

# Image Change Detection by Means of Discrete Fractional Fourier Transform

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## ABSTRACT

The proposed research paper shall analyze a method of image change detection based upon the Fractional Fourier transform (FrFT), which can provide results with good precision and better recall values obtained by optimizing its fractional order 'a'. The method is analyzed because, with extra degree of freedom provided by the Discrete Fractional Fourier Transform (DFrFT), we can get more accurate change regions as compared to other methods in the recent literature like Histogram based change detection or some fixed transformation technique like Discrete Cosine Transform (DCT). Among these three methods, change detection using DFrFT gives out improved results in terms of precision and recall parameters.

## Keywords

Image Change Detection, Discrete Fractional Fourier Transform

## 1. INTRODUCTION

Change detection is a technique used to determine change between two images which may arise due to various changing factors. It is an important process in monitoring natural resources, detecting out region of interest which are mainly the change objects in the two images taken at different instants of time. Change detection between images of the same scene taken at different times is important in various applications, such as the detection of abnormalities in medical or factory examination, event detection from surveillance images, and change detection from aerial/satellite images [1]. When images are taken consecutively with small time intervals, as with frame-rate video surveillance, the adaptive background subtraction strategy works well when using a series of previous frames [2-3]. However, there are also several situations in which images are taken with a long time interval. In such cases, it is not easy to discriminate "significant" changes, such as the appearance/disappearance of objects, from "insignificant" changes, such as those induced by illumination variation. For simple techniques which work directly on image data, like simple image differencing, the output is not very reliable except for artificially simplified cases or for highly controlled scenarios [4]. To improve reliability, one slowly moves from using direct image data to single or multi-step-derived information like information generated after segmentation or object classification. The transform domain techniques may include change detection using DCT, Discrete Fourier Transform (DFT) but we propose here a new change detection algorithm using the Fractional transforms. The FrFT, which is a generalization of the simple Fourier transform (FT). The FT is undoubtedly one of the most appreciated and frequently used tools in signal processing and analysis. Little need be said of the importance and ubiquity of the ordinary Fourier transform in many areas of science and engineering..FrFT was introduced in 1980 by Victor Namias [5] and it was established in the same

year that the other transforms could also be fractionalized [6]. McBride and Keer explored the refinement and mathematical definition in 1987 [7]. In a very short span of time, FrFT has established itself as a commanding tool for the analysis of time varying signals [8-9]. It has gained prominence in signal processing, optics and quantum mechanics during last two decades. But when FrFT is considered in discrete domain there are many definitions of Discrete Fractional Fourier Transform (DFrFT) [10-11]. It has been recently observed that DFrFT can be used in the field of image processing. The vital feature of Discrete Fractional Fourier domain Image compression aids from its extra degree of freedom that is provided by its fractional orders. The 1D DFrFT is useful in processing signals such as speech waveforms (one dimensional signal). For analysis of 2D signals such as images, a two dimensional version of DFrFT is required. For an  $M \times N$  matrix, the 2D DFrFT is computed in an unpretentious way. The 1D DFrFT is applied to each row of given matrix and then same is applied to each column of the result matrix. Thus, the generalization of the DFrFT to 2D is given by taking the DFrFT of the rows of the matrix i.e. image in a fractional domain and then taking the DFrFT of the subsequent column wise. In case of 2D DFrFT, two angles of rotation  $\alpha = \pi/2$  and  $\beta = \pi/2$  have to be taken. If one of these angles is zero, the 2D transformation kernel reduces to the 1D transformation Kernel. Image Change Detection using DFrFT involves a simple methodology, firstly the difference image is obtained and then by applying DFrFT of flexible order (0-1), we acquire different version of the difference image. Regions are marked on the change image with some criteria to find image change detection. The paper is organized as follows:

The definitions of FrFT along with its discrete version DFrFT are described in section 2. In section 3, we present methodology of Image change detection using DFrFT. Section 4 presents the results and discussions and Section 5 concludes the paper.

## 2. FRACTIONAL FOURIER TRANSFORM

The Fractional Fourier transform is a generalization of ordinary Fourier Transform with a tunable parameter  $a$ . This transform has been introduced for many years. It has much of prospective in signal processing and optics. Mathematically the  $a$ th order Fractional Fourier transform operator is the  $a$ th power of the ordinary Fourier transform operator. The fractional Fourier transform with parameter  $a = 1$  is the conventional Fourier transform. Note that the ordinary Fourier transform is a special case of a continuum of fractional Fourier domains. Essentially, the  $a$ th order fractional Fourier transform is an interpolation between a function  $f(u)$  and its Fourier transform  $F(u)$ . The 0th order transform is simply the function itself, whereas the 1st

order transform is its Fourier transform. The 0.5th order transform is something in between function and its Fourier transform. The transform is defined as a linear integral transform with kernel  $K_a(u, u')$ :

$$f_a(u) = \mathcal{F}^a[f(u)] = \int K_a(u, u') f(u') du' \quad (1)$$

where  $\mathcal{F}^a$  is the  $a$ th order fractional operator and  $f_a(u)$  is the  $a$ th order transform, where the kernel  $K_a(u, u')$  is

$$K_a(u, u') = \sqrt{1 - i \cot \phi} e^{i \pi [(\cot(\phi u^2) - 2 \csc(\phi u u') + \cot(\phi u'^2))]} ,$$

$$\phi = \frac{a\pi}{2} \quad (2)$$

The FrFT of a signal  $f(u)$  can be computed by four steps process:

1. Multiplying the function with a chirp.
2. Obtaining its Fourier transform.
3. Multiplying for a second time with a chirp.
4. Lastly multiplication with an amplitude factor.

## 2.1 Discrete Fractional Fourier Transform

A discrete version of a transform is analogy to the continuous version of transform. It is desirable for the discrete transform to usefully approximate the samples of the continuous transform, so that it can provide a basis for digital computation of the continuous transform. In the recent years, the FrFT has gained great significance as a tool for signal processing [10, 13-16]. Therefore, many attempts have been made to find the discrete version of the FrFT [10, 15]. The first work on discrete fractional Fourier transform (DFrFT) is claimed by Santhanam [17] in 1995. After that many researchers are trying to discretize this linear integral transform. Though quite a few different algorithms were suggested by various researchers, the most appropriate definition that agrees with the various properties of the FrFT and approximates to the FrFT is the one proposed by Candan et al. [15]. This is defined in terms of particular set of Eigen vectors. This Eigen vectors are discrete version of the continuous Hermite Gaussian functions. This definition fulfills all the vital properties such as unitary, index additive, reduction to DFT when order is equal to unity and approximation of Continuous FrFT. If we let  $\mathcal{F}^a$  be the  $N \times N$  matrix representing the discrete fractional Fourier transform, this definition can be stated as follows:

$$\mathcal{F}^a = \sum_{n=0}^3 e^{j \frac{3\pi}{4}(n-a)} \frac{\sin \pi(n-a)}{4 \sin \frac{\pi}{4}(n-a)} \mathcal{F}^n \quad (3)$$

where,  $\mathcal{F}^n$  is the  $n$ th (integer) power of the DFT matrix. Now the emphasis would be on finding an eigenvector set of the DFT matrix which can serve as discrete versions of the Hermite–Gaussian functions. The Hermite–Gaussian generating differential equation is

$$\frac{d^2 f(t)}{dt^2} - 4\pi t^2 f(t) = \lambda f(t) \quad (4)$$

Transform [12]. The FrFT is further explained by following definition.

Note that as  $h \rightarrow 0$  the difference equation following approximates

$$\frac{f(u+h) - 2f(u) + f(u-h)}{h^2} + \frac{2(\cos(2\pi hu) - 1)}{h^2} f(u) = \lambda f(u) \quad (5)$$

When  $h = \frac{1}{\sqrt{N}}$ , the difference equation (4) has periodic coefficients. Therefore, the solutions of the difference equation are periodic and can be jotted down as the eigenvectors of the following matrix [15], denoted by  $S$ :

$$S = \begin{bmatrix} 2 & 1 & 0 & \dots & 0 & 1 \\ 1 & 2\cos(2\pi/N) & 1 & \dots & 0 & 0 \\ 0 & 1 & 2\cos(2\pi/N) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & 1 & 2\cos(2\pi(n-1)/N) \end{bmatrix}$$

or, the difference equation can also be written as  $Sf = \lambda f$ . It can also be shown that  $S$  commutes with DFT matrix. Having got an appropriate set of eigenvectors, the DFrFT matrix can now be defined as follows:

$$\mathcal{F}^a = \begin{cases} \sum_{k=0, k \neq N-1}^N u_k e^{-j \frac{\pi k a}{2}} u_k^T, & \text{when } N \text{ even} \\ \sum_{k=0, k \neq N}^N u_k e^{-j \frac{\pi k a}{2}} u_k^T, & \text{when } N \text{ odd} \end{cases}$$

where,  $u_k$  corresponds to the eigenvector of the  $S$  matrix with  $k$  zero-crossings.

## 3. IMAGE CHANGE DETECTION USING DFrFT

Apart from DFrFT which is applied, other techniques which have been used in this paper for obtaining change detection include the Joint Intensity Histogram (JIH) technique and Discrete Cosine Transform (DCT) based change detection. While using DFrFT, we needed to vary the parameter ' $a$ ' to get the results. The following block diagram illustrates the working flow of proposed scheme:

**Step 1: Differencing:** In the initial change detection step, the simple difference image is obtained by algebraically subtracting the first image from the second image. The resultant simple difference image is itself a raw changed image that is to be used for further proceedings.

**Step 2: Transform:** This difference image is then Discrete fractionally Fourier transformed with different fractions in the frequency domain by varying parameter ' $a$ ' = ' $a_r$ ' (for rows) = ' $a_c$ ' (for columns).

**Step 3: Quantizer:** After applying Transform i.e. DFrFT, the equivalent transform matrix is passed through a quantizer,

which selects only appropriate coefficients based upon their values obtained after the quantization process.

**Step 4: Change detected image:** In order to retrieve the change image, IDFrFT is applied on the selected coefficients with parameter 'a' = -a. This task is performed with various values of 'a' to get consequent different change images.

**Step 5: Region marking on the image:** The changes in the image have been detected and then are categorized into regions by defining a particular region size according to the size of images. Hence proper change regions are obtained which are highlighted by drawing red rectangles.

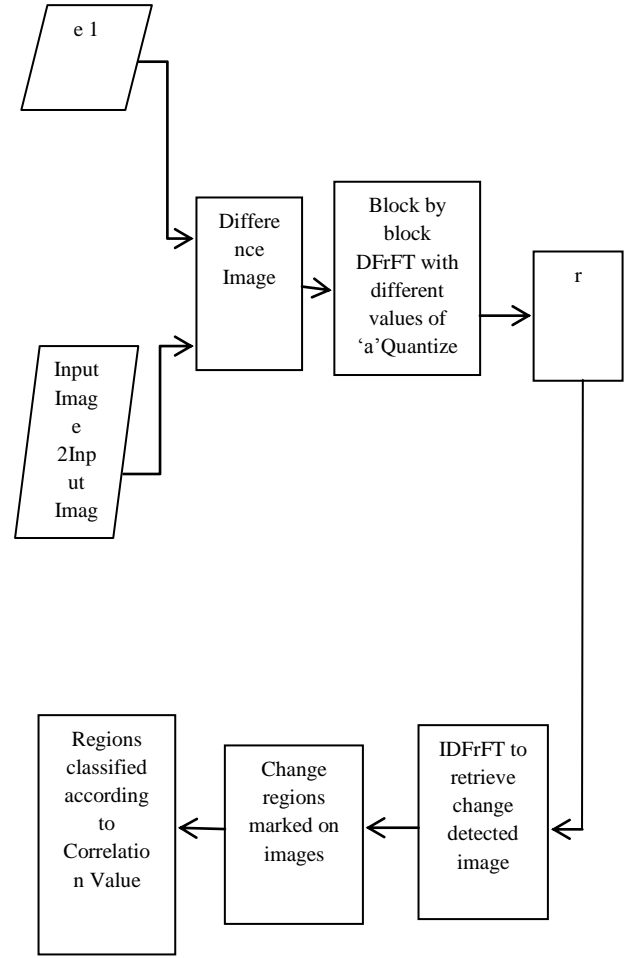
**Step 6: Classification of regions according to the correlation coefficient value:** We adopt gradient correlation checking as an additional filtering process for further discrimination of significant change regions obtained in above processes. A gradient measure of similarity, Se or correlation is defined for the particular region by taking gradient images in the above regions as in [18] and is given by:

$Se = \min (Ex, Ey)$

$$Ex = \frac{\sum (e_{1,x}(x) - \mu_{e_{1,x}})(e_{2,x}(x) - \mu_{e_{2,x}})}{(\max(\sigma_{e_{1,x}}, \sigma_{e_{2,x}}))^2}$$

$$Ey = \frac{\sum (e_{1,y}(y) - \mu_{e_{1,y}})(e_{2,y}(y) - \mu_{e_{2,y}})}{(\max(\sigma_{e_{1,y}}, \sigma_{e_{2,y}}))^2}$$

where  $\mu_{e_{i,k}}$  and  $\sigma_{e_{i,k}}$  represents the average and standard distribution of the gradient image in the kth direction of image in a candidate region and Ex and Ey are the gradient similarity measures of the input images in the candidate regions in the X and Y directions respectively and  $e_{i,k}(k)$  represents the pixel value in the kth direction of input images in candidate regions. The candidate regions are classified into the three classes according to Se: change with high certainty ( $0.0 \leq Se \leq 0.1$ ), change with low certainty ( $0.1 < Se \leq 0.3$ ), and no change ( $0.3 < Se \leq 1.0$ ) [18]. The partial movement of objects, such as waving trees, often occurs between images, and the detection of such movement is undesirable. When such partial translation occurs, a similar gradient pattern is observed in the locality of the corresponding position [18]. Therefore, if we take the maximum of Se while translating one region over the corresponding region in the other image, such regions should show large Se (high correlation) and thus can be rejected. The three classes, high certainty, low certainty and no change, are represented using red, yellow and green squares, respectively.



**Fig 1: Block diagram of image change detection using DFrFT**

## 4. RESULTS AND DISCUSSIONS

In order to find change detection between two images, simulations have been performed for three different set of images. After obtaining the change regions in the images, the correct detected regions ( $C_d$ ), false detected regions ( $F_d$ ) have been categorized and results are compared using two parameters viz. precision and recall.

**Precision:** Precision is the fraction of retrieved instances that are relevant [19]. In other words, precision can be seen as a measure of correctness or quality. Mathematically it is given by:

$$\text{Precision, } P = \frac{C_d}{(C_d + F_d)}, \text{ where}$$

$C_d$  = No. of correctly detected change regions.

$F_d$  = No. of false detected change regions.

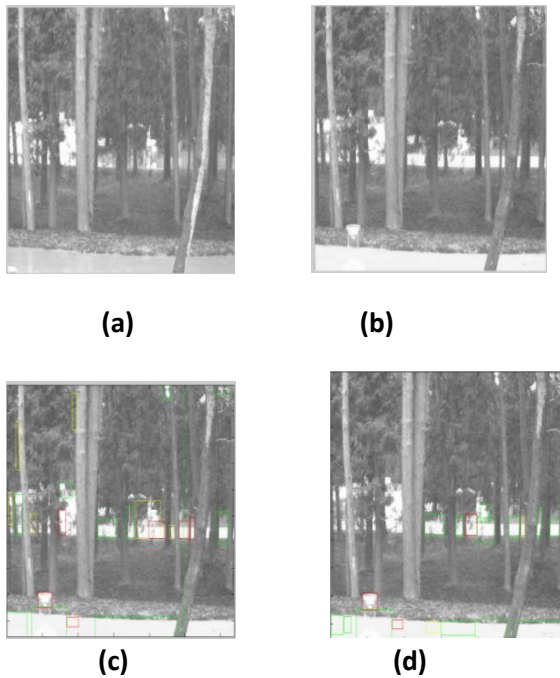
**Recall:** Recall is a measure of completeness or quantity or it is the fraction of relevant instances that are retrieved [19]. Formula for recall is given by:

$$\text{Recall, } R = \frac{C_d}{C_d + M_d}, \text{ where}$$

$C_d$  = No. of correctly detected change regions.

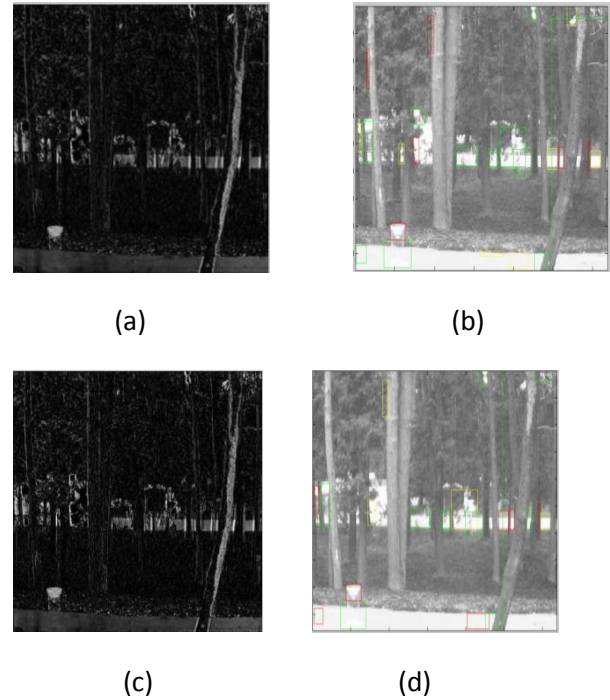
$M_d$  = No. of Missed Regions.

For e.g. suppose a scene change contain 9 objects. If the detection algorithm identifies 7 objects and among them 4 recognized are correct, then the precision value is  $4/7$  and recall value is  $4/9$ .



**FIGURE 2: [(a), (b) Original image sets to be change detected, (c) Change regions obtained using JIH, (d) Change regions using DCT]**

Figures 2(a) and 2(b) provide a typical example in which two images of the same scene do not show simple brightness changes. These images were taken at 7:06 AM and at 11:59 AM, respectively, using a Field Monitoring Server (FMS) camera used as a node of a sensor network [20]. Change Detection between these two images using joint intensity histogram technique with a threshold of  $\pm 30$  [18] and with a region size of area greater than 200 pixels is carried out and the result is shown in Figure 2(c). Figure 2(d) shows the Change Detection carried out using DCT (Discrete Cosine Transform) by using quantization threshold technique and having the same region size. In above figures 2(c) and 2(d), we show the regions classified as main change region (red rectangles), low certainty change regions (yellow rectangle), and no change or non-significant regions (green rectangle).



**FIGURE 3: [(a), (c): Change images obtained using DFrFT with  $a=0.2, 0.9$ , (b), (d): Change regions obtained using DFrFT with  $a=0.2, 0.9$ ]**

The same work is carried out using DFrFT, in which the output change region is obtained for various values of parameter 'a'. The region size boundary limit and the correlation classification values chosen are identical as taken for above two methods. Figure 3(a) and 3(c) shows the change image retrieved after applying IDFrFT and corresponding figures 3(b) and 3(d) shows the final change regions marked on the original image, i.e., figure 2(a). The same procedure is reiterated for three different image sets with varying size.

## 5. CONCLUSION

Image change detection is an important requisite for various applications like video surveillance, remote sensing, medical diagnosis and treatment, civil infrastructure, underwater sensing and driver assistance systems etc. In this paper Image change detection using fractional transform has been proposed. It provides a flexible approach by varying its fractional parameters so as to get the best possible results. Change regions were marked on difference image depending upon a particular size limit that may vary according to image dimensions. Further, a gradient correlation filtering technique was used for categorization of these obtained regions. These regions were classified into different classes depending upon gradient similarity measure value obtained in a particular change region. A global threshold technique for JIH and block by block quantization threshold technique was used for DCT to obtain results. The experiments were performed for three sets of change images and results of change detections were analyzed comparatively using values of precision and recall parameters. For image set (1), the precision and recall values obtained were 0.43 and 0.67 (using JIH [13]), 0.56 and 0.67 (using DCT) and with DFrFT (at parameter 'a' value=0.9), we obtained 0.62 and 0.87. Finally, the results show that image change detection using DFrFT improves the results by 30-80%.

Image sets	Method	T <sub>a</sub> (Total Actual Change Regions)	T <sub>d</sub> (Total Detected Regions)	C <sub>d</sub> (Correctly Detected Regions)	F <sub>d</sub> (False detected Regions)	M <sub>d</sub> (Regions missed)	P (Precision)	R (Recall)
Image set (1) / [638 x 479]	JIH	15	23	10	13	5	0.43	0.67
	DCT	15	18	10	8	5	0.56	0.67
	DFrFT(0.9)	15	21	13	8	2	0.62	0.87
Image set (2) / [500 x 500]	JIH	8	13	2	11	6	0.15	0.25
	DCT	8	9	3	6	5	0.33	0.38
	DFrFT(0.98)	8	10	6	4	2	0.60	0.75
Image set (3) / [241 x 304]	JIH	18	13	8	5	10	0.62	0.44
	DCT	18	15	9	6	9	0.60	0.50
	DFrFT(0.98)	18	20	14	6	4	0.70	0.78

**TABLE 1: Results comparing different change detection techniques**

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