Testing Hypothesis for New Class of Life Distribution

NBUFR-t0

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ABSTRACT
A new concept of ageing distribution, namely new better (worse) than used in failure rate at specific time \( t_0 \) (NBUFR-\( t_0 \) (NWUFR-\( t_0 \))) are introduced. The problem is investigated how to prove that after a specified time \( t_0 \) of operation the failure rate of an item is greater than the corresponding failure rate of a new item. This problem occurs in various areas like for instance in industry, when designing a maintenance policy. A test statistics that based on the goodness of fit method are derived for testing exponentially versus the NBUFR-\( t_0 \) alternatives. The percentiles and powers of this test statistic are tabulated. The asymptotic efficiencies for some alternatives are derived. A medical data is taken as a practical application.

Keywords
NBUFR-\( t_0 \); NWUFR-\( t_0 \); Mont Carlo method; Hypotheses testing; NBUFR; U-statistic; Life testing; Exponential distribution; Goodness of fit testing; Efficiency; Power of test.

1. INTRODUCTION
The notion of aging for engineering systems has been characterized by several classes of life distributions in reliability. Various classes of life distributions have been introduced to describe several types of deterioration (improvement) that accompany aging. Unfortunately these types of classes are restricted to one type of applications since they present positive aging or negative aging for their dual classes throughout their life span of the underlying components or their systems.

[16] have introduced the new better than used at specific age at \( t_0 \) (NBU-\( t_0 \)) and its dual (NWU-\( t_0 \)). They have listed some types of situation where (NBU-\( t_0 \)) aging or its dual might arise. [12] introduced the new better than used in convex ordering at age \( t_0 \) (NBUC-\( t_0 \)), and its dual.

There are many situation in real life where the components of the system gradually deteriorate up time \( t_0 \) which is warranty guarantee time provided by most manufacturers, then maintenance through repairs or spare part replacement take place after time \( t_0 \). Here maintenance is expected to improve the performance of the system but can not bring it back to a better situation than it was at age \( t_0 \). For some interesting examples, see [16].

One of the oldest problems in aging distributions is testing exponentiality against the known classes of aging distributions such as IFR, IFRA, NBU, NBARFR and others, for example see [23], [15], [2], [3] and [20] among others. With respect to the NBUFR class, see [6].

[16] introduced the concept of the class of life distributions called new better than used of specified age \( t_0 \). [11], [22] and [4] are investigated the testing of new better than used of specified age \( t_0 \) (NBU-\( t_0 \)) alternatives and others as [19] deal with the problem with respect to increasing failure rate (IFR) and new better than used (NBU) after specified time \( t_0 \). [21] introduce the new better (worse) than used renewal failure rate at specific time \( t_0 \) NBUFR-\( t_0 \) (NWURFR-\( t_0 \)).

In practice, one might be interested in the new better than used failure rate behaviour at specified age \( t_0 \). Therefore a new concept of ageing distribution namely new better (worth) than used failure rate of specified age \( t_0 \) (NBUFR-\( t_0 \) (NWUFR-\( t_0 \))). This class is generalization of NBUFR ([1]). We investigate the testing exponentiality problem of it alternatives here according to goodness of fit approach. The distributional properties of the test statistic are studied. The performances of a proposed test are studied in terms of powers for some of alternative life distributions. The proposed test is shown to be consistent for NBUFR-\( t_0 \) alternatives.

This paper is organized as follows. In Section 2 we will recall some definitions and facts about characteristics of lifetime distributions and about relationship between some classes of life distributions. Section 3 introduced main results for our study represented in the test statistic proposed for testing exponentially versus the NBUFR-\( t_0 \) alternatives.

2. DEFINITIONS and RELATIONSHIPS
Let X be a non-negative random variable representing the lifetime of a device, with distribution function \( F(x) = P[X \leq x] \), survival function \( \bar{F}(x) = 1 - F(x) \) and density function \( f(x) \) (if it exists), then the corresponding failure rate \( r(t) \), when the distribution is absolutely continuous, is defined by

\[
r(t) = \lim_{\delta \rightarrow 0} \frac{P[T \leq t + \delta t | T \geq t]}{\delta t}.
\]

[1] introduced the new better than used failure rate class of life distribution and its dual and discussed some of its properties as follows

Definition 2.1 An absolutely continuous probability distribution F on \([0, \infty)\) for which \( F(x) = \) as \( x \rightarrow 0 \) from above is said to be NBUFR(NWUFR) if there exists a version \( r \) of the failure function such that

\[
\bar{r}(0) \leq 2(r). \]
Authors state that: An absolutely continuous distribution F on \([0, \infty)\) for which \(F(x)/x\) has a limit as \(x \to 0\) from above has the NBUFR property if

\[
\int_0^t f(x) \, dx = \lim_{t \to 0^+} t^{-1} \int_0^t f(x) \, dx.
\]

The random variable \(X\), or its distribution function \(F(x)\), is said to be (or to have) new better (worse) than used (NBU (NUW)) if

\[
\overline{F}(x + t) \leq e^{-r(t)} \overline{F}(x) \text{ for all } x, t \geq 0
\]

where

\[
\overline{F}(0) = \lim_{t \to 0^+} t^{-1} \int_0^t f(x) \, dx.
\]

Definition 2.2 The random variable \(X\), or its distribution function \(F(x)\), is said to be (or to have) new better (worse) than used (NBU (NUW)) if

\[
\overline{F}(x + t) \leq e^{-r(t)} \overline{F}(x) \text{ for all } x, t \geq 0
\]

[9] introduced the class of new better than used in convex ordering (NBUFR distributions). [10] suggested the class of new better than used of second order as follows:

\[
\int_0^t f(x + s) \, ds \leq \int_0^x f(s) \, ds \text{ for all } x, t \geq 0
\]

Definition 2.3 The random variable \(X\), or its distribution function \(F(x)\), is said to be

new better than used in convex order (NBUFR) if

\[
\int_0^x f(s) \, ds \leq \int_0^t f(s) \, ds \text{ for all } x, t \geq 0
\]

Definition 2.4 A non-negative random variable \(X\) is said to be new better than used of age \(t_0\) if

\[
\overline{F}(t_0 + x) \leq \overline{F}(t_0) \overline{F}(x) \text{ for all } x \geq 0
\]

The NBU property states that a used item of any age has stochastically smaller residual life length than does a new item, whereas the NBU-\(t_0\) property states that a used item of age \(t_0\) has stochastically smaller residual life length than does a new item.

[12] introduced the concept of new better than used in convex ordering and new better than used of second order classes of age \(t_0\) as follows:

Definition 2.5 A non-negative random variable \(X\) is said to be

new better than used in convex ordering of age \(t_0\) (NBU(t0)) if

\[
\int_x^\infty \overline{F}(t_0 + y) \, dy \leq \overline{F}(t_0) \int_x^\infty \overline{F}(y) \, dy \text{ for all } x \geq 0
\]

new better than used of second order of age \(t_0\) (NBU(2)-\(t_0\) ) if

\[
\int_0^x \overline{F}(t_0 + y) \, dy \leq \overline{F}(t_0) \int_0^x \overline{F}(y) \, dy \text{ for all } x \geq 0
\]

It is seen from the above definitions that that

\[
\text{NBU} - t_0 \equiv \text{NBU} - t_0 \rightarrow \text{NBU} (2) - t_0
\]

and

\[
\text{NBU} - t_0 \downarrow \text{NBU} - t_0
\]

By the same concept, we can introduce new class of life distributions at specific age \(t_0\). Namely new better than used in failure rate at specific age \(t_0\) (NBUFR-\(t_0\) (NUFR-\(t_0\) ) as follows:

Definition 2.6 An absolutely continuous distribution \(F\) on \([0, \infty)\) is said to be new better (worse) than used in failure rate at specific time \(t_0\) (NBUFR-\(t_0\) (NUFR-\(t_0\) ) if there exist a version \(r\) of the failure rate functions such that for specified \(t_0\)

\[
r(t_0) \leq r(t + t_0), \quad \text{for all } t \geq 0
\]

Note that the border class where \(r(t_0) = r(t + t_0)\) include only the following members:

all exponential distribution.

all life distribution defined on \([0, t_0]\) .

all life distribution defined freely on \([0, t_0]\) and defined as

\[
\overline{F}(t_0) \overline{F}(x - j t_0) \text{ for all } j t_0 \leq x \leq (j + 1) t_0, \quad j \geq 1.
\]

3. The U-STATISTIC TEST PROCEDURE

In this section we consider the problem of testing \(H_0: \overline{F}\) belongs to border distributions against \(H_1: \overline{F}\) is strictly NBUFR-\(t_0\) by using a goodness of fit approach. For more detail see [3]. The following is the form of our new class that called new better than used failure rate of specified age \(t_0\)

\[
r_F(t_0) \leq r_F(t + t_0) \quad t \geq 0 \text{ for fixed } t_0.
\]

that is equivalent to

\[
\frac{f(t_0)}{\overline{F}(t_0)} \leq \frac{f(t + t_0)}{\overline{F}(t + t_0)}, \quad t \geq 0 \text{ and fixed } t_0.
\]

In contrast to goodness of fit problems, where the test statistic is based on a measure of departure from \(H_0\) and \(H_1\), we refer to the departure between the two sides of inequality by

\[
\Delta_F = \int_0^\infty \frac{\overline{F}(t)}{\overline{F}(t_0)} f(x + t_0) - f(t_0) \overline{F}(x + t_0) \, dF_0(x)
\]

A nonparametric test statistic for this class is introduced.

This test is designed to test the hypothesis whether a
NBUFR- \( t_0 \) elements are strictly or not. The following lemma is essential for the development of our test statistic.

**Lemma** Let \( X \) be a random variable with NBUFR- \( t_0 \) distribution function \( F \) then

\[
\Delta_F = \bar{F}(t_0)E\left[ e^{-X-t_0}I(X > t_0) \right] - f(t_0)E\left[ e^{-(X-t_0)}I(X > t_0) \right]
\]

**Proof** Since

\[
\int_{t_0}^{\infty} \bar{F}(t + t_0) dF(t) = \int_{0}^{\infty} I(X > x + t_0) e^{-x} dx
\]

and

\[
\int_{t_0}^{\infty} I(X > x) e^{-(a-t_0)} dx = \bar{F}(t_0) E\left[ I(X > t_0) \right] \left[ 1 - e^{-(X-t_0)} \right]
\]

Then the result is simple to obtain.

\( \Delta_F = 0 \) under \( H_0 \) and \( \Delta_F > 0 \) under \( H_1 \). To estimate \( \Delta_F \) let \( X_1, X_2, X_3, \ldots, X_n \) be a random sample from \( F \), then \( F(t) \) will be empirically estimated by \( \hat{F}_n(t) \). So the empirical form of \( \Delta_F \) is as follows

\[
\hat{\Delta}_F = \frac{1}{n^2} \sum_{i,j=1}^{n} \left\{ I(X_i > t_0) I(X_j > t_0) e^{-(X_j-t_0)} \right\}
\]

where \( \hat{F}_n(t_0) \) is as follows

\[
\hat{F}_n(t_0) = \frac{1}{na_n} \sum_{j=1}^{n} K \left( \frac{t_0 - X_j}{a_n} \right)
\]

For more details about \( K \) and the sequence \( \{a_n\} \) see [13]. If we choose

\[
\phi(X_1, X_2) = I(X_1 > t_0) I(X_2 > t_0) e^{-(X_2-t_0)}
\]

then \( \hat{\Delta}(F) \) equivalents the following U-statistic cf [18].

\[
U_n = \frac{1}{n^2} \sum_{i,j=1}^{n} \varphi(X_i, X_j)
\]

Where, the summation over all arrangements of \( X_{i1}, X_{i2}, \ldots, i: 1, 2, 3, \ldots, n \). The proof of the following theorem follows from the standard theory of U-statistic, which obtained the asymptotic distribution of \( \Delta_F \).

**Theorem** As \( n \to \infty \), the random variable \( \sqrt{n}(\hat{\Delta}_F - \Delta_F) \) is asymptotically normal with mean 0 and variance \( \sigma^2 \) where \( \sigma^2 \) is given in (3.1). Under \( H_0 \), \( \sigma^2 \) is given by (3.2).

**Proof** Using standard U-statistics theory, cf- [18], we need only the asymptotic variance which is equal to

\[
\sigma^2 = V(E[\phi(X_1, X_2) | X_1]) + E[\phi(X_1, X_2) | X_2]
\]

Now since

\[
E[\phi(X_1, X_2) | X_1] = I(X_1 > t_0) + f(t_0) E[I(X_2 > t_0) e^{-X_2}]
\]

and

\[
\phi(X_1, X_2) | X_2 = \bar{F}(t_0) I(X_2 > t_0) e^{-(X_2-t_0)}
\]

Hence,

\[
\sigma^2 = V\left( \hat{F}_n(t_0) I(X > t_0) e^{-(X-t_0)} \right)
\]

Under \( H_0 \),

\[
\sigma^2_0 = \frac{1}{4} \int_{-\infty}^{t_0} F^4(t_0) - \frac{1}{12} F^3(t_0)(-6 + 7 F^{-1}(t_0))
\]

This completes the proof. It is easy to explain that our proposed test is consistent and unbiased. To perform above test, calculate \( \hat{\Delta}_F \) and reject \( H_0 \) if this value exceeds \(-Z_\alpha \), the lower \( \alpha \)- percentile of the standard normal variate.

The asymptotic Pitman efficacy of a statistic \( T_n \) is defined by

\[
\text{eff}(T_n) = \frac{\varphi(T_n, t)}{\varphi(T_0, t)}
\]

The following families of alternatives are used for efficacy comparisons:

- Linear failure rate distribution: \( \exp(-t - \theta t^2/2) \), \( t \geq 0, \theta \geq 0 \)
- Makeham distribution: \( e^{-t-\theta(1+e^{-t})} \), \( t > 0, \theta > 0 \)
- \( e^s \) distribution

\[
\bar{F}(t) = \begin{cases} e^{-t-\theta t^2/2} & 0 \\ e^{-t-\theta t^2} & t > 0, \theta \leq 1 \end{cases}
\]

the null exponential is attained at \( \theta = 0 \).

Carrying out efficacy calculations for the above alternatives, we get

\[
\frac{\sqrt{5} e^{-2t_0} \left[ 1 - 2t_0 - \frac{1}{1 + 2t_0 + 2t_0^2} \right]}{4 \sqrt{e^{-2t_0} - 3 + 7e^{2t_0} - e^{-3t_0}}}
\]

and

\[
\frac{1}{\sqrt{3} \sqrt{e^{-2t_0} - 3 + 7e^{2t_0}}}
\]
respectively. For sample sizes 5, 6, 7…50 and using 5000 replications we tabulate the lower and upper percentiles for $\hat{\Delta}_n$ for $t_0 = 0.25$ in table (Appendix). Its powers for sample sizes 10, 20 and 30 for the linear failure rate, weibull and gamma families are given in the following two tables respectively.

### Power estimates of $\hat{\Delta}_n$ ($t_0 = 0.25$)

<table>
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<th>Distribution</th>
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<th>n</th>
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<tr>
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<tr>
<td></td>
<td>4</td>
<td>1.000 1.000 1.000 1.000</td>
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</table>

Thus we have shown that, based on Monte Carlo methods, our test statistic has a full powers for all alternatives.

Example 3.3 The data of this example consist of the survival times of 40 patients suffering from leukemia (Leukemia) from one of the hospitals in Saudi Arabia, [14]. The orderd life times (in days) are:

\[
\begin{align*}
115.0 & \\
181.0 & \\
255.0 & \\
418.0 & \\
441.0 & \\
461.0 & \\
516.0 & \\
739.0 & \\
743.0 & \\
789.0 & \\
807.0 & \\
865.0 & \\
924.0 & \\
983.0 & \\
1024.0 & \\
1062.0 & \\
1063.0 & \\
1165.0 & \\
1191.0 & \\
1222.0 & \\
1122.0 & \\
1251.0 & \\
1277.0 & \\
1290.0 & \\
1357.0 & \\
1369.0 & \\
1408.0 & \\
1455.0 & \\
1478.0 & \\
1549.0 & \\
1578.0 & \\
1578.0 & \\
1599.0 & \\
1603.0 & \\
1605.0 & \\
1696.0 & \\
1735.0 & \\
1759.0 & \\
1815.0 & \\
1852.0 & \\
\end{align*}
\]

To apply the test we choose $t_0 = 1.137$ ($\approx 3$ years). We obtain the value of our test for the blood cancer data with a corresponding one side and 0.05 level of significant hence $H_0$ is not rejected, agreeing with the conclusion of [14] and [3].

### References


Appendix

Table Percentiles for $\hat{\Delta}_F (t_0 = 0.25)$

<table>
<thead>
<tr>
<th>n</th>
<th>.01</th>
<th>.05</th>
<th>.10</th>
<th>.90</th>
<th>.95</th>
<th>.98</th>
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<td>0.01</td>
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