Ranking of Hexagonal Fuzzy Numbers for Solving Multi Objective Fuzzy Linear Programming Problem

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ABSTRACT
In this paper a ranking Procedure based on Hexagonal Fuzzy numbers, is applied to a Multi-objective Linear programming problem (MOLPP) with fuzzy coefficients. By this ranking method any Multiobjective Fuzzy Linear Programming problem (MOFLPP) can be converted in to a crisp value problem to get an optimal solution. This method provides an insight for the planner due to uncertain environment in an organizational Economics. In an organization, where a number of alternatives and variables such as production, inventory, financial management, costing and various other parameters are involved, this ranking procedure serves as an efficient method wherein a numerical example is also taken and the inference is given.

Keywords
Ranking, Hexagonal fuzzy numbers, MOFLPP, Decision making.

1. INTRODUCTION
Ranking fuzzy number is used in decision- making process in an economic environment. In an organization various activities such as planning, execution, and other process takes place continuously. This requires careful observation of various parameters which are all in uncertain in nature due the competitive business environment globally. In fuzzy environment ranking fuzzy numbers is a very important decision making procedure. The idea of fuzzy set was first proposed by Bellman and Zadeh [1], as a mean of handling uncertainty that is due to imprecision rather than randomness. The concept of fuzzy linear programming (FLP) was first introduced by Tanaka et al. [11, 12] Zimmerman [17] introduced fuzzy linear programming in fuzzy environment. Multi-objective linear Programming was introduced by Zeleny [16]. Lai Y.J –Hawng C.L [5] considered MOLPP with all parameters having a triangular possibility distribution. They used an auxiliary model and it was solved by MOLPP. Zimmerman [18] applied their approach to vector maximum problem by transforming MOFLP problem to a single objective linear programming problem. Qiu- Peng Gu, and Bing-Yuan Cao [7] solved Fuzzy linear programming problems based on Fuzzy numbers distance. Tong Shaocheng [13] focused on the fuzzy linear programming with interval numbers. Chanas [2] proposed a fuzzy programming in multi objective linear programming. Verdegay [14] have proposed three methods for solving three models of fuzzy integer linear programming based on the representation theorem and on fuzzy number ranking method. In particular, the most convenient methods are based on the concept of comparison of fuzzy numbers by the use of ranking functions. Nasseri [6] has proposed a new method for solving FLP problems in which he has used the fuzzy ranking method for converting the fuzzy objective function into crisp objective function.

2. PRELIMINARIES

2.1Definition:
Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A: X \rightarrow [0,1]$, where A(x) is interpreted as the degree of membership of element x in fuzzy A for each $x \in X$.

2.2 Interval Number:
Let R be the set of real numbers. Then closed interval [a, b] is said to be an interval number, where a, b Є R, a≤ b.

2.3 Distance between interval numbers:
Let a = $[a_1,a_2]$, b= $[b_1,b_2$] be two interval numbers. Then the distance between (a, b) denoted by d (a, b), is defined by

$$d(a,b) = \int_{\frac{-1}{2}}^{\frac{1}{2}} \left\| \left[\frac{a_1 + a_2}{2} + x(a_2 - a_1) \right] - \left[\frac{b_1 + b_2}{2} + x(b_2 - b_1) \right] \right\| dx$$

2.4 Fuzzy number:
A fuzzy set $\widetilde{A}$ of the real line R with membership function $\mu_{\widetilde{A}}(x): R \rightarrow [0,1]$ is called fuzzy number if

i) A must be normal and convex fuzzy set;

ii) the support of $\widetilde{A}$, must be bounded

iii) α_A must be a closed interval for every $\alpha \in [0,1]$

2.5 Support:
The support of a fuzzy set Ã ,S (Ã), is the crisp set of all xЄX such that $\mu_{\widetilde{A}}(X) \succ 0$

2.6 Distance between fuzzy numbers:
Let $\widetilde{A}, \widetilde{B}$ be two fuzzy numbers. Then the distance between the fuzzy numbers $\widetilde{A}$ *and* $\widetilde{B}$ is defined by

$$D(\widetilde{A}, \widetilde{B}) = \int_0^1 d\left(\widetilde{A}_\lambda, \widetilde{B}_\lambda\right) d\lambda$$

3. HEXAGONAL FUZZY NUMBER
A fuzzy number $\widetilde{A}_H$ is a hexagonal fuzzy number denoted by $\widetilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6)$ *where* $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu\widetilde{A}_H(x)$ is given below.

$$\mu_{\tilde{A}_H}(X)=\begin{cases}0 & for\ \chi < a_1\\ \frac{1}{2}\left(\frac{\chi-a_1}{a_2-a_1}\right) & for\ a_1\le\chi\le a_2\\ \frac{1}{2}+\frac{1}{2}\left(\frac{\chi-a_2}{a_3-a_2}\right) & for\ a_2\le\chi\le a_3\\ 1 & for\ a_3\le\chi\le a_4\\ 1-\frac{1}{2}\left(\frac{\chi-a_4}{a_5-a_4}\right) & for\ a_4\le\chi\le a_5\\ \frac{1}{2}\left(\frac{a_6-\chi}{a_6-a_5}\right) & for\ a_5\le\chi\le a_6\\ 0 & for\ \chi > a_6\end{cases}$$

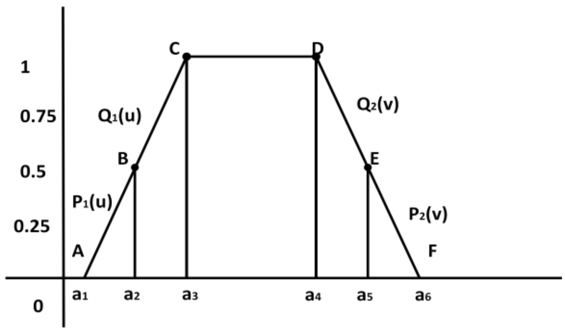

Figure 1 Graphical representation of a hexagonal fuzzy number

3.1 Arithmetic Operations on Hexagonal Fuzzy numbers

Let $\tilde{A}_H=(a_1,a_2,a_3,a_4,a_5,a_6)$ *and* $\tilde{B}_H=(b_1,b_2,b_3,b_4,b_5,b_6)$ be two hexagonal fuzzy numbers, then

(i) $\tilde{A}_H(+)\tilde{B}_H=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5,a_6+b_6)$

(ii) $\tilde{A}_H(-)\tilde{B}_H=(a_1-b_1,a_2-b_2,a_3-b_3,a_4-b_4,a_5-b_5,a_6-b_6)$

(iii) $\tilde{A}_H(*)\tilde{B}_H=(a_1*b_1,a_2*b_2,a_3*b_3,a_4*b_4,a_5*b_5,a_6*b_6)$

3.2. Ranking of Hexagonal Fuzzy Numbers:

A number of approaches have been proposed for the ranking of fuzzy numbers. In this paper for a hexagonal fuzzy number $\tilde{A}_H=(a_1,a_2,a_3,a_4,a_5,a_6)$, a ranking method is devised based on the following formula.

$$R(\tilde{A}_H)=\left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5}{18}\right)-------(1)$$

Let $\tilde{A}_H=(a_1,a_2,a_3,a_4,a_5,a_6)$ *and* $\tilde{B}_H=(b_1,b_2,b_3,b_4,b_5,b_6)$ be two hexagonal fuzzy numbers then

$$\tilde{A}_H\approx\tilde{B}_H\Leftrightarrow R(\tilde{A}_H)=R(\tilde{B}_H)$$

$$\tilde{A}_H\ge\tilde{B}_H\Leftrightarrow R(\tilde{A}_H)\ge R(\tilde{B}_H)$$

$$\tilde{A}_H\le\tilde{B}_H\Leftrightarrow R(\tilde{A}_H)\le R(\tilde{B}_H)$$

4. PROPOSED RANKING ALGORITHM

The above procedure (3.2) can be used to develop ranking for hexagonal fuzzy numbers. Based on this ranking procedure, a ranking algorithm is developed for a hexagonal fuzzy numbers. Moreover, it is applied to MOLPP under constraints with fuzzy coefficients.

4.1 Algorithm:

Step1: Consider the fuzzy numbers $\tilde{A}_H$ & $\tilde{B}_H$ of hexagonal fuzzy number

Step2: Find Supremum M= Sup(**s** $(\tilde{A}_H)$ U s $(\tilde{B}_H)$), where **s** $(\tilde{A}_H)$=Support set of $\tilde{A}_H$ and s $(\tilde{B}_H)$=Support set of $\tilde{B}_H$.

Step3: Take $\tilde{A}_H=(a_1,a_2,a_3,a_4,a_5,a_6)$ *and* $\tilde{B}_H=(b_1,b_2,b_3,b_4,b_5,b_6)$ as hexagonal fuzzy numbers

Step4: Calculate

$$D(\tilde{A}_H,M)=M-\left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\ \&$$

$$D(\tilde{B}_H,M)=M-\left(\frac{2b_1+3b_2+4b_3+4b_4+3b_5+2b_6}{18}\right)$$

Step5: If

$$D(\tilde{A}_H,M)>D(\tilde{B}_H,M)\ then\ \tilde{A}_H>\tilde{B}_H$$

$$D(\tilde{A}_H,M)<D(\tilde{B}_H,M)\ then\ \tilde{A}_H<\tilde{B}_H$$

$$D(\tilde{A}_H,M)=D(\tilde{B}_H,M)\ then\ \tilde{A}_H=\tilde{B}_H$$

Step 6: Stop.

5. METHOD OF SOLVING MULTI-OBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM

In this paper, we discuss a Multi-Objective Fuzzy Linear Programming Problem in constraint conditions with fuzzy coefficients. Moreover, the objectives considered in this paper are mixed with both maximization and minimization types. We discuss a model whose standard form is

Maximize $z_1 = c_1 x$

Minimize $z_2 = c_2 x$

Subject to $\widetilde{A}_H X \le \widetilde{b}, X \ge 0$

Where $c_{ij} = (c_{i1}, c_{i2}, ...c_{in})$ is an n- dimensional crisp row vector, $\widetilde{A}_H = \widetilde{a}_{ij}$ is an m x n fuzzy matrix,

$\widetilde{b} = \left(\widetilde{b}_1, \widetilde{b}_2, \widetilde{b}_3, ...\widetilde{b}_m\right)^T$ is an m-dimensional fuzzy line vector and $X = (x_1, x_2, x_3, x_4 ....x_n)^T$ is an n-dimensional decision variable vector.

We now consider a bi-objective Fuzzy linear programming Problem with constraints having fuzzy coefficients is given by

Maximize $z_1 = c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + ... + c_{1n}x_n$
Minimize $z_2 = c_{21}x_1 + c_{22}x_2 + c_{23}x_3 + ... + c_{2n}x_n$

Subject to $\widetilde{a}_{i1}x_1 + \widetilde{a}_{i2}x_2 + \widetilde{a}_{i3}x_3 + ... + \widetilde{a}_{in}x_n \le \widetilde{b}_i$
$x_1, x_2, x_3, ...x_n \ge 0, i = 1,2, ...m$ where fuzzy numbers are hexagonal, where

$$\widetilde{a}_{i1} = \widetilde{a}_{i11}, \widetilde{a}_{i12}, \widetilde{a}_{i13}, \widetilde{a}_{i14}, \widetilde{a}_{i15}, \widetilde{a}_{i16},$$
$$\widetilde{a}_{i2} = \widetilde{a}_{i21}, \widetilde{a}_{i22}, \widetilde{a}_{i23}, \widetilde{a}_{i24}, \widetilde{a}_{i25}, \widetilde{a}_{i26}$$

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$$\widetilde{a}_{in} = \widetilde{a}_{in1}, \widetilde{a}_{in2}, \widetilde{a}_{in3}, \widetilde{a}_{in4}, \widetilde{a}_{in5}, \widetilde{a}_{in6},$$

$$\widetilde{b}_i = \widetilde{b}_{i1}, \widetilde{b}_{i2}, \widetilde{b}_{i3}, \widetilde{b}_{i4}, \widetilde{b}_{i5}, \widetilde{b}_{i6}$$

By the ranking Algorithm, the above MOFLPP is transformed into a MOLPP is as follows:

Maximize $z_1 = c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + ... + c_{1n}x_n$

Minimize $z_2 = c_{21}x_1 + c_{22}x_2 + c_{23}x_3 + ... + c_{2n}x_n$

Subject to,

$$\begin{aligned}
&2\left(a_{i11}x_1 + a_{i21}x_2 + ... + a_{in1}x_n\right) + \\
&3\left(a_{i12}x_1 + a_{i22}x_2 + ... + a_{in2}x_n\right) + \\
&4\left(a_{i13}x_1 + a_{i23}x_2 + ... + a_{in3}x_n\right) + \\
&4\left(a_{i14}x_1 + a_{i24}x_2 + ... + a_{in4}x_n\right) + \\
&3\left(a_{i15}x_1 + a_{i25}x_2 + ... + a_{in5}x_n\right) + \\
&2\left(a_{i16}x_1 + a_{i26}x_2 + ... + a_{in6}x_n\right)
\end{aligned}$$

$$\le 2b_{i1} + 3b_{i2} + 4b_{i3} + 4b_{i4} + 3b_{i5} + 2b_{i6}$$

$$x_1, x_2, x_3 .... x_n \ge 0, i = 1,2, ... m \quad --------(2)$$

Using (2), this can be converted into a single objective problem subject to the constraints with transformed crisp number coefficients and hence solved accordingly.

Similarly ,multi-objective problems with more than two objectives can also be solved using the above procedure, here in the very first stage itself the problem is transformed into a crisp problem and afterwards there will be no more fuzziness in the constraints as well as in the problem.

6. NUMERICAL EXAMPLE

The below mentioned example is taken from Rajarajeswari. P and SahayaSudha A [8] in a production planning process.

Maximize $Z_1 = 75x_1 + 90x_2$

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$\begin{aligned}
\widetilde{a}_{11}x_1 + \widetilde{a}_{12}x_2 &\le \widetilde{b}_1 \\
\widetilde{a}_{21}x_1 + \widetilde{a}_{22}x_2 &\le \widetilde{b}_2 \\
x_1, x_2 &\ge 0
\end{aligned}$$

Where

$$\widetilde{a}_{11} = (180,190,200,200,210,220)$$

$$\widetilde{a}_{12} = (230,240,250,250,260,270)$$

$$\widetilde{a}_{21} = (280,290,300,300,310,320)$$

$$\widetilde{a}_{22} = (180,190,200,200,210,220)$$

$$\widetilde{b}_1 = (12000,13000,14000,14000,15000,16000)$$

$$\widetilde{b}_2 = (11500,12000,12500,12500,13000,13500)$$

Maximize $Z_1 = 75x_1 + 90x_2$

Subject to

$$\begin{aligned}
&(180,190,200,200,210,220)x_1 \\
&+ (230,240,250,250,260,270)x_2 \\
&\le (12000,13000,14000,14000,15000,16000)
\end{aligned}$$

$$\begin{aligned}
&(280,290,300,300,310,320)x_1 \\
&+ (180,190,200,200,210,220)x_2 \\
&\le (11500,12000,12500,12500,13000,13500)
\end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

By (2), this can be transformed into a crisp LPP as

Maximize $Z_1 = 75x_1 + 90x_2$,

Subject to

$$2(180x_1 + 230x_2) + 3(190x_1 + 240x_2) + 4(200x_1 + 250x_2) + 4(200x_1 + 250x_2) + 3(210x_1 + 260x_2) + 2(220x_1 + 270x_2)$$
$$\leq 2(12000) + 3(13000) + 4(14000) + 4(14000) + 3(15000) + 2(16000)$$
$$2(280x_1 + 180x_2) + 3(290x_1 + 190x_2) + 4(300x_1 + 200x_2) + 4(300x_1 + 200x_2) + 3(310x_1 + 210x_2) + 2(320x_1 + 220x_2)$$
$$\leq 2(11500) + 3(12000) + 4(12500) + 4(12500) + 3(13000) + 2(13500)$$

Case **(i):**

We consider the problem with the maximization objective

Maximize $Z_1 = 75x_1 + 90x_2$

Subject to

$$3600x_1 + 4500x_2 \leq 252000$$

$$5400x_1 + 3600x_2 \leq 225000$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

The solution is $x_1 = 9.29, x_2 = 48.57$, and Max $z_1 = 5067.86$

Next we proceed to solve the problem with minimization objective

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$3600x_1 + 4500x_2 \leq 252000$$

$$5400x_1 + 3600x_2 \leq 225000$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

The solution is $x_1 = 9.29, x_2 = 48.57$, and Min $z_2 = 4246.43$

Case (ii): Consider the problem with the maximization objective and an additional constraint

Maximize $Z_1 = 75x_1 + 90x_2$

Subject to

$$3600x_1 + 4500x_2 \leq 252000$$

$$5400x_1 + 3600x_2 \leq 225000$$

$$9.29x_1 + 48.57x_2 \leq 5067.88$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

The solution is $x_1 = 9.29, x_2 = 48.57$ and Max $z_1 = 5067.86$

Next we proceed to solve the problem with minimization objective with additional constraint

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$3600x_1 + 4500x_2 \leq 252000$$

$$5400x_1 + 3600x_2 \leq 225000$$

$$9.29x_1 + 48.57x_2 \leq 5067.88$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

The solution is $x_1 = 9.29, x_2 = 48.57$ and Min $z_2 = 4246.43$

Case (iii):

The feasibility of the solution will be explained as follows

When all the fuzzy numbers are in lesser priority values, the MOLPP becomes

Maximize $Z_1 = 75x_1 + 90x_2$,

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$930x_1 + 1180x_2 \leq 63000$$
$$1430x_1 + 930x_2 \leq 59000$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

The solution is $x_1 = 13.4, x_2 = 42.82$ and

Max $z_1 = 4859.64,$ Min $z_2 = 4083.2$

Case **(iv):**

When all the fuzzy numbers are in priority values, the MOLPP becomes

Maximize $Z_1 = 75x_1 + 90x_2$,

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$1600x_1 + 2000x_2 \le 112000$$
$$2400x_1 + 1600x_2 \le 100000$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

The solution is $x_1 = 9.29, x_2 = 48.57$ and Max $z_1 = 5067.86$, Min $z_2 = 4246.43$

Case (v):

When all the fuzzy numbers are in the higher priority values, the MOLPP becomes

Maximize $Z_1 = 75x_1 + 90x_2$,

Minimize $Z_2 = 60x_1 + 75x_2$

Subject to

$$1070x_1 + 1320x_2 \le 77000$$
$$1570x_1 + 1070x_2 \le 66000$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

The solution is $x_1 = 5.10, x_2 = 54.20$ and

Max $z_1 = 5260.43$, Min $z_2 = 4396.44$

Table 6.1: Comparison of results obtained by using existing and proposed method.

Method	**Existing method proposed by Qiu-Peng, Bing-Yuan Cao[7] and used by** Rajerajeswari.P and Sahaya Sudha A [8]				**Proposed method** Hexagonal ranking			
Objective with constraints	x_1	x_2	$Max z_1$	$Min z_2$	x_1	x_2	$Max z_1$	$Min z_2$
Max and Min objective with same constraints	9.29	48.57	5067.86	4246.46	9.29	48.57	5067.86	4246.46
Max and Min objective with same additional constraints	9.29	48.57	5067.86	4246.46	9.29	48.57	5067.86	4246.46
Lesser Priority	7.94	49.21	5023.81	4206.35	13.4	42.82	4859.64	4083.2
Priority	9.29	48.57	5067.86	4246.46	9.29	48.57	5067.86	4246.46
Higher Priority	10.39	48.05	5103.90	4279.22	5.10	54.20	5260.43	4396.44

7. RESULTS AND DISCUSSIONS

As per the above Table (6.1), we have obtained the same results from both the existing as well as the proposed method and the feasibility of the solution is also from the lowest to higher intervals.

In the above problem the profit maximization is from [4859.64, 5260.43] and the cost of production ranges from [4083.2, 4396.44].This new method reduces the ambiguity in the solution. This forms an optimal solution for a manager to take a decision whether to produce a particular product or not, or else any alternate changes can be done .This method can be used where a particular problem cannot be solved by triangular or trapezoidal method. This method also can be used even if the number of variables and parameters are increased this method is far more efficient and easy when compared to the earlier method .This can be extended to any number of input parameters.

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