Application of Fuzzy Soft Set Theory in Day to Day Problems

Krishna Gogoi
Devicharan Barua Girls College, Jorhat, Assam

Alock Kr. Dutta
Bahona College, Bahona, Jorhat, Assam

Chandra Chutia
Jorhat Institute of Science and Technology, Jorhat, Assam

1. INTRODUCTION
The most appropriate theory for dealing with uncertainty in the theory of Fuzzy Sets developed by Zadeh [1] in 1965. A Fuzzy Set is defined by its membership function, whose values are defined on the interval [0,1].

Pawlak introduced rough set theory [3] in 1982, which is another significant approach to modeling vagueness. This theory has been successfully applied to many fields such as machine learning, data mining, data analysis, medicine etc.


Rosenfeld [14] proposed the concept of Fuzzy group in order to establish the algebraic structure of Fuzzy Sets. Rough groups were defined by Biswas et al. [15] and have studied the algebraic properties of rough sets. Before going to the main objectives of our study we need the following notations for better understanding the topic

1.1 Important Notations
Let us recall the notation of Fuzzy Sets as follows :

For every set $A \subset X$, define its indicator function $\mu_A$

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \not\in A \end{cases}$$

A Fuzzy Set $F$ is described by its membership function $\mu_F$. For every $x \in X$, this function associates a real number $\mu_F(x)$ in the interval [0,1]. The number $\mu_F(x)$ is interpreted for the point as a degree of belonging $x$ to the Fuzzy Set $F$.

Let $F$ and $G$ be two Fuzzy Set and $\mu_F, \mu_G$ are their membership functions. The complement $CF$ is defined by its membership functions as $\mu_{CF}(x) = 1 - \mu_F(x)$.

The union $F \cup G$ can be defined by any one of the following membership functions.

$$\mu_{F \cup G}(x) = \max \{ \mu_F(x), \mu_G(x) \}$$

The intersection $F \cap G$ can be defined by any one of the following membership function.

$$\mu_{F \cap G}(x) = \min \{ \mu_F(x), \mu_G(x) \}$$

Though the Fuzzy Set is progressing rapidly, but there exists a difficulty; how to set the membership function in each particular case.

To avoid such difficulties, Molodtsov [2] use an adequate parameterization. $U$ be an initial universal set and $E$ be a set of parameters. The pair $(F, E)$ is called a Soft Set (over $U$) if and only if $F$ is a mapping of $E$ into the set of all subsets of $U$. Hence the Soft Set is a parameterized family of subsets of the set $U$. Every set $F(e), e \in E$, from this family may be considered as the set of - element of the Soft Set $(F, E)$.

2. APPLICATION OF SOFT SET THEORY
Molodtsov [2] presented some applications of the soft set theory in several directions viz. study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. In this section, we present an application of soft set theory in a decision making problem with the help of rough approach [3]. Let us now formulate our problem as follows:

**Problem:** Let $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ be the set of seven cars and $E = \{\text{expensive, fuel efficiency, spacious, maintenance free, ecofriendly, high security measure}\}$ be set of parameters . Let $(F, P)$ be a soft set representing the “suitable cars” given by $(F, P) = \{\text{expensive cars} = \{c_1, c_2, c_3, c_4\}, \text{fuel efficiency} = \{c_1, c_3, c_5, c_6\}, \text{spacious} = \{c_4, c_5, c_6, c_7\}, \text{maintenance free} = \{c_2, c_6, c_7\}, \text{ecofriendly} = \{c_1, c_2, c_3, c_4, c_5, c_6\}, \text{high security measure} = \{c_6, c_7\}\}$.

Suppose that Mr X wants to buy a car consisting the parameter fuel efficiency, spacious, ecofriendly, high security measure which forms the subset...
P={fuel efficiency, spacious, ecofriendly, high security measure} of the set E.

The problem is to select the car which is suitable with the choice parameters set by Mr X.

Solution:
Let us first make a tabular representation of the problem:

Consider the soft set (F, P) where P is the choice parameter of Mr X.

Suppose \( h_i = 1 \) if \( h_i \in F(\notin) \)
\( = 0 \) if \( h_i \notin F(\notin) \)

<table>
<thead>
<tr>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
<th>( e_4 )</th>
<th>choice value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>( C_4 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>( C_5 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( C_7 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Here \( p_4=p_7 \)

DECISION: Mr X can buy the car \( C_4 \)

Weighted table of a soft set
Lin [16] introduced the theory of “W-soft set” which is weighted soft set. The weighted choice value of an object \( c_i \) \( \in U \) is \( Wp_i \) where

\[ Wp_i = \sum d_{ij} \] such that \( d_{ij} = w_j \times e_{ij} \)

The revised Algorithm for selection of car ---

1. Input the softset (F,E).
2. Input the set P of choice parameter for Mr X which is a subset of E
3. Find all reduced soft set of (F,P)
4. Choose one reduced-softset say(F,Q) of (F,P)
5. Find weighted table of the softset (F,Q) according to the weights decided by Mr X.
6. Find K for which \( Wp_k = \max Wp_i \)

Then \( c_k \) is the optional choice object. If \( K \) has more than one value, then any one of them could be chosen by Mr X by using his option.

Let us try to solve the original problem by using revised algorithm.
Suppose Mr X sets the weights for the parameter as follows
fuel efficiency———-\( w_1=0.9 \)
spacious———————-\( w_2=0.7 \)
ecofriendly———-\( w_3=0.6 \)
high security measure———\( w_4=0.5 \)

Using these weights the reduce-softset can be tabulated as —
DECISION : Mr X can buy the car c4

Mr X is facing a problem for choosing the suitable course for his son among the available courses medical, engineering, computer application, pure science and humanities which are denoted by m, e, c, p and h respectively. He seeks advice from four counseling agencies a1, a2, a3, a4.

The four agencies provided the information about the courses considering the parameters availability of seat, future prospects, affordability, student-friendly curriculum, job security and eligibility of the student which are denoted by s1, s2, s3, s4, s5 and s6 respectively.

U={m, e, c, p, h} be the set of courses for study and S={s1, s2, s3, s4, s5, s6} be the set of parameters.

The information provided by the counseling agencies forms the fuzzy soft sets (F1,S),(F2,S),(F3,S) and (F4,S) over U where F1,F2,F3,F4 are mappings from S into I_U where I_U is the set of all fuzzy subsets of U given by the counseling agencies.

Here max Wp=Wp4

**ALGORITHM**

We shall follow the following algorithm for the solution of the problem discussed above.

1. Input the performance evaluation of courses by different counseling agencies as matrix.
2. Find the average of the corresponding entries of all the matrices in step 1.
3. Multiply the weightage of different courses of the guardian to the corresponding entries of each row to get the comprehensive decision matrix.
4. Formulate the comparison table.
5. Find the row-sums and column-sums of the comparison table.
6. Obtain the score for each product and the product of with maximum score is recommended as the best choice.

Now the matrix representation of above four fuzzy soft set (F1,S),(F2,S),(F3,S) and (F4,S) are as follows

<table>
<thead>
<tr>
<th></th>
<th>w1 \times x</th>
<th>w2 \times x</th>
<th>w3 \times x</th>
<th>w4 \times x</th>
<th>Choice value</th>
<th>Wp_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>0.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>c2</td>
<td>0.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>c3</td>
<td>0.9</td>
<td>0</td>
<td>0.5</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c4</td>
<td>0.9</td>
<td>0.7</td>
<td>0.5</td>
<td>2.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c5</td>
<td>0</td>
<td>0.7</td>
<td>0</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c6</td>
<td>0</td>
<td>0.7</td>
<td>0.5</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c7</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F_1(s_1) = m \cdot \frac{0.55}{0.63} \cdot e \cdot \frac{0.54}{0.75} \cdot c \cdot \frac{0.75}{0.91} \cdot p \cdot \frac{0.91}{0.50} \cdot h
F_1(s_2) = m \cdot \frac{0.9}{0.86} \cdot e \cdot \frac{0.70}{0.50} \cdot c \cdot \frac{0.86}{0.70} \cdot p \cdot \frac{0.70}{0.50} \cdot h
F_1(s_3) = m \cdot \frac{0.29}{0.33} \cdot e \cdot \frac{0.48}{0.52} \cdot c \cdot \frac{0.29}{0.33} \cdot p \cdot \frac{0.52}{0.85} \cdot h
F_1(s_4) = m \cdot \frac{0.29}{0.33} \cdot e \cdot \frac{0.48}{0.52} \cdot c \cdot \frac{0.29}{0.33} \cdot p \cdot \frac{0.52}{0.85} \cdot h

F_2(s_1) = m \cdot \frac{0.58}{0.67} \cdot e \cdot \frac{0.66}{0.75} \cdot c \cdot \frac{0.75}{0.86} \cdot p \cdot \frac{0.86}{0.91} \cdot h
F_2(s_2) = m \cdot \frac{0.89}{0.97} \cdot e \cdot \frac{0.60}{0.45} \cdot c \cdot \frac{0.89}{0.97} \cdot p \cdot \frac{0.60}{0.45} \cdot h
F_2(s_3) = m \cdot \frac{0.89}{0.97} \cdot e \cdot \frac{0.60}{0.45} \cdot c \cdot \frac{0.89}{0.97} \cdot p \cdot \frac{0.60}{0.45} \cdot h
F_2(s_4) = m \cdot \frac{0.89}{0.97} \cdot e \cdot \frac{0.60}{0.45} \cdot c \cdot \frac{0.89}{0.97} \cdot p \cdot \frac{0.60}{0.45} \cdot h

F_3(s_1) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
F_3(s_2) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
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F_4(s_1) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
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<tr>
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<td></td>
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</tr>
<tr>
<td>c7</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F_1(s_4) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
F_2(s_4) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
F_3(s_4) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h
F_4(s_4) = m \cdot \frac{0.5}{0.6} \cdot e \cdot \frac{0.5}{0.6} \cdot c \cdot \frac{0.5}{0.6} \cdot p \cdot \frac{0.5}{0.6} \cdot h

(F_4,S)=

<table>
<thead>
<tr>
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<td>0</td>
<td>0.9</td>
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<tr>
<td>c2</td>
<td>0.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.5</td>
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<tr>
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<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values in the table are normalized.
Taking the average of the above four fuzzy soft sets we get the performance evaluation matrix as

$$A = \begin{bmatrix}
0.54 & 0.62 & 0.55 & 0.82 & 0.92 \\
0.84 & 0.80 & 0.66 & 0.45 & 0.33 \\
0.16 & 0.25 & 0.39 & 0.64 & 0.87 \\
0.29 & 0.36 & 0.46 & 0.47 & 0.78 \\
0.87 & 0.85 & 0.74 & 0.63 & 0.24 \\
0.13 & 0.23 & 0.33 & 0.53 & 0.82 \\
\end{bmatrix}$$

Suppose that Mr X sets the preference weightage for the different courses as

$$W = \begin{bmatrix}
m & e & c & p & h \\
0.3 & 0.275 & 0.1 & 0.025 \\
\end{bmatrix}$$

such that $\sum w = 1$

The comprehensive decision matrix $D$ can be obtained by multiplying $A$ row by row by the weightage matrix $W$ and the transposing it as follow

$$D = \begin{bmatrix}
0.54 & 0.84 & 0.16 & 0.29 & 0.87 & 0.13 \\
0.62 & 0.80 & 0.25 & 0.36 & 0.85 & 0.23 \\
0.55 & 0.66 & 0.39 & 0.46 & 0.74 & 0.33 \\
0.82 & 0.45 & 0.64 & 0.47 & 0.63 & 0.53 \\
0.92 & 0.33 & 0.87 & 0.78 & 0.24 & 0.82 \\
\end{bmatrix}$$

The comparison table of the above comprehensive decision matrix is

<table>
<thead>
<tr>
<th></th>
<th>m</th>
<th>e</th>
<th>c</th>
<th>p</th>
<th>h</th>
<th>Row-sum</th>
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<td>e</td>
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<td>6</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>p</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>h</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>Column-sum</td>
<td>15</td>
<td>11</td>
<td>12</td>
<td>22</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

The row-sum and the column-sum from the comprehensive decision matrix and the score for each $c_i$ are as below

$$\begin{bmatrix}
\text{Row-sum} & \text{Column-sum} & \text{Score} \\
\hline
m & 21 & 15 & 6 \\
 e & 25 & 11 & 14 \\
c & 24 & 12 & 12 \\
p & 14 & 22 & -8 \\
h & 6 & 30 & -24 \\
\end{bmatrix}$$
Here we find that the maximum score is obtain by e, that is by the course ENGINEERING.
Hence Mr X can choose the course ENGINEERING for his son.

3. CONCLUSION
Thus we see that Fuzzy soft set theory is very much interesting and useful for solving the day to day problems. It helps to take decision making in a critical situation.

4. REFERENCE