

Inverse Rayleigh Software Reliability Growth Model

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ABSTRACT

A Non Homogenous Poisson Process (NHPP) with its mean value function generated by the cumulative distribution function of inverse Rayleigh distribution is considered. It is modeled to assess the failure phenomenon of a developed software. When the failure data is in the form of number of failures in a given interval of time the model parameters are estimated by the maximum likelihood method. The performance of the model using four data sets is discussed in comparison with existing models.

General Terms:

NHPP- non homogenous poisson process
SRGM- software reliability growth model
MLE- maximum likelihood estimation
MSE- mean square error
IRD- inverse Rayleigh distribution

Keywords:

IRD,MLE,MSE,NHPP,SRGM

1. INTRODUCTION

It is well-known that computers are used in diverse areas for various applications. The growing importance of software dictates that a reliable software is by all means essential. A software itself does not fail unless the faults within the software result in its failure. Generally, software faults are more difficult to handle. All design faults are present from the time the software is installed in the computer. A software fault inherent in a program is not dangerous unless and until it results in a failure of software. Accordingly, the concept of software reliability is rather dependent on the failure of a software and its frequency rather than the unknown number of faults latent in the software. Therefore, the term software reliability may be defined as the probability of failure free functioning of a software rather than the faults contained in it. However we cannot risk out the fact that software reliability depends on the number of faults also. In this regard, theory of probability and hence statistical analysis have become essential in the development of a model that

can be used to evaluate the reliability of real world software systems. Quantifying the software quality in terms of reliability is attempted through the study of software reliability growth models. Software reliability models are statistical models which can be used to make predictions about a software system's failure rate, given the failure history of the system. The models make assumptions about a fault discovery and removal process. These assumptions determine the form of the model and the meaning of the model's parameters. Some recent works in this regard are by Akaike(1974) [1], Yamada *et al*(1986) [15], Huang *et al*(1999) [10], Pham *et al*(1999) [13], Huang *et al*(2000) [11], Kapur *et al*(2002) [3], Huang and Kuo(2002) [6], Pham and Zhang(2003) [18], Yamada *et al*(2003) [20], Yamada and Inoue(2004) [22], Huang(2005) [7], Huang and Lyu(2005) [8], Kapur *et al*(2005) [2], Pham(2005) [5], Quadri *et al*(2006) [16], Huang *et al*(2007) [23], Lan and Leemis(2007) [12]. With this backdrop, we study the modeling of software reliability as a Non Homogenous Poisson Process (NHPP) with mean value function based on inverse Rayleigh distribution. Similar attempts based on Pareto distribution is made by Kantam and Subbarao(2009) [9] and that based on half logistic distribution is given by Srinivasa Rao *et al*(2011) [19]. The genesis and the development of the model with the necessary input about a Non Homogenous Poisson Process are presented in Section 2. Maximum likelihood (ML) estimation of the parameters of the developed software reliability growth model (SRGM) is discussed in Section 3. The proposed SRGM is then compared with other software reliability growth models in Section 4. The concept of cost aspect in developing a software, associated randomness and the optimum release time of a developed software with respect to cost aspect are given in Section 5. Summary and Conclusions are given in Section 6.

2. SRGM AS A NON HOMOGENOUS POISSON PROCESS

Suppose that we are interested in observing the occurrences of a repeatable event over a period of time. The situation relevant here can be the number of times a developed software fails in a given period of testing/operational time. As failures do not occur in a predictable way such a failure process can be identified with a random counting process, generally defined as a count of number

of events that have occurred in a specified interval of time. Let it be denoted by $N(t)$, where t is any non negative real number. $N(t)$ indicates the number of random occurrences in the interval $[0, t]$. A counting process is said to be a Poisson process if the failure has stationary independent increments and the number of failures in any time interval of length s has a Poisson distribution with mean λs given by

$$P(N(t+s) - N(t) = y) = \frac{e^{-\lambda s} (\lambda s)^y}{y!}, y = 0, 1, 2, \dots \quad (1)$$

This mathematical model indicates that the changes in $N(t)$ from one time period to another time period say $[t, t+s]$ depend only on the length of the interval s but not on the extremities $t, t+s$ of the interval. λ is called the failure intensity. In the above equation $E[N(t)] = \lambda t, \forall t$. If we think of a Poisson process whose mean depends on the starting t and also the length of the interval s such a Poisson process can be explained by an equation as

$$P(N(t) = y) = \frac{e^{-m(t)} (m(t))^y}{y!}, y = 0, 1, 2, \dots \quad (2)$$

In this equation $m(t)$ is a positive valued, non decreasing, continuous function of t , generally tending to a finite limit 'a' as $t \rightarrow \infty$. $m(t)$ is called the mean value function and its derivative with respect to t is the intensity function $\lambda(t)$. Equation (2) is called a Non Homogenous Poisson Process. If a software system when put to use fails with probability $F(t)$ before time t , if 'a' stands for the unknown eventual number of failures that it is likely to experience, then the average number of failures expected to be experienced before time t is $aF(t)$. Hence $aF(t)$ can be taken as the mean value function of an NHPP. In the theory of probability, $F(t)$ is called the cumulative distribution function (CDF) of a continuous non negative valued random variable. Thus an NHPP designed to study the failure process of a software can be constructed as a Poisson process with mean value function based on the cumulative distribution function of a continuous positive valued random variable. Since a number of distributions is available in statistical science, one can think of a number of NHPP models each based on a CDF. The first and foremost of such models is due to Goel and Okumoto(1979) [4] which is based on the well-known exponential distribution. Later many such models have been suggested and studied by various researchers that can be found in Wood(1996) [21], Pham(2000) [17] and Huang *et al* (2007) [23] and references therein. The probability density function (pdf) of inverse Rayleigh distribution (IRD) with scale parameter b is

$$f(x) = \frac{2b}{x^3} e^{(-\frac{b}{x^2})}, \quad x > 0, b > 0 \quad (3)$$

Its cumulative distribution function (cdf) is given by

$$F(x) = e^{(-\frac{b}{x^2})}, \quad x > 0, b > 0 \quad (4)$$

We consider an NHPP with the mean value function given in terms of the CDF of inverse Rayleigh distribution (IRD) given as

$$m(t) = e^{-\frac{b}{t^2}} \times a \quad a > 0, t > 0 \quad (5)$$

It can be seen that $m(t)$ tends to 'a' as $t \rightarrow \infty$ and $m(t)$ is a positive valued non decreasing function of t .

The reliability in the software system with the above modeling is the probability of no failures in the time interval $[0, t]$ and is given by

$$R(t) = P\{N(t) = 0\} = e^{-m(t)} \quad (6)$$

In general, the reliability $R(x/t)$ the probability that there are no failures in the interval $[t, t+x]$ is given by

$$R(x/t) = P\{N(t+x) - N(t) = 0\} = e^{-[m(t+x) - m(t)]} \quad (7)$$

Generally, the expression given in Equation (7) is called software reliability based on NHPP and this is also called as software reliability growth model (SRGM). If the mean value function is completely specified with its parameters we can have the value of the software reliability at any time of our choice. If the parameters of the mean value function are not known they need to be estimated by a software failure data in the form of failure counts which can be used to get an estimate of the software reliability in order to assess the software quality. We present the ML estimation of parameters in an NHPP based on inverse Rayleigh distribution in section 3.

3. ML ESTIMATION

Suppose that software failure data are given in the form of (y_i, t_i) $i=1, 2, \dots, n$ where y_i is the number of failures observed in the interval $[0, t_i]$ $i=1, 2, \dots, n$ with $0 < t_1 < t_2 < \dots < t_n$. Such a data is called failure count data. The log likelihood function to get the estimates of parameters of the NHPP shall be of the form

$$LLF = \sum_{i=1}^n (y_i - y_{i-1}) \log[m(t_i) - m(t_{i-1})] - m(t_n) \quad (8)$$

$$\frac{\partial}{\partial a} (LLF) = 0, \Rightarrow a = y_n \times e^{\frac{b}{t_n^2}} \quad (9)$$

$$\frac{\partial}{\partial b} (LLF) = 0, \Rightarrow \sum_{i=1}^n \frac{t_{i-1}^2 e^{\frac{-b}{t_{i-1}^2}} - t_i^2 e^{\frac{-b}{t_i^2}}}{e^{\frac{-b}{t_i^2}} - e^{\frac{-b}{t_{i-1}^2}}} (y_i - y_{i-1}) - y_n t_n^2 = 0 \quad (10)$$

Solving the equations (9) and (10) simultaneously for a given sample data we get the ML estimates of a and b . However, these two equations admit only iterative solutions. The ML estimates for four different data sets published in Wood(1996) [21] given in Table 3.1 using the equations (9) and (10) are given in the first part of Table 4.1.

Table 3.1

	Data Set 1		Data Set 2		Data Set 3		Data Set 4	
Test Week	CPU hours	defects found	CPU hours	defects found	CPU hours	defects found	CPU hours	defects found
1	519	16	384	13	162	6	254	1
2	968	24	1186	18	499	9	788	3
3	1,430	27	1471	26	715	13	1054	8
4	1,893	33	2236	34	1137	20	1393	9
5	2,490	41	2772	40	1799	28	2216	11
6	3,058	49	2967	48	2438	40	2880	16
7	3,625	54	3812	61	2818	48	3593	19
8	4,422	58	4880	75	3574	54	4281	25
9	5,218	69	6104	84	4234	57	5180	27
10	5,823	75	6634	89	4680	59	6003	29
11	6,539	81	7229	95	4955	60	7621	32
12	7,083	86	8072	100	5053	61	8783	32
13	7,487	90	8484	104	—	—	9604	36
14	7,846	93	8847	110	—	—	10064	38
15	8,205	96	9253	112	—	—	10560	39
16	8,564	98	9712	114	—	—	11008	39
17	8,923	99	10083	117	—	—	11237	41
18	9,282	100	10174	118	—	—	11243	42
19	9,641	100	10272	120	—	—	11305	42
20	10,000	100	—	—	—	—	—	—

4. COMPARATIVE STUDY

The present SRGM based on NHPP model can be compared with other models also w.r.t some criteria of preference. The standard models that are considered here are those based on the

- Exponential cumulative distribution function.
- Half logistic cumulative distribution function.
- Gamma cumulative distribution function with shape parameter 2.

in succession. The first NHPP is called Goel -Okumoto model (1979) [4]. The second NHPP is software reliability growth model based on half logistic model(2011) [19]. The third NHPP is called Yamada S-shaped software reliability growth model (1983) [14]. For a ready reference,given below are the mean value functions and associated results of differentiation useful to get the ML estimates of the parameters in our proposed model and the three competitive models.

- Inverse Rayleigh Distribution(The proposed model):

$$\sum_{i=1}^n \frac{t_i^2 e^{-\frac{b}{t_i^2}} - t_{i-1}^2 e^{-\frac{b}{t_{i-1}^2}}}{e^{-\frac{b}{t_i^2}} - e^{-\frac{b}{t_{i-1}^2}}} (y_i - y_{i-1}) + y_n t_n^2 = 0$$

$$a = y_n \times e^{\frac{b}{t_n^2}}$$

- Exponential Distribution(Goel-Okumoto(1979) [4] Model):

$$\sum_{i=1}^n \frac{t_i e^{-bt_i} - t_{i-1} e^{-bt_{i-1}}}{e^{-bt_{i-1}} - e^{-bt_i}} (y_i - y_{i-1}) - \frac{y_n t_n e^{-bt_n}}{1 - e^{-bt_n}} = 0$$

$$a = \frac{y_n}{1 - e^{-bt_n}}$$

- Half Logistic Distribution:

$$\sum_{i=1}^n \frac{\frac{t_i e^{-bt_i}}{(1+e^{-bt_i})^2} - \frac{t_{i-1} e^{-bt_{i-1}}}{(1+e^{-bt_{i-1}})^2}}{\frac{e^{-bt_i}}{(1+e^{-bt_i})^2} - \frac{e^{-bt_{i-1}}}{(1+e^{-bt_{i-1}})^2}} (y_i - y_{i-1}) - \frac{t_n e^{-bt_n}}{(1+e^{-bt_n})^2} = 0$$

$$a = y_n \times \frac{1+e^{-bt_n}}{1 - e^{-bt_n}}$$

- Gamma Distribution (Yamada(1983) [14] Model):

$$\sum_{i=1}^n \frac{t_i^2 e^{-bt_i} - t_{i-1}^2 e^{-bt_{i-1}}}{\hat{a}[(1+bt_{i-1})e^{-bt_{i-1}} - (1-bt_i)e^{-bt_i}]} (y_i - y_{i-1}) - \hat{a} t_n^2 e^{-bt_n} = 0$$

$$y_n = a[1 - (1 + bt_n)e^{-bt_n}]$$

Now, we adopt calculation of mean square error(MSE) for model comparison. The formula is defined as

$$MSE = \frac{\sum_{i=1}^n (y_i - \hat{m}(t_i))^2}{n - N}$$
 where $\hat{m}(t)$ stands for MLE of $m(t)$. For four sets of Wood(1996), the MLE of the parameters and the estimators of the mean value function are computed and thereby the values of MSE for various models. The results are given in the table 4.1.

Table 4.1 Estimated values of parameters and MSE			
Inverse Rayleigh distribution			
	a	b	MSE
DS 1	107.0339	27.1805	102.4486
DS 2	156.6663	96.2521	700.3687
DS 3	68.21619	16.1002	20.2086
DS 4	47.7392	46.2382	8.4156
Exponential distribution			
	a	b	MSE
DS 1	104.4582	0.1577	91.5324
DS 2	122.8602	0.1979	208.8905
DS 3	61.1117	0.5253	504.8928
DS 4	43.7504	0.1694	157.2457
Half Logistic distribution			
	a	b	MSE
DS 1	101.8768	0.2339	71.3439
DS 2	122.8363	0.2342	196.5170
DS 3	63.2061	0.3358	103.4418
DS 4	43.0577	0.2309	42.0143
Gamma distribution			
	a	b	MSE
DS 1	299.6177	0.0595	914.2080
DS 2	126.3762	0.2492	15.7047
DS 3	216.8435	0.0872	201.9281
DS 4	45.8767	0.2157	1.2239

5. OPTIMAL RELEASE POLICY

The cost of developing software leads to considerable expenses in a software system development. The quality of a software system usually depends upon the length of testing time. The more

the testing time the more reliable the software is. However, the total cost of software development is also expected to increase. On the other hand, if the testing time is too short, though the cost of software development would be reduced we cannot avoid the customer's risk of receiving unreliable software which in turn leads to increase in cost during the operational phase. Testing is an efficient way to remove faults in software products but testing of all possible executable paths in a general program is impractical. To determine when to stop testing or when to release the software to customers keeping the expected total software cost at a minimum subject to warranty and risk is considered as an optimal release policy.

A cost model is essential to define important software cost factors. It should help software developers in scheduling of resources for prompt delivery. Moreover with a reasonably sufficient reliability the model should contribute to decide an appropriate release time of the software. With these objectives several software cost models are suggested (Pham(2000) [17], Chapter 6). In this section a software cost model with risk factor as discussed in Pham(2000) [17] is adapted and the model is presented in the following lines for a ready reference. A software cost generally consists of the following components.

- (i) cost to perform testing
- (ii) cost incurred in removing errors during testing phase
- (iii) risk cost due to software failure.

Testing cost is denoted by $C_1 t$, where t is the total test time. C_1 is software test cost per unit time. If $N(t)$ stands for number of errors detected by time t , expected time to remove all these errors is given by

$$E\left(\sum_{i=1}^{N(t)} Y_i\right) = E[N(t)]E[Y_i] = m(t)\mu_y \quad (11)$$

where Y_i is time to remove the i^{th} error during testing phase, $m(t)$ is expected number of errors detected by time t , μ_y is expected time to remove an error during testing phase also called $E(Y)$. Therefore the expected cost to remove all errors is given by $C_2 m(t)\mu_y$ where C_2 is cost of removing each error per unit time during testing. The risk cost due to software failure, after releasing the software is

$$E(t) = C_3[1 - R(x/t)] \quad (12)$$

where C_3 is cost due to software failure and $R(x/t)$ is survival probability of the software by x units of time given it is tested for t units of time. Therefore the total expected cost of software is given by

$$E(t) = C_1 t + C_2 m(t)\mu_y + C_3[1 - R(x/t)] \quad (13)$$

The value of t that minimizes the expected total cost in Equation (13) is to be calculated. Such an optimal value of t is called optimal release time. In the expression for $m(t)$ in Equation (13) is taken and the mean value function as given by IRD and t has to be solved. The formula for such a t has to be compared with the value of t for a similar NHPP model say Goel & Okumoto(1979) [4], half logistic model (2011) [19], Yamada(1983) [14] etc. The expected cost function given in equation(13) will show an increasing trend and falls down at a certain time and then increases from there. The time instant at which the change in the trend is observed is taken as the optimal time at which the testing is to be stopped and the product is ready for release. This methodology of locating optimum release time is explained with the data set given in Table 5.1.

Table 5.1 Cumulative number of defects given in Pham(2000) [17]	
Test week	cumulative defects found
1	27
2	43
3	54
4	64
5	75
6	82
7	84
8	89
9	92
10	93
11	97
12	104
13	106
14	111
15	116
16	122
17	122
18	127
19	128
20	129
21	131
22	132
23	134
24	135
25	136

For the above data, the parametric values of IRD are $a=136.8905$ and $b=13.2174$. After estimating these values, the goodness of fit for 25 observations showed that $R=0.8057$. For the above data containing count of cumulative failures, let us start at an arbitrary choice of cumulative failures, say, let us note down the time by which 100 cumulative failures are experienced. In the present example it is.. "97 cumulative failures are observed within 11 weeks". From that time onwards using the data on time t_i , cumulative number of failures y_i , the estimate of mean value function with the help of MLEs of the parameters which are given in Table 5.2, is calculated. For the sake of explanation let us take the specified costs C_1, C_2, C_3 as $C_1 = 25, C_2 = 200, C_3 = 7000$, the choice μ_y be kept at $\mu_y = 0.1$ (as considered by Pham (2000) [17]). These specifications would help us to get the values of expected total software cost as given by Equation (13). For various times and cumulative failures of the data set, our chosen time is "11th week onwards". Therefore from 11th week onwards in the data set at each time point $E(t)$ can be calculated. These are given in Table 5.3, which searches for a trend in $E(t)$ from a rise to a fall and a rise after 11th week onwards say 11.5 etc. It shows that $E(t)$ gives the desired trend of rise-fall-rise at 15.5. Therefore, to release the software after 15th week before 16th week based on IRD is suggest. The same data based on Goel-Okumoto model suggest to release after 20th week as worked out in Pham(2000) [17]. Based on half logistic model suggest to release after 17th week as worked out in Srinivas *et al* (2011) [19] This example also indicates that IRD based NHPP suggests an earlier release than Goel-Okumoto and half logistic models at an optimal expected cost.

Table 5.2 Parametric estimates of testing times		
Test time T	\hat{a}	\hat{b}
11	105.7758	10.4800
12	118.2575	18.5001
13	118.2696	18.5101
14	125.9659	24.7903
15	133.3727	31.4005
16	143.0224	40.6990
17	140.4543	40.7090
18	145.7193	44.5484
19	144.8155	44.5583
20	144.2046	44.5683
21	144.9345	44.5784
22	144.7382	44.5883
23	146.4692	47.0679
24	146.5202	47.1679
25	146.8135	47.8178

Table 5.3 Expected total cost at testing times	
Test time t	E(t)
11.5	2681.4664
12.5	2638.8900
13.5	2614.4390
14.5	2602.6788
15.5*	2599.9557
16.5	2603.7715
17.5	2612.3903
18.5	2624.5862
19.5	2639.4804
20.5	2656.4342
21.5	2674.9769
22.5	2694.7579
23.5	2715.5124
24.5	2737.0389

6. CONCLUSIONS

The well known inverse Rayleigh distribution of the statistical science to develop a SRGM through NHPP is considered. Its suitability and preferability over three reliability growth models is exemplified with the help of four live data sets. The delay in releasing a software product whose failure phenomenon is approximated by our model, can be reduced in comparison with three competitive models Goel-Okumoto(1979) [4], half logistic (2011) [19] and Yamada(1983)[14].

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