Asymptotic Stability of Stochastic Impulsive Neutral Partial Functional Differential Equations

R. Maheswari
Department of Mathematics
Sri Eshwar College of Engineering,
Coimbatore – 642 109, Tamilnadu, India

S. Karunanithi
Department of Mathematics,
Kongunadu Arts and Science College(Autonomous),
Coimbatore – 641 029, Tamilnadu, India

ABSTRACT
In this paper the authors study the existence and asymptotic stability in p-th moment of mild solutions to stochastic neutral partial differential equation with impulses. Their method for investigating the stability of solutions is based on the fixed point theorem.

Keywords
Asymptotic stability, mild solution, stochastic, neutral, impulse

1. INTRODUCTION
The study of existence, uniqueness and stability of mild solutions of stochastic partial functional differential equations due to their range of applications in various sciences such as physics, mechanical, engineering, control theory and economics and many significant results have been obtained [1-4]. However many dynamical system not only depend on present and past states but also involve derivatives with delays. Neutral functional differential equations are often used to describe such systems. In addition, impulsive phenomena can be found in a wide variety of evolution processes, for example medicine and biology, economics, mechanics, electronics and telecommunication etc., in which sudden and abrupt changes occur instantaneously, in the form of impulses. Many interesting results have been obtained, for example [5-7]. There are only few works on the stability of mild solutions to stochastic neutral partial functional differential equations [8, 9]. In [10] authors studied the asymptotic stability of impulsive stochastic neutral partial differential equations with infinite delays and in [11] Global attracting set and exponential stability of stochastic neutral partial functional differential equations with impulses. Motivated by the above papers, the authors study the existence and asymptotic stability of impulsive stochastic neutral partial differential equations.

2. PRELIMINARIES
Let X and Y be two real separable Hilbert spaces and $L(Y,X)$ be the space of bounded linear operators from E intoX, equipped with the usual operator norm $\|\cdot\|$. Let (Ω,Γ,P) be a complete probability space furnished with a normal filtration $\{\Gamma_t\}_{t\geq 0}$ generated by the Q -Wiener process w on (Ω,Γ,P) with the linear bounded covariance operator Q such that $\mathrm{tr}Q<\infty$. In order to define stochastic integrals with respect to the Q -Wiener process $w(t)$, we introduce the subspace $Y_0 = Q^{1/2}Y$ of Y which, endowed with the inner product $\langle u,v\rangle_{E_0} = \langle Q^{-1/2}u, Q^{-1/2}v\rangle_E$, is a Hilbert space. We assume that there exist a complete orthonormal system $\{e_i\}_{i\geq 1}$in E, a bounded sequence of nonnegative real numbers $\{\lambda_i\}$ such that $Qe_i = \lambda_i e_i$, $i = 1,2,3,\ldots$, and a sequence $\{\beta_i\}$ of independent Brownian motions such that

$$\langle w(t), e\rangle = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\,\langle e_i, e\rangle \beta_i(t)\ ,\ e\in Y$$

and $\Gamma_t = \Gamma_t^w$,where $\Gamma_t^w$is the sigma algebra generated by$\{w(s); t\geq 0\}$. Let $L_2^0 = L_2(Q^{1/2}Y;X)$ denote the space of all Hilbert –Schmidt operators from $Q^{1/2}Y$ to X with the inner product $\langle\psi,\phi\rangle_{L_2^0} = \mathrm{tr}(\psi Q\phi *)$. Let $L^p(\Omega,\Gamma_t,X)$ is the Hilbert space of all Γ_t-measurable square integrable random variables with values in a Hilbert space X.

Consider the following stochastic neutral partial differential equation with impulses of the form

$$d[x(t) + u(t,x_t)] = [Ax(t) + f(t,x_t)]dt + g(t,x_t)dw(t)\ ,\ t\geq 0, t\neq t_k,$$

$$x_0(s) = \phi(s) \in C^b_{\mathcal{F}_0}([-\tau,0];X)$$

$$\Delta x(t_k) = x(t_k^{+}) - x(t_k^{-}) = I_k(x(t_k^{-}))\ , \qquad (1)$$

where$A: D(A)\subset X\to X$ is the infinitesimal generator of an analytic semigroup of linear operator $(S(t))_{t\geq 0}$ on a Hilbert space X; $f,u: R^+\times X\to X$ and $g: R^+\times X\to L(Y,X)$ are all Borel measurable.$I_k: X\to X$ and $0<t_1<\cdots<t_k<\lim_{k\to\infty} t_k = \infty$; $x(t_k^{+})$and $x(t_k^{-})$ represent the right and left limits of $x(t)$ at $t = t_k, k = 1,2,\ldots$, respectively. Here I_k represents the size of the jump. Let $\tau>0$ and $C([-\tau,0];X)$ denote the family of all continuous X -valued random functions ϕ from $[-\tau,0]$ to X with the norm $\|\phi\|_C = \sup_{t\in[-\tau,0]} E\|\phi(t)\|_X$. Let $C^b_{F_0}([-\tau,0];X)$ be the family of all almost surely bounded, Γ_0-measurable, $C([-\tau,0];X)$ - valued random variables.

Definition 1. A stochastic process $\{\mathbf{x(t)}, \mathbf{t}\in[\mathbf{0},\mathbf{T}]\}$, $\mathbf{0}\leq\mathbf{T}<\infty$, is called a mild solution of the system (1), if

(i) $x(t)$ is adapted to $\Gamma_t, t\geq 0$.

(ii) $x(t)\in X$ had càdlàg paths on $t\in[0,T]$ almost surely, and for each $t\in[0,T]$, $x(t)$ satisfies the integral equation

$$\begin{aligned} x(t) = {} & S(t)[\phi(0) + u(0,\phi)] - u(t,x_t) \\ & - \int_0^t AS(t-s)\,u(s,x_s)ds \\ & + \int_0^t S(t-s)\,f(s,x_s)ds + \int_0^t S(t-s)\,g(s,x_s)dw(s) \\ & + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^{-})) \end{aligned} \qquad (2)$$

For any $x_0(\cdot) = \phi\in C^b_{\mathcal{F}_0}([-\tau,0];X)$. To prove the stability of mild solution of (1) the following assumptions are impose

(H1) A is the infinitesimal generator of a semigroup of bounded linear operators $\{S(t); t\geq 0\}$, in X such that $0\in\rho(A)$, the resolvent set of $-A$, and

$\|S(t)\| \leq Me^{-at}, t \geq 0$ for some constants $M \geq 1$ and $0 < a \in R^+$.

(H2) There exist a constant K for any $x, y \in X$ and $t \geq 0$ such that

$\|f(t,x) - f(t,y)\| \leq K\|x - y\|$,

$\|g(t,x) - g(t,y)\| \leq K\|x - y\|$.

(H3) There exists an $\alpha \in (0, 1]$ and $K_1 > 0$ such that for any $x, y \in X$ and $t \geq 0$ $u(t,x) \in D((-A)^\alpha)$ and

$\|(-A)^\alpha u(t,x) - (-A)^\alpha u(t,y)\| \leq K_1\|x - y\|$.

(H4)There exists a constant q_k such that

$\|I_k(x) - I_k(y)\| \leq q_k\|x - y\|$ for each $x, y \in X$, $k = (1,2,3, \dots,)$. Moreover it is assumed that

$u(t, 0) = f(t, 0) = g(t, 0) = I_k(t, 0) = 0$.

LEMMA 1. [12] For any $\mathbf{r} \geq \mathbf{1}$ and for arbitrary $\mathbf{L_2^0}$- valued predictable process $\boldsymbol{\phi}(\cdot)$ such that

$$\sup_{s\in[0,t]} E\left\|\int_0^s \phi(u)dw(u)\right\|_X^{2r} \leq \left(r(2r-1)\right)^r \left(\int_0^t \left(E\|\phi(s)\|_{L_2^0}^{2r}\right)^{1/r} ds\right)^r.$$

LEMMA 2. [13] If $(\mathbf{H1})$ holds then for any $\boldsymbol{\beta} \in (\mathbf{0}, \mathbf{1}]$

(i) For each $x \in D\left((-A)^\beta\right)$, $S(t)(-A)^\beta x = (-A)^\beta S(t)x$

(ii) $\left\|(-A)^\beta S(t)\right\| \leq M_\beta t^{-\beta} e^{-at}$, $t > 0$

3. ASYMPTOTIC STABILITY OF THE MILD SOLUTION

In this section the asymptotic stability in p-th moment of mild solutions of (1) by using the contraction mapping principle is considered. Let H be the space of all $\mathcal{F}_0$- adapted process $\psi(t,\widehat{w}): [-\tau, \infty) \times \Omega \to R$ which is almost surely continuous in t for fixed $\widehat{w} \in \Omega$. Moreover $\psi(s,\widehat{w}) = \phi(s)$ for each $s \in [-\tau, 0]$ and $E\|\psi(t.\widehat{w})\|^p \to 0$ as $t \to \infty$.

THEOREM 1. If $(\mathbf{H1})$-$(\mathbf{H4})$ hold for some $\boldsymbol{\alpha} \in (\mathbf{1}/\mathbf{p}, \mathbf{1}]$, $\mathbf{p} \geq \mathbf{2}$, and the inequality

$$5^{p-1}[K_1^p\|(-A)^{-\alpha}\|^p + K_1^p a^{-p\alpha}\left(\Gamma\left(1 + \frac{p(\alpha-1)}{p-1}\right)\right)^{p-1} M_{1-\alpha}^p + M^p K^p a^{-p} + M^p C_p K^p a^{-1} + M^p \hat{L}] < 1$$

is satisfied, then the mild solution of (1) is asymptotically stable in pth moment.Where $C_p = \left(\frac{2a(p-1)}{p-2}\right)^{1-\frac{p}{2}}$

PROOF. Define an operator $\pi: H \to H$ by

$$\pi(x)(t) = S(t)[\phi(0) + u(0,\phi)] - u(t,x_t) - \int_0^t AS(t-s)\, u(s,x_s)ds + \int_0^t S(t-s)\, f(s,x_s)ds + \int_0^t S(t-s)\, g(s,x_s)dw(s) + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)) = \sum_{i=1}^6 I_i(t). \quad (3)$$

In order to prove the asymptotic stability, it is enough to prove that the operator π has a fixed point in H.To prove this result, the contraction mapping principle is used. To apply the contraction mapping principle, first the mean square continuity of π on $[0, \infty)$ is verified.

Let $x \in H$, $t_1 \geq 0$ and $|r|$ is sufficiently small then

$$E\|\pi(x)(t_1 + r) - \pi(x)(t_1)\|_X^p \leq 6^{p-1}\sum_{i=1}^6 E\|I_i(t_1 + r) - I_i(t_1)\|_X^p$$

It is seen that $E\|I_i(t_1 + r) - I_i(t_1)\|_X^p \to 0$, $i = 1,2,3,4,6$ as $r \to 0$. Moreover by using Holder's inequality and lemma 1,

$$E\|I_5(t_1 + r) - I_5(t_1)\|_X^p \leq 2^{p-1}C_p\left[\int_0^{t_1}\left(E\left\|\left(S(t_1 + r - s) - S(t_1 - s)\right)g(s,x_s)\right\|^p\right)^{(2/p)} ds\right]^{(p/2)}$$

$$+2^{p-1}C_p\left[\int_{t_1}^{t_1+r}(E\|S(t_1 + r - s)g(s,x_s)\|^p)^{(2/p)} ds\right]^{(p/2)}$$

$\to 0$ as $r \to 0$.

Thus, π is continuous in p-th moment on$[0, \infty)$.

Next, it is shown that $\pi(H) \subset H$. From (3)

$$E\|(\pi x)(t)\|_X^p \leq 6^{p-1}E\|S(t)[\phi(0) + u(0,\phi)]\|_X^p - 6^{p-1}E\|u(t,x_t)\|_X^p - 6^{p-1}E\left\|\int_0^t AS(t-s)\, u(s,x_s)ds\right\|_X^p + 6^{p-1}E\left\|\int_0^t S(t-s)\, f(s,x_s)ds\right\|_X^p + 6^{p-1}E\left\|\int_0^t S(t-s)\, g(s,x_s)dw(s)\right\|_X^p + 6^{p-1}\sum_{0<t_k<t} E\left\|S(t-t_k)I_k\left(x(t_k^-)\right)\right\|_X^p \quad (4)$$

Now, the terms on the right hand side of (3) are estimated using(H1), (H3), (H4) and lemma 2. Then

$$6^{p-1}E\|S(t)[\phi(0) + u(0,\phi)]\|^p \leq 6^{p-1}M^p e^{-apt}(1 - K_1\|(-A)^{-\alpha}\|)^p\|\phi\|^p \to 0 \text{ as } t \to \infty, \quad (5)$$

$$6^{p-1}E\|u(t,x_t)\|^p \leq 6^{p-1}{K_1}^p\|(-A)^{-\alpha}\|^p E\|x_t\|^p \to 0 \text{ as } t \to \infty, \quad (6)$$

$$6^{p-1}\sum_{0<t_k<t} E\|S(t-t_k)I_k(x(t_k^-))\|^p \leq 6^{p-1}M^p e^{-ap(t-t_k)}q_k^p E\|x(t_k)\|^p \to 0 \text{ as } t \to \infty, \quad (7)$$

By lemma 2, (H3) and Holder's inequality, for H3 ,

$1/p + 1/q = 1$, $1 < q \leq 2$

$$6^{p-1}E\left\|\int_0^t -AS(t-s)\, u(s,x_s)ds\right\|^p \leq 6^{p-1}E\left\|\int_0^t(-A)^{1-\alpha}S(t-s)\, (-A)^\alpha u(s,x_s)ds\right\|^p$$

$$\leq 6^{p-1}M_{1-\alpha}^p {K_1}^p\left(\int_0^t e^{-a(t-s)}(t-s)^{q\alpha-q}ds\right)^{p/q}$$

$$\times \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

$$\leq 6^{p-1} M_{1-\alpha}^p K_1{}^p a^{p(1-\alpha)-\frac{p}{q}} \left(\Gamma(1+q\alpha-q)\right)^{p/q}$$

$$\times \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

For any $x(t) \in H$ and $t > 0$, there exists $t_1 > 0$ such that $E\|x(t)\|^p < t$ for $t \geq t_1$. Hence

$$6^{p-1} E \left\| \int_0^t -AS(t-s)\, u(s, x_s) ds \right\|^p$$

$$\leq 6^{p-1} M_{1-\alpha}^p K_1{}^p a^{p-p\alpha-\frac{p}{q}} \left(\Gamma(1+q\alpha-q)\right)^{p/q}$$

$$\times \int_0^{t_1} e^{-a(t-s)} E\|x_s\|^p ds$$

$$+6^{p-1} M_{1-\alpha}^p K_1{}^p a^{-p\alpha} \left(\Gamma(1+q\alpha-q)\right)^{p/q} \varepsilon \qquad (8)$$

It is seen that $e^{-at} \to 0$ as $t \to \infty$. By condition of theorem 1, there exists $t_2 \geq t_1$ such that for $t \geq t_2$

$$6^{p-1} M_{1-\alpha}^p K_1{}^p a^{p-p\alpha-\frac{p}{q}} \left(\Gamma(1+q\alpha-q)\right)^{p/q}$$

$$\times \int_0^{t_1} e^{-a(t-s)} E\|x_s\|^p ds$$

$$\leq \varepsilon - 6^{p-1} M_{1-\alpha}^p K_1{}^p a^{-p\alpha} \left(\Gamma(1+q\alpha-q)\right)^{p/q} \varepsilon \qquad (9)$$

From (8) and (9) for any $t \geq t_2$,

$$6^{p-1} E \left\| \int_0^t -AS(t-s)\, u(s, x_s) ds \right\|^p \leq \varepsilon$$

That is $6^{p-1} E \left\| \int_0^t -AS(t-s)\, u(s, x_s) ds \right\| \to 0$ as $t \to \infty$.

(10)

Now by (H1),(H2), the Holder's inequality, and lemma 1,

$$6^{p-1} E \left\| \int_0^t S(t-s)\, f(s, x_s) ds \right\|^p$$

$$\leq 6^{p-1} M^p E \left\| \int_0^t e^{-a(t-s)} K x_s ds \right\|^p$$

$$\leq 6^{p-1} M^p K^p \left(\int_0^t e^{-a(t-s)} ds \right)^{p-1} \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

$$\leq 6^{p-1} M^p K^p a^{(1-p)} \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

For any $x(t) \in H$ and $t > 0$, there exists $t_1 > 0$ such that $E\|x_s\|^p < \varepsilon$ for $t \geq t_1$, then

$$6^{p-1} E \left\| \int_0^t S(t-s)\, f(s, x_s) ds \right\|^p$$

$$\leq 6^{p-1} M^p K^p a^{(1-p)} \int_0^{t_1} e^{-a(t-s)} E\|x_s\|^p ds$$

$$+6^{p-1} M^p K^p a^{-p} \varepsilon$$

It is seen that $e^{-at} \to 0$ as $t \to \infty$. By condition of theorem 1, there exists $t_2 \geq t_1$ such that for $t \geq t_2$

$$6^{p-1} M^p K^p a^{(1-p)} \int_0^{t_1} e^{-a(t-s)} E\|x_s\|^p ds$$

$$\leq \varepsilon - 6^{p-1} M^p K^p a^{-p} \varepsilon$$

For any $t \geq t_2$,

$$6^{p-1} E \left\| \int_0^t S(t-s)\, f(s, x_s) ds \right\|^p < \varepsilon$$

ie, $6^{p-1} E \left\| \int_0^t S(t-s)\, f(s, x_s) ds \right\|^p \to 0$ as $t \to \infty$ (11)

and

$$6^{p-1} E \left\| \int_0^t S(t-s)\, g(s, x_s) dw(s) \right\|^p$$

$$\leq 6^{p-1} M^p K^p \left(\frac{p(p-1)}{2} \right)^{p/2} \left(\int_0^t e^{-ap(t-s)} \left(E\|g(s, x_s)\| \right)^{2/p} ds \right)^{\frac{p}{2}}$$

$$\leq 6^{p-1} M^p C_p K^p \left(\int_0^t e^{-\left(\frac{2(p-1)}{p-2}\right) a(t-s)} ds \right)^{\frac{p}{2}-1}$$

$$\times \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

$$\leq 6^{p-1} M^p C_p K^p \left(\frac{2a(p-1)}{p-2} \right) \int_0^t e^{-a(t-s)} E\|x_s\|^p ds$$

Similarly $6^{p-1} E \left\| \int_0^t S(t-s)\, g(s, x_s) dw(s) \right\|^p \to 0$ as $t \to \infty$

(12)

Thus from (5)-(7) and (10)-(12),

$E\|(\pi x)(t)\|_X^p \to 0$ as $t \to \infty$. Hence $\pi(H) \subset H$.

Next it is proved that π is a contraction mapping. To see this, Let $x, y \in H$,

$$\sup_{s \in [0,t]} E\|(\pi x)(t) - (\pi y)(t)\|_X^p$$

$$\leq 5^{p-1} \sup_{s\in[0,t]} E\|u(t, x_t) - u(t, y_t)\|^p$$

$$+5^{p-1} \sup_{s\in[0,t]} E \left\| \int_0^t -AS(t-s) \left(u(s, x_s) - u(s, y_s) \right) ds \right\|^p$$

$$+ 5^{p-1} \sup_{s\in[0,t]} E \left\| \int_0^t S(t-s) \left(f(s, x_s) - f(s, y_s) \right) ds \right\|^p$$

$$+5^{p-1} \sup_{s\in[0,t]} E \left\| \int_0^t S(t-s) \left(g(s, x_s) - g(s, y_s) \right) dw(s) \right\|^p$$

$$+5^{p-1} \sup_{s\in[0,t]} E \left\| \sum_{0<t_k<t} S(t-t_k) \left(I_k(x(t_k^-)) - I_k(y(t_k^-)) \right) \right\|^p$$

$$\leq 5^{p-1} \left(K_1^p + K_1^p a^{-p\alpha} \left(\Gamma\left(1 + \frac{p(\alpha-1)}{p-1}\right) \right)^{p-1} M_{1-\alpha}^p + M^p K^p a^{-p} + M^p C_p K^p a^{-1} + M^p \hat{L}] \right)$$

$$\times \sup_{s \in [0,t]} E\|x(t) - y(t)\|_X^p$$

where $\hat{L} = e^{-apT} E \left(\sum_{k=1}^{m} \|q_k\|^p \right)$

Thus, π is a contraction mapping and hence there exist a unique fixed point $x(t)$ in H which is the solution of the equation (1) with $E\|x(t)\|_X^p \to 0$ as $t \to \infty$. This completes the proof.

COROLLARY 1. If hypothesis (H1)– (H4) hold for some $\alpha \in \left(\frac{1}{2}, 1\right]$, then the impulsive stochastic system (1) is mean square asymptotically stable if

$$\left[5(K_1^2\|(-A)^{-\alpha}\|^2 + K_1^2 a^{-2\alpha}\left(\Gamma(2\alpha-1)\right)M_{1-\alpha}^2 + M^2K^2a^{-2} + M^2K^2a^{-1} + M^2e^{-2aT}E\left(\sum_{k=1}^{m}\|q_k\|^2\right)\right] < 1$$

4. EXAMPLE

The following neutral stochastic partial functional differential equation with impulses is considered.

$$d[x(t) + a_1x_t] = \left[\frac{\partial^2}{\partial z^2}x(t) + a_2x_t\right]dt + a_3x_t dw(t);$$
$$0 \le z \le \pi, t \ge 0, t \ne t_k,$$

$$\Delta x(t_k) = I_k(x(t_k^-) = \frac{b_1}{k^2}\,x(t_k^-), t = t_k, k = 1,2,3,\ldots,$$
$$x_0(s) = \varphi(s) \in C_{\mathcal{F}_0}^b([-\tau, 0], L^2[0,\pi]),$$
$$x(t,0) = x(t,\pi) = 0, \quad -\tau \le s \le 0$$

where $a_i > 0, i = 1,2,3, b_1 \le 0$ are constants and $w(t)$denotes the standard one dimensional Brownian motion.

Let $H = L^2[0,\pi], H_1 = W^{2,2}[0,\pi] \cap W_0^{1,2}[0,\pi]$

Define bounded linear operator $A\colon H_1 \to H$ by

$$A_x = \frac{\partial^2 x}{\partial z^2} \in H; \text{for all } x \in H$$

It is well known that $\|S(t)\| \le e^{-\pi^2 t}$ for each $t \ge 0$.

$$\|(-A)^{-\alpha}\| \le \left(\Gamma(\alpha)\right)^{-1}\int_0^\infty t^{\alpha-1}\|S(t)\|dt \le \pi^{-2\alpha}.$$

Obviously, (H1) – (H4) hold. By corollary 1, we obtain that if

$$a_1^2\pi^{-4\alpha} + a_1^2\pi^{-2\alpha}\left(\Gamma(2\alpha-1)\right) + a_2^2\pi^{-2} + a_3^2\pi^{-1} + e^{-2\pi T}E\left(\sum_{k=1}^{m}\|b_k\|^2\right) < \frac{1}{5}$$

then the mild solution is mean square asymptotically stable.

5. CONCLUSION

This paper investigates the asymptotic stability in p-th moment of mild solutions to stochastic impulsive neutral partial functional differential equations by fixed point theory. In addition an example is provided to illustrate the theory, in which some earlier results are generalized and improved.

6. REFERENCES

[1] Caraballo, T., Liu, K., 1999. Exponential stability of mild solutions of stochastic partial differential equations with delays. Stoch. Anal. Appl. 17, 743-763.

[2] Luo, J. W., 2008b. Stability of stochastic partial differential equations with infinite delays. J. Comput. Appl. Math. 222, 364-371.

[3] Taniguchi, T., Liu, K., Truman, A., 2002. Existence, uniqueness and asymptotic behavior of mild solutions to stochastic functional differential equations in Hilbert spaces, J. differential equations. 181, 72-91.

[4] Wan, L., Duan, J., 2008. Exponential stability of non-autonomous stochastic partial differential equations with finite memory. Statist. and Probab. Lett. 78, 490-498.

[5] Anguraj, A., Vinodkumar, A., 2010a. Existence, uniqueness and stability of impulsive stochastic partial neutral functional differential equations with infinite delays. J. Appl. Math. Inform. 28 (3-4), 739-751.

[6] Chen, H. B., 2010. Impulsive-integral inequality and exponential stability for stochastic partial differential equations with delays. Statist. Probab. Lett. 80, 50-56.

[7] Sakthivel, R., Luo, J., 2009a. Asymptotic stability of impulsive stochastic partial differential equations with infinite delays. J. Math. Anal. Appl. 356, 1-6.

[8] Cui, J., Yan, L.T., Sun, X.C., 2011. Exponential stability for neutral stochastic partial differential equations with delays and Poisson jumps. Statist. Probab. Lett. 81, 1970-1977.

[9] Govindan, T. E., 2005. Almost sure exponential stability for stochastic neutral partial functional differential equations. Stochastics .77, 139-154.

[10] Feng Jiang and Yi Shen. 2012. Stability of impulsive stochastic neutral partial differential equations with infinite delays. Asian J Control. vol. 14, No. 6, 1706-1709.

[11] Runping Ye., 2012. Global attracting set and stability of stochastic neutral partial functional differential equations with impulses. Statist. and Probab. Lett. 82, 1699-1709.

[12] Da prato, G., and J. Zabczyk, Stochastic equations in infinite dimensions. Cambridge University Press, Cambridge 1992.

[13] Pazy, A., 1983, Semigroups of Linear Operators and Applications to Partial Differential Equations. Springer-Verlag, New York.