square Error (MMSE) approach sought to find a coefficient \( W \) that minimizes the following criterion:

\[ E[(Wy - x)(Wy - x)^H]. \]  

(9)

\[ W = [H^H H + N_0 I]^{-1} H^H. \]

When comparing the equation Zero Forcing equalizer, apart from the expression \( N_0 I \), both equations are comparable. Indeed, when the noise term is zero, the MMSE equalizer reduced to zero forcing equalizer.

**2.4.2 The matrix Equivalent**

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\[ y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1}h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1. \]  

(10)

The signals received on the second receiving antenna and the following form:

\[ y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1}h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2. \]  

(11)

Or: \( y_1, y_2 \) sont les Symboles reçu sur la première et la deuxième antenne de réception respectivement.

- \( h_{1,2} \) is the channel for the first antenna transmitting the worm first receiving antenna.
- \( h_{2,1} \) is the channel for the first antenna transmitting the worm 2nd receiving antenna.
- \( h_{2,2} \) is the channel for the second antenna transmitting the worm 2nd receiving antenna.
- \( x_1, x_2 \) are symbols transmitted.
- \( n_1, n_2 \) are the Puits on the two receiving antennas.

We assume that \( h_{1,1}, h_{1,2}, h_{2,1}, h_{2,2} \) are known and the receiver also knows \( y_1, y_2 \). Then the unknown is \( x_1, x_2 \). Two equations and two unknowns may have solved? Yes.

For convenience, the above equation can be represented in matrix notation as follows:

\[ y = Hx + n. \]
2.5.1 Other assumptions ML (maximum likelihood)
The channel is flat fading - In simple terms, this means that the multi-channel has a single tap. Thus the convolution operation reduces to a simple multiplication. For a more thorough discussion on the Flat fading and frequency selectivity, it was admitted that:
- Channel use each transmit antenna is independent of the chain by other antennas sent.
- For the transmitting antenna, each symbol transmitted is multiplied by a random variable complex $h_{ij}$. As the channel under consideration is a Rayleigh channel, the real and imaginary parts $h_{ij}$ are Gaussian distributed.

An average $\mu_{h_{ij}} = 0$ and Variance $\sigma^2_{h_{ij}} = \frac{1}{2}$

- examine the channel between each transmit antenna and reception (issued and received) is an independent random time varying.

- On the receiving antenna, the noise $n$ characterize the Gaussian probability density function $p(n)$ with:

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}} \quad (12)$$

with $\mu = 0$ and $\sigma^2 = \frac{h_{ij}}{2}$

- The channel $h_{ij}$ is known at the receiver.

2.5.2 Receiver maximum likelihood (ML):
The maximum likelihood receiver tries to find $\hat{x}$ which minimizes

$$J = [Y - H\hat{x}]^2 \quad (4.13)$$

Since the BPSK modulation, the possible values are $+1$ or $-1$. $x_1$ same axis $x_2$, predevalue $+1$ or $-1$. And for the solution of maximum likelihood, we need to find the minimum of all four combinations $x_1$ and $x_2$.

The estimation of transmitted symbols is chosen according to the minimum value of these four values, so say:

- If the minimum is $+1$, $+1 \Rightarrow [1 \ 1]$. (4.10)
- If the minimum is $-1$, $-1 \Rightarrow [1 \ 0]$. (4.11)
- If the minimum is $+1$, $-1 \Rightarrow [0 \ 1]$. (4.12)
- If the minimum is $+1$, $-1 \Rightarrow [0 \ 0]$. (4.13)

The results of $(2 \times 2)$ MIMO equalizer with Maximum Likelihood (ML) helped to achieve a performance close to the corresponding system $(1,2)$, one transmit antenna and two receive antennas in the case (MRC).

If we use a higher order such as 64QAM constellation, then the calculation of the maximum likelihood equalizer could become too complex. 64QAM with 2 spatial streams and we need to find the minimum of $64^2 = 4096$ combinations.

2.6. MIMO zero forcing equalizer with sic and optimal control (ZF -SIC)

We will explore a variant of ZF -SIC called zero forcing successive interference cancellation with optimal control. We assume that the channel Rayleigh multipath and flat-fading channel and BPSK modulation.

![Fig 5: BER performance for $2 \times 2$ MIMO channel equalized (ZF-SIC).](image)

To finish the graph if we compare the two equalization criteria: zero forcing (ZF) and (ZF-SIC), the data adds the result of improvements near 4.0dB SNR optimal control corresponding to the point $BER = 10^{-3}$.

2.7 MIMO with MMSE-SIC and optimal control

We extend the notion of (MMSE-SIC) called Minimum Mean Square Error successive interference cancellation with optimal control and simulate their performance. We assume that the channel Rayleigh multipath and flat-fading channel and BPSK modulation.
To decode the simplistic decoding, the received signal is:

$$y = x + n$$

where:

- $y$ is the received symbol.
- $x$ is the transmitted symbol (taking values $+\sqrt{E_b} \text{ and } -\sqrt{E_b}$).
- $n$ is the additive white Gaussian noise (AWGN).

Our discussion for BPSK in AWGN, we know that the probability of bit error is:

$$p_b = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{E_b}}{\sqrt{N_0}} \right).$$

2.9 Evaluation of BER in an AWGN channel with Nr receive antennas

2.9.1 Case of a single antenna transmission

Let us begin the discussion with an antenna (1 transmission), sending signals with energy $v$ and a receiving antenna. Since we are considering only the BPSK modulation, the signals that are sent are:

$$\pm \sqrt{E_b} \text{ and } -\sqrt{E_b}.$$  

When there is a single receiving antenna with a thermal noise (AWGN) with average

$$\mu = 0 \text{ and } \text{variance } \sigma^2 = \frac{N_0}{2}.$$  

La probability density function of the noise is:

$$p(n) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}.$$  

The received signal is of the form:

$$y = x + n$$

where:

- $y$ is the received symbol.
- $x$ is the transmitted symbol (taking values $+\sqrt{E_b} \text{ and } -\sqrt{E_b}$).
- $n$ is the additive white Gaussian noise (AWGN).

Our discussion for BPSK in AWGN, we know that the probability of bit error is:

$$p_b = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{E_b}}{\sqrt{N_0}} \right).$$

2.9.2 Case with two (02) of transmit antennas

Now, consider the case where we have two receiving antennas each having noise (AWGN) with average $\mu = 0$ and variance $\sigma^2 = \frac{N_0}{2}$. As the noise on each antenna is independent of each other, in the signal processing, we can say that the noise on each antenna are independent, that is to say, independent and identically distributed. The transmitter is still sending power symbols $E_b$.

The received signal is of the form:

$$y_2 = x + n_2$$

with:

- $y_1, y_2$ are symbols of receive antenna received 1, 2, respectively.
- $y_1, y_2$ are symbols of receive antenna received 1, 2, respectively.
- $x$ is the transmitted (taking values $+\sqrt{E_b}$) symbol.
- $n_1, n_2$ is the additive white Gaussian noise (AWGN) on the receiving antennas 1, 2, respectively.

For simplicity, we assume that the signal $+\sqrt{E_b}$ was transmitted. At the receiver, we now have:

$$y_1 = \sqrt{E_b} + n_1 \text{ and } y_2 = \sqrt{E_b} + n_2.$$  

To decode the simplistic (and better in this case) is to take the average of $\hat{y}_1 \text{ and } \hat{y}_2$ and make the hard decision decoding that is

$$\hat{y}_e = \frac{y_1 + y_2}{2}$$

and if $\hat{y}_e \geq 0$ implies that the transmitted bit is $1$ and $\hat{y}_e \leq 0$, implies that the bit transmitted and equals $0$.

Now, let us know if there is a gain by the reception diversity. Performing this average $\hat{y}_e$ splitting in terms of signal and noise:

$$\hat{y}_e = \frac{y_1 + y_2}{2}.$$
The transmitted symbol $x$ is the additive white Gaussian noise (AWGN) on the receiving antennas $1, 2, \ldots, N$, respectively, for demodulation, one calculates $y_i$, which is the average of all the $N$ received symbols, is $y_i \geq 0$ the transmitted bit is 1, so $y_i \leq 0$ implies that the bit 0 is transmitted.

The variance of the expression $\frac{\sum_{r=1}^{N} n_r^2}{N}$ is the noise $\sigma^2$.

The signal to noise ratio bit Effective N receiving antenna is $N$ times the signal to noise ratio for a single antenna (single):

$$\frac{E_b}{N_0^{\text{eff}, N}} = \frac{E_b}{N_0^{\text{eff}, 1}} N$$

Thus, the probability of error for a case of two receive antennas is:

$$p_b = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

As desired, the simulation results show good agreement with theory.

With a system with $N$ receive antenna gain relative to the case of a single receive antenna is about $10 \log_2(N)$. For example, in the figure above, for the bit error rate of $10^{-4}$.

With two receiving antennas lowest $\frac{E_b}{N_0}$ is 3dB.

With four receiving antennas lowest $\frac{E_b}{N_0}$ is 6dB.

Hopefully this has helped you start with diversity reception.

3. CONCLUSION

The multi-antenna systems (MIMO) are booming thanks to their potential in terms of throughput and robustness to fading channel. The cost of production of complex systems is high. It is therefore interesting to predict, before completion, the important parameters to have the best performance. Throughout this paper, we studied the performance of MIMO systems equalized reception thanks to the knowledge of the channel (CSI). Precoding gives an extra freedom makes possible the optimization of transmission according to certain criteria (capacity, quality of service, etc). For these systems, we have established theoretically their bit error probability (PEB) for a Rayleigh channel, and bit error rate (BER) through simulations differ. We could then identify key parameters. We have focused on the selection of antenna reception (which is a technique provided by MIMO) and in particular the implementation of an important technique that offers the possibility of correcting the received signals. This technique and call it EQ Based on the following, the transmission channel and rethinking impulse is why we gave them a large portion of this work. Then, as an application of equalization, we proposed a MIMO system with two transmit antennas and two receiving antennas. Finally, simulations...
have shown that the use of deferent types of equalization with MIMO technique helps to increase the bit rate and the correction of the channel effect.

4. REFERENCES


