

Strong Dominating Sets of Some Arithmetic Graphs

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ABSTRACT

Graph Theory has been realized as one of the most flourishing branches of modern Mathematics finding widest applications in all most all branches of Sciences, Social Sciences, Engineering, Computer Science, etc. Number Theory is one of the oldest branches of Mathematics, which inherited rich contributions from almost all greatest mathematicians, ancient and modern. Using the number theoretic function Euler totient function an Euler totient Cayley graph is defined and in this paper we study the Strong domination parameters of Euler totient Cayley graphs and Arithmetic V_n Graphs.

Keywords

Euler totient Cayley graphs, Arithmetic V_n Graphs, Strong domination, Cayley Graph

Subject Classification: 68R10

1. INTRODUCTION

Nathanson [1] was the pioneer in introducing the concepts of Number Theory, particularly, the 'Theory of Congruences' in Graph Theory, thus paved the way for the emergence of a new class of graphs, namely "Arithmetic Graphs". Inspired by the interplay between Number Theory and Graph Theory several researchers in recent times are carrying out extensive studies on various Arithmetic graphs in which adjacency between vertices is defined through various arithmetic functions.

Cayley Graphs are another class of graphs associated with elements of a group. If this group is associated with some Arithmetic function then the Cayley graph becomes an Arithmetic graph. The Cayley graph associated with Euler totient function is called an Euler totient Cayley graph.

2. EULER TOTIENT CAYLEY GRAPH

For each positive integer n , let Z_n be the additive group of integers modulo n and S be the set of all numbers less than n and relatively prime to n . The Euler Totient Cayley graph $G(Z_n, \varphi)$ is defined as the graph whose vertex set V is given by $Z_n = \{0, 1, 2, \dots, n-1\}$ and the edge set is given by $E = \{(x, y) / x - y \in S\}$.

Some properties of Euler totient Cayley graphs can be found in [2] and the domination parameters of these graphs are presented in [3].

The Euler Totient Cayley graph $G(Z_n, \varphi)$ is a complete graph if n is a prime and it is $\Phi(n)$ - regular.

3. ARITHMETIC V_n GRAPH AND ITS PROPERTIES

Let n be a positive integer such that $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$. Then the Arithmetic V_n graph is defined as the graph whose vertex set consists of the divisors of n and two vertices u, v are adjacent in V_n graph if and only if $\text{GCD}(u, v) = p_i$, for some prime divisor p_i of n .

In this graph vertex 1 becomes an isolated vertex. Hence we consider Arithmetic V_n graph without vertex 1 as the contribution of this isolated vertex is nothing when domination parameters are studied.

Clearly, V_n graph is a connected graph. If n is a prime, then V_n graph consists of a single vertex. Hence it is connected. In other cases, by the definition of adjacency in V_n , there exist edges between prime numbers, their prime powers and also to their prime products. Therefore each vertex of V_n is connected to some vertex in V_n .

It is observed that the domination parameters these graphs are functions of k , where k is the core of n , that is the number of distinct prime divisors of n [4].

Let $G(V_n)$ denote the V_n graph throughout this paper.

Strong domination

Let $G(V, E)$ be a graph and $u, v \in V$. Then, u strongly dominates v if (i) $uv \in E$ and (ii) $\deg u \geq \deg v$. A set $D \subset V$ is called a strong-dominating set (sd-set) of G if every vertex in $V - D$ is strongly dominated by at least one vertex in D . The strong domination number γ_s of G is the minimum cardinality of a strong dominating set.

Some results on strong domination can be seen in [5].

In the following sections we find minimum strong dominating sets of $G(Z_n, \varphi)$ and $G(V_n)$ graphs and obtain their strong domination number in various cases.

4. STRONG DOMINATING SETS OF EULER TOTIENT CAYLEY GRAPH

Theorem 4.1: If n is a prime, then the strong domination number of $G(Z_n, \varphi)$ is 1.

Proof: Let n be a prime. Then $G(Z_n, \varphi)$ is a complete graph and hence every vertex is of degree $n-1$. So, any single vertex dominates all other vertices in $G(Z_n, \varphi)$. Hence every vertex in $V - D$ is adjacent with a vertex in $D = \{t\}$, where t is any vertex in V . Obviously the degree of every vertex in $V - D$ of $G(Z_n, \varphi)$ is equal to the degree of the vertex $t \in D$ in $G(Z_n, \varphi)$. Thus $D = \{t\}$ is a strong dominating set with minimum cardinality 1.

Hence $\gamma_s(G(Z_n, \varphi)) = 1$.

Theorem 4.2: The strong domination number of $G(Z_n, \varphi)$ is 2, if $n = 2p$ where p is an odd prime.

Proof: Let us consider $G(Z_n, \varphi)$ for $n = 2p$, p is an odd prime. Then the vertex set is given by $V = \{0, 1, 2, \dots, 2p-1\}$. Let V be decomposed into the following disjoint subsets.

1. The set of odd integers which are less than n and relatively prime to n .
 But this set is nothing but S , since $n = 2p$,
2. The set M of non-zero even integers,
3. The set D of integers 0 and p .

We now show that $D = \{0, p\}$ is a dominating set of $G(Z_n, \varphi)$. By the definition of edges in $G(Z_n, \varphi)$, it is clear that the vertices in S are adjacent with the vertex 0 and hence they are dominated by the vertex 0.

Now consider the elements of M . By the definition of M , it contains non-zero even numbers. So if $v \in M$ then $v - p$ is an odd number, because p is an odd prime. Hence $v - p$ or $p - v \in S$. That is the vertex v is adjacent with the vertex p . This implies that the vertices of M are dominated by the vertex p . Thus every vertex of $V - D$ is dominated by the vertices of D . Therefore D is a dominating set of $G(Z_n, \varphi)$. Now we show that $D = \{0, p\}$ is a strong dominating set of $G(Z_n, \varphi)$.

By the properties of $G(Z_n, \varphi)$, we know that the graph $G(Z_n, \varphi)$ is $\varphi(n)$ -regular. So for $n = 2p$, $G(Z_n, \varphi)$ is $(p - 1)$ -regular. That is every vertex of $G(Z_n, \varphi)$ is of degree $p - 1$.

That is the degree of the vertices $\{0, 1, 2, \dots, p - 1, p + 1, \dots, 2p - 1\} \in V - D$ of $G(Z_n, \varphi)$ is equal to the degree of vertices in $D = \{0, p\}$ in $G(Z_n, \varphi)$.

So the vertices in S are strongly dominated by the vertex $0 \in D$ and the vertices in M are strongly dominated by the vertex $p \in D$.

Thus every vertex in $V - D$ is strongly dominated by at least one vertex in D .

Therefore D is a strong dominating set of $G(Z_n, \varphi)$. Since the graph $G(Z_n, \varphi)$ is $(p - 1)$ -regular, a single vertex cannot dominate the rest of $p - 1$ vertices. Therefore $D = \{0, p\}$ becomes a minimal strong dominating set with cardinality 2.

Thus $\gamma_s(G(Z_n, \varphi)) = 2$.

Theorem 4.3: Suppose n is neither a prime nor $2p$. Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$, where p_1, p_2, \dots, p_k are primes and $\alpha_1, \alpha_2, \dots, \alpha_k$ are integers ≥ 1 . Then the strong domination number of $G(Z_n, \varphi)$ is given by $\gamma(G(Z_n, \varphi)) = \lambda + 1$, where λ is the length of the longest stretch of consecutive integers in V , each of which shares a prime factor with n .

Proof: Suppose $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$ and n is neither a prime nor $2p$. Let us consider the vertex set V of $G(Z_n, \varphi)$ given by $V = \{0, 1, 2, \dots, n - 1\}$. Then the set V falls into the following disjoint subsets.

1. The set S of integers relatively prime to n ,
2. The set $X = \{S_i\}$, where S_i is a collection of consecutive integers in V such that for every x in S_i , $\text{GCD}(x, n) > 1$,
3. The singleton set $\{0\}$.

By the definition of adjacency in $G(Z_n, \varphi)$, it is obvious that the vertices of S are dominated by the vertex 0. Let S_λ be a subset in X with maximum cardinality λ . Suppose $S_\lambda = \{x_1, x_2, x_3, \dots, x_\lambda\}$ where $\text{GCD}(x_i, n) > 1$ for $i = 1, 2, 3, \dots, \lambda$.

Since $x_1, x_2, x_3, \dots, x_\lambda$ are in consecutive order, we have $\text{GCD}(x_1 - 1, n) = 1$, $\text{GCD}(x_2 - 2, n) = 1, \dots$, $\text{GCD}(x_\lambda - \lambda, n) = 1$. Therefore it follows that $x_1 - 1, x_2 - 2, x_3 - 3, \dots, x_\lambda - \lambda \in S$. So the vertices $x_1, x_2, \dots, x_\lambda$ are adjacent with the vertices $1, 2, \dots, \lambda$

respectively. Hence the vertices in S_λ are dominated by the vertices $1, 2, \dots, \lambda$.

In a similar way, we can show that all vertices of each collection S_i in X are dominated by the vertices $1, 2, \dots, \lambda$. Thus the set $D = \{0, 1, 2, \dots, \lambda\}$ becomes a dominating set in $G(Z_n, \varphi)$ as it dominates every vertex in $V - D$. Now we show that D is a strong dominating set of $G(Z_n, \varphi)$.

We know that the graph $G(Z_n, \varphi)$ is $\varphi(n)$ -regular. So, the degree of every vertex in $V - D$ of $G(Z_n, \varphi)$ is equal to the degree of every vertex in D . Thus every vertex in $V - D$ is strongly dominated by at least one vertex in D .

Therefore D is a strong dominating set of $G(Z_n, \varphi)$.

We claim that D is minimal. Suppose we delete 0 from D and let $D' = \{1, 2, \dots, \lambda\}$. Then D' cannot be a dominating set of $G(Z_n, \varphi)$, since the vertices of S are dominated only by the vertex 0 and $0 \notin D'$.

Suppose we delete any other vertex, say j from $\{1, 2, \dots, \lambda\}$. Let $D'' = \{0, 1, 2, \dots, j - 1, j + 1, \dots, \lambda\}$. Suppose D'' is a dominating set of $G(Z_n, \varphi)$. Consider the vertex x_j in S_λ , then the vertex x_j is dominated by some vertex k in D'' and $k \neq 0$. This implies that $x_j - k \in S$. Now $x_1, x_2, x_3, \dots, x_\lambda$ are in consecutive order implies that $x_j - k = x_{j-k}$. So $x_{j-k} \in S$, for all j and k such that $1 \leq j \leq \lambda$ and $1 \leq k \leq \lambda - 1$. For $j = \lambda$, $k = \lambda - 1$, we get $x_{j-k} = x_1 \in S$, a contradiction because $\text{GCD}(x_1, n) > 1$. Therefore it follows that D'' cannot be a dominating set of $G(Z_n, \varphi)$ and hence D is minimal.

If a minimal dominating set is formed in any other way, then the cardinality of such a set is not smaller than that of D . This follows from the properties of the prime divisors of a number.

Hence D becomes a strong dominating set of $G(Z_n, \varphi)$ with minimum cardinality.

Therefore $\gamma_s(G(Z_n, \varphi)) = |D| = \lambda + 1$.

5. STRONG DOMINATING SETS OF ARITHMETIC V_n GRAPH

Theorem 5.1: If $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$, where p_1, p_2, \dots, p_k are primes and $\alpha_1, \alpha_2, \dots, \alpha_k$ are integers ≥ 1 , then the strong domination number of $G(V_n)$ is given by

$$\gamma_s(G(V_n)) = \begin{cases} k - 1 & \text{if } \alpha_i = 1 \text{ for more than one } i, \\ k & \text{otherwise.} \end{cases}$$

where k is the core of n .

Proof: Suppose $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$. Consider $G(V_n)$ with vertex set V . Now we have the following possibilities.

Case 1: Suppose $\alpha_i > 1$ for all i . Now we show that the set $D = \{p_1, p_2, \dots, p_k\}$ becomes a dominating set of $G(V_n)$.

By the definition of $G(V_n)$ graph, the vertices in $G(V_n)$ are primes p_1, p_2, \dots, p_k , their powers and their products.

All the vertices $u \in V - D$, for which $\text{GCD}(u, p_i) = p_i$ are adjacent with the vertex p_i in D for all $i = 1, 2, \dots, k$. Since every vertex in $V - D$ has atleast one prime factor viz., p_1, p_2, \dots, p_k (as they are divisors of n), every vertex in $V - D$ is adjacent with at least one vertex in D . Thus D becomes a dominating set of $G(V_n)$.

Now we show that D is a strong dominating set of $G(V_n)$.

By the definition of edges in $G(V_n)$, we see that the degree of p_i^r , $r < \text{degree of } p_i, i = 1, 2, 3, \dots, k; r > 1$ and the degree of $p_i > \text{the degree of } p_i^r p_j^s$, the degree of $p_i^r p_j^s p_l^t, \dots$, the degree of $p_i^r p_j^s \dots p_k^u, r = 1, 2, \dots, \alpha_i$;

$s = 1, 2, \dots, \alpha_j; \dots; u = 1, 2, \dots, \alpha_k$ in $G(V_n)$.

So, the degree of $p_i, i = 1, 2, \dots, k$ is greater than the degree of all other vertices in $G(V_n)$. Thus every vertex in $V - D$ is strongly dominated by at least one vertex in D .

Therefore $D = \{p_1, p_2, \dots, p_k\}$ is a strong dominating set of $G(V_n)$.

We now prove that D is minimal. Suppose we remove any p_i from D . Then the vertices of the form $p_i^r, r > 1$ are not dominated by any other vertex p_j in D as $\text{GCD}(p_i^r, p_j) = 1$ for $i \neq j$. Therefore every $p_i, i = 1, 2, \dots, k$ must be included into D .

If we form a minimum dominating set in any other manner, then the order of such a set is not smaller than that of D . This follows from the properties of prime divisors of a number. Hence $\gamma_s(G(V_n)) = k$.

Case 2: Suppose $\alpha_i = 1$ for only one i . That is, p_i is the only prime divisor of n with exponent 1. Then

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots p_{i-1}^{\alpha_{i-1}} \cdot p_i \cdot p_{i+1}^{\alpha_{i+1}} \dots p_k^{\alpha_k}.$$

Let $D = \{p_1, p_2, \dots, p_{i-1}, p_i p_j, p_{i+1}, \dots, p_k\}$. The vertex p_i is adjacent only with the vertex $p_i p_j$ in D , for any j , as $\text{GCD}(p_i, p_i p_j) = p_i$. Now the degree of $p_i < \text{the degree of } p_i p_j$. Hence it follows as in Case 1 that the vertices in D strongly dominate the vertices of $V - D$.

Therefore $D = \{p_1, p_2, \dots, p_{i-1}, p_i p_j, p_{i+1}, \dots, p_k\}$ is a strong dominating set of $G(V_n)$ with cardinality k .

$$\text{Hence } \gamma_s(G(V_n)) = k.$$

Case 3: Suppose $\alpha_i = 1$ for more than one i .

We have the following sub cases.

Sub case (i): Suppose $\alpha_i = 1$ for all i . That is

$$n = p_1 p_2 \dots p_k.$$

Here we have the following possibilities.

Suppose $k = 2$. Then the vertex set of $G(V_n)$ is $\{p_1, p_2, p_1 p_2\}$. Obviously $D = \{p_1 p_2\}$ is a dominating set of $G(V_n)$ and this set is also a strong dominating set with cardinality 1 as $\text{deg}(p_1 p_2) > \text{deg}(p_1), \text{deg}(p_2)$.

Suppose $k = 3$.

Then $V(G(V_n)) = \{p_1, p_2, p_3, p_1 p_2, p_1 p_3, p_2 p_3, p_1 p_2 p_3\}$. It is clear that $D = \{p_1, p_2 p_3\}$ is a strong dominating set of $G(V_n)$, as $\text{deg}(p_2 p_3) > \text{deg}(p_2), \text{deg}(p_3)$ and $\text{deg}(p_2 p_3) = \text{deg}(p_1 p_3), \text{deg}(p_1 p_2); \text{deg}(p_1) = \text{deg}(p_1 p_2 p_3)$. Therefore D is a strong dominating set of $G(V_n)$ with cardinality 2.

Suppose $k > 3$. Now we show that the set

$D = \{p_1, p_1 p_2, p_2 p_3, \dots, p_{k-3} p_{k-2}, p_{k-1} p_k\}$ is a dominating set of $G(V_n)$. In this case the vertices of $G(V_n)$ are primes p_1, p_2, \dots, p_k and their products.

All the vertices $u \in V - D$, for which $\text{GCD}(u, p_1) = p_1$ are adjacent with the vertex p_1 in D , the vertices p_2, \dots, p_k are adjacent with the vertices $p_1 p_2$ and $p_2 p_3; p_2 p_3$ and $p_3 p_4; \dots; p_{k-4} p_{k-3}$ and $p_{k-3} p_{k-2}; p_{k-1} p_k$ respectively.

The vertices

$p_2 p_3, p_2 p_4, p_2 p_5, \dots, p_2 p_k, p_2 p_3 p_4, \dots, p_2 p_3 p_k$ and so on are adjacent with the vertex $p_1 p_2$, the vertices $p_3 p_1, p_3 p_4, p_3 p_5, p_3 p_6, \dots, p_3 p_k, p_3 p_4 p_5, \dots, p_3 p_4 p_k$ and so on are adjacent with the vertex $p_2 p_3$.

Similarly we can show that every vertex in $V - D$ shares a common factor p_1 or p_2 or \dots or p_k with some vertex in D .

Thus every vertex in $V - D$ is adjacent with at least one vertex in D .

Therefore D becomes a dominating set of $G(V_n)$.

By the definition of edges in $G(V_n)$, we see that $\text{deg}(p_i p_j) > \text{deg}(p_i p_j p_l) > \dots > \text{deg}(p_1 p_2 p_3 \dots p_k)$; $\text{deg}(p_i) < \text{deg}(p_i p_j)$; $\text{deg}(p_i) > \text{deg}(p_i p_j p_l), \dots, \text{deg}(p_i) > \text{deg}(p_1 p_2 p_3 \dots p_k), i = 1, 2, \dots, k$. Now the vertex $p_1 p_2 \dots p_k$ is adjacent with the vertex $p_1 \in D$ only.

Thus every vertex in $V - D$ is strongly dominated by at least one vertex in D .

Therefore $D = \{p_1, p_1 p_2, p_2 p_3, \dots, p_{k-3} p_{k-2}, p_{k-1} p_k\}$ is a strong dominating set of $G(V_n)$.

Now we show that D is minimum. Suppose we delete the vertex p_i from D , then the vertex $p_1 p_2 \dots p_k$ is not adjacent with any other vertex in D , because $\text{GCD}(p_1 p_2, p_1 p_2 \dots p_k) = p_1 p_2$; $\text{CD}(p_2 p_3, p_1 p_2 \dots p_k) = p_2 p_3; \dots$; $\text{GCD}(p_{k-3} p_{k-2}, p_1 p_2 \dots p_k) = p_{k-3} p_{k-2}$; $\text{GCD}(p_{k-1} p_k, p_1 p_2 \dots p_k) = p_{k-1} p_k$.

Similar is the case with the deletion of any vertex $p_i p_j$ from D , for $i = 1, 2, \dots, k - 3, k - 1$;

$$j = 2, 3, \dots, k - 2, k.$$

Thus $D = \{p_1, p_1 p_2, p_2 p_3, \dots, p_{k-3} p_{k-2}, p_{k-1} p_k\}$ is a minimum strong dominating set of $G(V_n)$.

$$\text{Hence } \gamma_s(G(V_n)) = k - 1.$$

Sub case (ii): Suppose $\alpha_1 = \alpha_2 = 1$. That is

$$n = p_1 \cdot p_2 \cdot p_3^{\alpha_3} \cdot p_k^{\alpha_k}.$$

Now we show that the set $D = \{p_3, p_4, \dots, p_k, p_1 p_2\}$ becomes a dominating set of $G(V_n)$.

The vertices p_1, p_2 are adjacent only with the vertex $p_1 p_2$ in D , as $\text{GCD}(p_1, p_1 p_2) = p_1$; $\text{GCD}(p_2, p_1 p_2) = p_2$. Now $\text{deg}(p_1 p_2) > \text{deg}(p_1), \text{deg}(p_2)$, Hence it follows as in Case 1 that the vertices in D strongly dominate the vertices of $V - D$.

Therefore $D = \{p_3, p_4, \dots, p_k, p_1 p_2\}$ is a strong dominating set of $G(V_n)$ with cardinality $k - 1$.

$$\text{Hence } \gamma_s(G(V_n)) = k - 1.$$

Sub case (iii): Suppose $\alpha_1 = \alpha_2 = \alpha_3 = 1$. That is

$$n = p_1 \cdot p_2 \cdot p_3 \cdot p_4^{\alpha_4} \cdot \dots \cdot p_k^{\alpha_k}.$$

Let $D = \{p_4, p_5, \dots, p_k, p_1, p_2, p_3\}$. Then we can show as in Subcase (ii) that D is a strong dominating set of $G(V_n)$ with cardinality $k - 1$.

Hence $\gamma_s(G(V_n)) = k - 1$.

Sub case (iv): Suppose $\alpha_j = 1$ for $j = 1, 2, \dots, i$ and $\alpha_j > 1$ for $j = i + 1, i + 2, \dots, k$.

Then $n = p_1 \cdot p_2 \cdot p_3 \dots p_i \cdot p_{i+1}^{\alpha_{i+1}} \dots p_k^{\alpha_k}$.

We can prove as in Case 1 and Sub case (i) of Case 3 that the set $D = \{p_{i+1}, p_{i+2}, \dots, p_k, p_1, p_1 p_2, \dots, p_{i-3} p_{i-2}, p_{i-1} p_i\}$ forms a minimum strong dominating set of (V_n) .

Hence $\gamma_s(G(V_n)) = k - 1$.

6. CONCLUSION

Using Number theory, it is interesting to study the strong dominating sets of these Arithmetic graphs. This work gives the scope for the study of strong dominating sets of product graphs of these graphs and the authors have also studied this aspect.

ILLUSTRATIONS

Euler totient Cayley Graphs

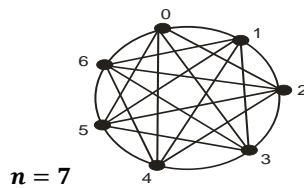


Fig.1
 $G(Z_7, \varphi)$

Strong Dominating Set = $\{0\}$.

$$n = 14 = 2 \times 7$$

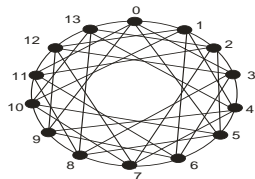


Fig.2
 $G(Z_{14}, \varphi)$

Strong Dominating Set = $\{0, 7\}$.

$$n = 8 = 2^3$$

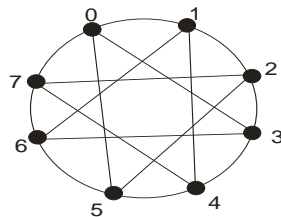


Fig.3
 $G(Z_8, \varphi)$

Strong Dominating Set = $\{0, 1\}$.

Arithmetic V_n Graphs

$$n = 100 = 2^2 \times 5^2$$

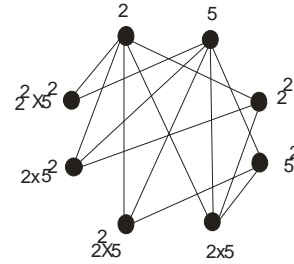


Fig.4
 $G(V_{100})$

Strong Dominating Set = $\{2, 5\}$.

$$n = 180 = 2^2 \times 3^2 \times 5$$

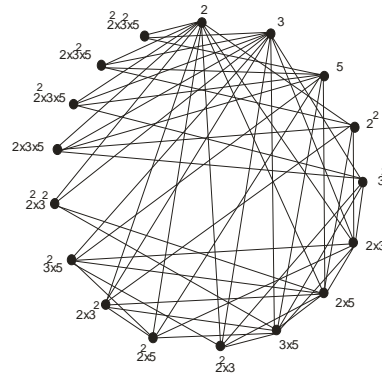


Fig.5
 $G(V_{180})$

Strong Dominating Set = $\{2, 3, 15\}$

$$n = 10 = 2 \times 5$$

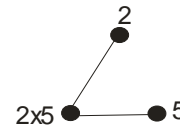


Fig.6
 $G(V_{10})$

Strong Dominating Set = $\{10\}$.

$$n = 30 = 2 \times 3 \times 5$$

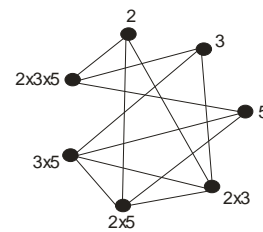


Fig.7
 $G(V_{30})$

Strong Dominating Set = $\{2, 15\}$.

$$n = 210 = 2 \times 3 \times 5 \times 7$$

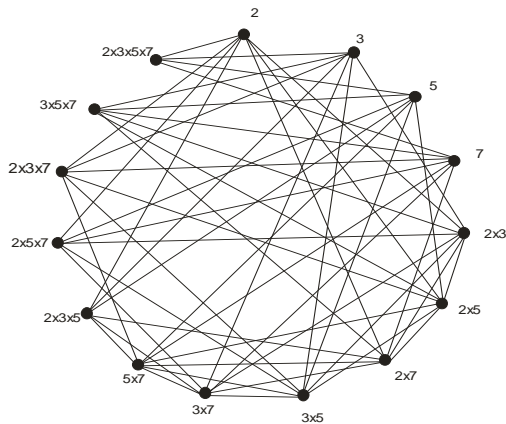


Fig.8
 $G(V_{210})$

Strong Dominating Set = {2,6,35}.

$$n = 60 = 2^2 \times 3 \times 5$$

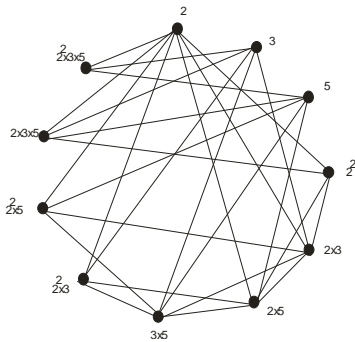


Fig.9
 $G(V_{60})$

Strong Dominating Set = {2, 15}.

$$n = 420 = 2^2 \times 3 \times 5 \times 7$$

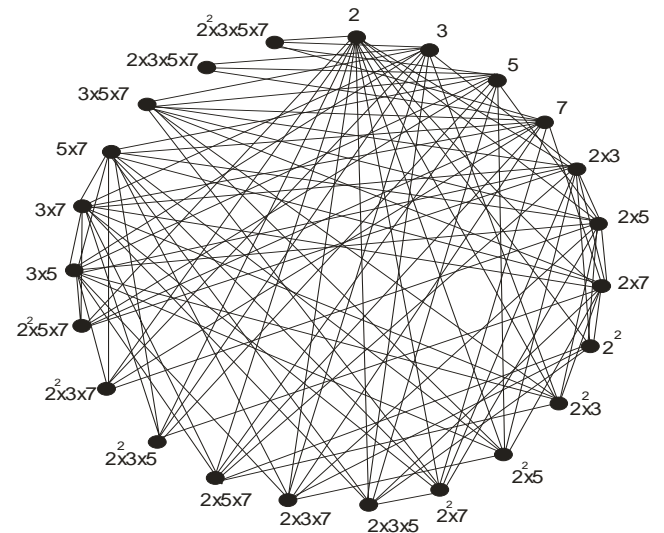


Fig.10
 $G(V_{420})$

Strong Dominating Set = {2,3,35}.

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