Stochastic modeling of a Single-Unit Repairable System with Preventive Maintenance under Warranty

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ABSTRACT

In this paper a stochastic model for a single unit repairable system with Preventive Maintenance (PM) under warranty is analyzed in details by using supplementary variable technique.The cost of repair during warranty is born by the manufacturers, but warranty does not apply to product failure due to user-induced damage such as cracked screen or cracked casing, accident, misuse, physical damage, damage due to liquid and unauthorized modifications etc. within warranty period. Unit goes under PM during warranty and works as new after PM. There is a single server who always remains with the system. The time to failure of the system follows negative exponential distribution while PM and repair time distributions are taken as arbitrary. The expressions for some economic measures such as reliability, mean time to system failure (MTSF) and availability have been derived.A particular case is considered to highlight the behaviour of reliability and profit function.

Keywords: Stochastic model, Single-Unit System, Warranty, Preventive Maintenance and Reliability.

1. INTRODUCTION

Single-unit systems have been widely studied in the literatue of reliability due to their frequent use in mordern bussiness and industries. Many researchers including Arekar et al. [1], Kadyan et al. [3], Kadyan and Promila [4], Kharoufeh J.P. et al. [5], Malik et al. [6], Proctor and Singh [7], Shakuntla et al. [8] and Uematsu and Nishida [9] have analysed single-unit sysytems under a common assumption that the unit works continuously till failure without undergoing PM.

The continued operation of the systems may reduce performance and reliability of the system.Therefore, PM of the unit is necessary after a specific period of time at any stage of operation to improve the reliability and availability of the system because the cost to repair the system after its failure is greater than the cost of maintaining the system before its failure. Jin et al. [10] have studied reliability models with PM and without considering any warranty and service contract. But, warranty acts as an insurance in the event of an early failure of the product. Better warranty terms serve as an indicator of the reliability of the product and may increase sales. However, the concept of single-unit system with PM under warranty has not appeared in the literature so far

Thus, in view of the above observations here we analyzed a single unit repairable system with PM under warranty by using supplementary variable technique.The cost of repair during warranty is born by the manufacturers, but warranty does not apply to product failure due to user-induced damage such as cracked screen or cracked casing, accident, misuse, physical damage, damage due to liquid and unauthorized modifications etc. within warranty period. Unit goes under PM during warranty and works as new after PM. There is a single server who always remains with the system. The time to failure of the system follows negative exponential distribution while PM and repair time distributions are taken as arbitrary. The expressions for some economic measures such as reliability, MTSF and availability have been derived. A particular case is considered to highlight the behaviour of reliability and profit function.

2. MODEL ASSUMPTIONS

(1) The system has a single unit
(2) There is single server, which is always available with the system.
(3) The repair of the unit within warranty is born by the manufacturer.
(4) Warranty does not apply to product failure due to user-induced damage within warranty period.
(5) PM is made during warranty period.
(6) The unit works as new after repair and PM.
(7) The distribution of failure time is taken as negative exponential while the PM and repair time are considered as arbitrary.
(8) Switching is perfect.

3. MODEL DEVELOPMENT

3.1. State-Specification

S_0 / S_1 The unit is operative under warranty period/ beyond warranty period.

S_3 / S_4 The unit is in failed state under warranty period/ beyond warranty period.

S_2 The unit is under PM.

3.2. Notations

λ / λ_1 Constant failure rate of the unit within warranty period/beyond warranty period.

α Constant rate of completion of warranty period.

λ_m Transition rate which transits the operative unit under PM.

$\mu(x), s(x) / \mu_1(x), s_1(x)$ Repair rate of the unit and probability density function, for the elapsed repair time 'x' in warranty period/ beyond warranty period.

$\mu_2(y), s_2(y)$ PM rate of the unit and probability density function, for the elapsed PM time 'y'.

$p_0(t) / p_1(t)$ The Probability that at time t the system is in good state in warranty period/ beyond warranty period.

$p_3(x,t)\Delta / p_4(x,t)\Delta$ The Probability that at time t the system is in failed state in warranty period/ beyond warranty period, the repair time lies in the interval (x,x+Δ).

$p_2(y,t)\Delta$ The Probability that at time t the system is under PM, the elapsed PM time is 'y'.

$p(s)$ Laplace transform of function $p(t)$

$s_2(y)$ $\mu_2(y)e^{\left[-\int_0^y \mu_2(y)dy\right]}$

$s_1(x)$ $\mu_1(x)e^{\left[-\int_0^x \mu_1(x)dx\right]}$

$s(x)$ $\mu(x)e^{\left[-\int_0^x \mu(x)dx\right]}$

$\int$ Definite integral from 0 to ∞

Using the probabilistic arguments and limiting transitions, we have the following difference-differential equations (Cox D.R. [2]):

$$\left[\frac{\partial}{\partial t}+\lambda+\alpha+\lambda_m\right]p_0(t)=\int\mu(x)p_3(x,t)dx+\int\mu_2(y)p_2(y,t)dy \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\lambda_1\right]p_1(t)=\alpha p_0(t)+\int\mu_1(x)p_4(x,t)dx \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x)\right]p_3(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_2(y)\right]p_2(y,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]p_4(x,t)=0 \quad (5)$$

The boundary and initial conditions to be satisfied are given below

Boundary conditions

$$p_3(0,t)=\lambda p_0(t) \quad (6)$$

$$p_2(0,t)=\lambda_m p_0(t) \quad (7)$$

$$p_4(0,t)=\lambda_1 p_1(t) \quad (8)$$

Initial conditions

$$p_i(0)=1;\ \text{when } i=0$$

$$p_i(0)=0;\ \text{when } i\neq 0 \quad (9)$$

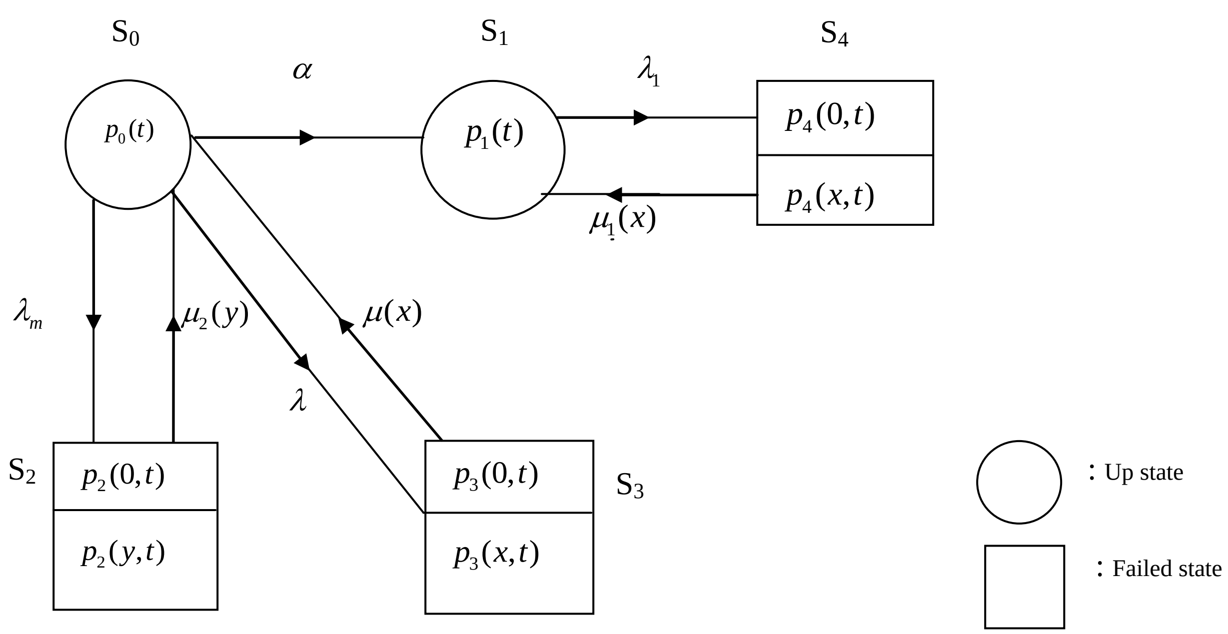

Fig 1: Transition diagram of the model

4. MODEL ANALYSIS

4.1. Solution of the equations

Taking Laplace transforms of equations (1)-(8) and using (9) we obtain

$$[s+\lambda+\alpha+\lambda_m]\,p_0(s)=1+\int \mu(x)p_3(x,s)dx+\int \mu_2(y)p_2(y,s)dy \quad (10)$$

$$[s+\lambda_1]\,p_1(s)=\alpha p_0(s)+\int \mu_1(x)p_4(x,s)dx \quad (11)$$

$$\left[\frac{\partial}{\partial x}+s+\mu(x)\right]p_3(x,s)=0 \quad (12)$$

$$\left[\frac{\partial}{\partial y}+s+\mu_2(y)\right]p_2(y,s)=0 \quad (13)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_1(x)\right]p_4(x,s)=0 \quad (14)$$

$$p_3(0,s)=\lambda p_0(s) \quad (15)$$

$$p_2(0,s)=\lambda_m p_0(s) \quad (16)$$

$$p_4(0,s)=\lambda_1 p_1(s) \quad (17)$$

Integrating equation (12) and further using (15) we get

$$p_3(x,s)=p_3(0,s)e^{\left[-sx-\int_0^x \mu(x)dx\right]} \quad (18)$$

Similarly integrating equation (13) and further using (16) we get

$$p_2(y,s)=p_2(0,s)e^{\left[-sy-\int_0^y \mu_2(y)dy\right]} \quad (19)$$

Similarly integrating equation (14) and further using (17) we get

$$p_4(x,s)=p_4(0,s)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]} \quad (20)$$

Using equations (16) and (19), equation (10) yields

$$[s+\lambda+\alpha+\lambda_m]\,p_0(s)=1+p_3(0,s)\int \mu(x)p_3(x,s)e^{\left[-sx-\int_0^x \mu(x)dx\right]}dx+\int p_2(0,s)e^{\left[-sy-\int_0^x \mu_2(y)dy\right]}\mu_2(y)dy=1+\lambda p_0(s)S(s)+\lambda_m p_0(s)S_2(s) \quad (21)$$

$$p_0(s)=\frac{1}{T(s)} \quad (22)$$

where $T(s)=s+\alpha+\lambda(1-S(s))+\lambda_m(1-S_2(s))$ (23)

Using equations (16)-(17) and (19)-(20), equation (11) yields

$$[s+\lambda_1]\,p_1(s)=\alpha p_0(s)+p_4(0,s)\int \mu_1(x)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]}dx=\alpha p_0(s)+\lambda_1 p_1(s)S_1(s) \quad (24)$$

$$p_1(s)=\frac{A(s)}{T(s)} \quad (25)$$

where $A(s)=\dfrac{\alpha}{(s+\lambda_1-\lambda_1 S_1(s))}$ (26)

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$p_3(s)=\int p_3(s,x)dx=\lambda p_0(s)\frac{(1-S(s))}{s}$$

$$p_3(s)=\frac{\lambda B(s)}{T(s)} \quad (27)$$

where $B(s)=\dfrac{(1-S(s))}{s}$ (28)

Similarly $p_2(s)=\int p_2(s,y)dy=\lambda_m p_1(s)\dfrac{(1-S_2(s))}{s}$

$$p_2(s)=\frac{(\lambda_m A(s)C(s))}{T(s)} \quad (29)$$

where $C(s)=\dfrac{(1-S_2(s))}{s}$ (30)

Similarly $p_4(s)=\int p_4(s,x)dx=\lambda_1 p_1(s)\dfrac{(1-S_1(s))}{s}$

$$p_4(s)=\frac{(\lambda_1 A(s)D(s))}{T(s)} \quad (31)$$

where $D(s)=\dfrac{(1-S_1(s))}{s}$ (32)

It is worth noticing that

$$p_0(s)+p_1(s)+p_2(s)+p_3(s)+p_4(s)=\frac{1}{s} \quad (33)$$

4.2. Evaluation of Laplace transforms of up anddown state probabilities

The Laplace transforms of the probabilities that the system is in up (i.e. good) and down (i.e. failed) state at time "*t*" are as follows

$Av(s)$ or $P_{up}(s)=p_0(s)+p_1(s)$

$$Av(s)=\frac{(1+A(s))}{T(s)} \quad (34)$$

$$P_{down}(s)=p_2(s)+p_3(s)+p_4(s)$$

$$P_{down}(s)=\frac{(\lambda B(s)+\lambda_1 A(s)D(s)+\lambda_m A(s)C(s))}{T(s)} \quad (35)$$

4.3. Steady-State Probabilities

Using Abel's Lemma in Laplace transforms, viz.

$\lim_{s\to 0} s[Z(s)]=\lim_{n\to\infty}[Z(t)]=Z(say),$

Provided the limit on the right hand side exists, the following time independent probabilities have been obtained.

$$Av=\frac{1}{(1-\lambda_1 S_1^{'}(0))} \quad (36)$$

$$P_{down}=\frac{(-\lambda_1 S_1^{'}(0))}{(1-\lambda_1 S_1^{'}(0))} \quad (37)$$

4.4. Reliability Indices

In order to obtain system reliability, consider repair rates $(i.e., \mu(x), \mu_1(x))$ and PM rate $\mu_2(y)$ equal to zero. Using the method similar to that in section 3, the differential–difference equations are:

$$\left[\frac{\partial}{\partial t}+\lambda+\alpha+\lambda_m\right]p_0(t)=0 \quad (38)$$

$$\left[\frac{\partial}{\partial t}+\lambda_1\right]p_1(t)=\alpha p_0(t) \quad (39)$$

Theorem 1. The reliability of the system is given by

$$R(t)=e^{-(\lambda+\alpha+\lambda_m)t}\left[\frac{(\lambda-\lambda_1+\lambda_m)}{(\lambda-\lambda_1+\lambda_m+\alpha)}\right]$$

$$e^{-(\lambda_1)t}\left[\frac{\alpha}{(\lambda-\lambda_1+\lambda_m+\alpha)}\right]$$

Proof. Taking Laplace transforms of (38) and (39) and using (9) we get

$$[s+\lambda+\alpha+\lambda_m]\,p_0(s)=1$$

$$[s+\lambda_1]\,p_1(s)=\alpha p_0(s)$$

Using the initial conditions, the solution can be written as

$$p_0(s)=\frac{1}{(s+\alpha+\lambda+\lambda_m)}$$

$$p_1(s)=\frac{\alpha}{(s+\alpha+\lambda+\lambda_m)(s+\lambda_1)}$$

$$R(s)=p_0(s)+p_1(s)$$

$$=\frac{1}{(s+\alpha+\lambda+\lambda_m)}+\frac{\alpha}{(s+\alpha+\lambda+\lambda_m)(s+\lambda_1)}$$

Taking inverse Laplace transform, we get

$$R(t)=e^{-(\lambda+\alpha+\lambda_m)t}\left[\frac{(\lambda+\lambda_m-\lambda_1)}{(\lambda+\lambda_m-\lambda_1+\alpha)}\right]+\left[\frac{\alpha}{(\lambda+\lambda_m-\lambda_1+\alpha)}\right]e^{-(\lambda_1)t}$$

Corollary 1. The mean time to system failure (MTSF) is:

$$MTSF=\left[\frac{(\lambda-\lambda_1+\lambda_m)}{(\lambda-\lambda_1+\lambda_m+\alpha)(\lambda+\alpha+\lambda_m)}\right]$$

$$+\left[\frac{\alpha}{(\lambda-\lambda_1+\lambda_m+\alpha)(\lambda_1)}\right]$$

Proof. Calculating $MTSF=\int_0^\infty R(t)dt$

$$MTSF=\int_0^\infty\left\{e^{-(\lambda+\alpha+\lambda_m)t}\left(\frac{(\lambda+\lambda_m-\lambda_1)}{(\lambda+\lambda_m-\lambda_1+\alpha)}\right)+\left(\frac{\alpha}{(\lambda+\lambda_m-\lambda_1+\alpha)}\right)e^{-(\lambda_1)t}\right\}dt$$

$$MTSF=\left[\frac{(\lambda-\lambda_1+\lambda_m)}{(\lambda-\lambda_1+\lambda_m+\alpha)(\lambda+\alpha+\lambda_m)}\right]$$

$$+\left[\frac{\alpha}{(\lambda-\lambda_1+\lambda_m+\alpha)(\lambda_1)}\right]$$

5. THE WARRANTY COST FOR THE MANUFACTURER AND COST FOR THE USER

Suppose that the useful life of the system is L and the warranty period [0, W) includes the second and third state, in this case we compute the warranty cost for the manufacturer and cost for the user as follows.

(1) The warranty cost for the manufacturer can be represented by

$$C_M=C_R\int_0^W(\,failure\quad rate\quad for\quad the\quad third\quad state)dt$$

$$+C_{PM}\int_0^W(\,failure\quad rate\quad for\quad the\quad second\quad state)dt$$

$$C_M=C_R\int_0^W\lambda dt+C_{PM}\int_0^W\lambda_m dt=W\left[C_{PM}\lambda_m+C_R\lambda\right] \tag{40}$$

where C_M is the cost for the manufacturer, C_R is the repair cost and C_{PM} is the PM cost.

(2) Cost for the user can be represented by

$$C_B=C_R\int_W^L(\,failure\quad rate\quad for\quad the\quad fourth\quad state)dt$$

$$C_B=C_R\int_W^L\lambda_1 dt=C_R\lambda_1(L-W) \tag{41}$$

where C_B is the cost for the user.

6. SPECIAL CASES

6.1. Availability

When repair follows exponential time distribution

Setting $S(s)=\frac{\mu}{(s+\mu)}, S_1(s)=\frac{\mu_1}{(s+\mu_1)}$ and $S_2(s)=\frac{\mu_2}{(s+\mu_2)}$

where μ and μ_1 are constant repair rates and μ_2 is constant PM rate. Putting these values in equations (22)-(26) we get

$$p_0(s)=\frac{1}{I(s)} \tag{42}$$

where

$$I(s)=\frac{\left(s^3+s^2(\lambda+\alpha+\mu+\lambda_m+\mu_2)+s(\lambda_m\mu+\mu\alpha+\mu_2\alpha+\mu\mu_2+\alpha\mu_2)+\alpha\mu\mu_2\right)}{(s+\mu)(s+\mu_2)} \tag{43}$$

$$p_1(s)=\frac{E(s)}{I(s)} \tag{44}$$

where $$E(s)=\left[\frac{\alpha(s+\mu_1)}{s(s+\lambda_1+\mu_1)}\right] \tag{45}$$

$Av(s)$ or $P_{up}(s)=p_0(s)+p_1(s)$

$$=\left[\frac{\left(s^4+b_3s^3+b_2s^2+b_1s+b_0\right)}{s(s+\lambda_1+\mu_1)\left(s^3+s^2a_2+sa_1+a_0\right)}\right] \tag{46}$$

Where $b_3=(\lambda_1+\mu+\alpha+\mu_1+\mu_2)$,

$b_2=(\lambda_1\mu+\mu\alpha+\alpha\mu_1+\mu_1\mu_2+\mu\mu_2+\mu\mu_1+\lambda_1\mu_2+\mu_2\alpha)$,

$b_1=(\mu\alpha\mu_1+\alpha\mu_1\mu_2+\mu\mu_1\mu_2+\lambda_1\mu\mu_2+\mu_2\alpha\mu)$

and $b_0=(\alpha\mu\mu_1\mu_2)$

and $a_2=(\lambda_m+\mu+\mu_2+\alpha+\lambda)$

$a_1=(\lambda_m\mu+\mu\alpha+\mu\mu_2+\alpha\mu_2+\lambda\mu_2)$ and $a_0=(\alpha\mu\mu_2)$

Taking inverse Laplace transforms of equations (46) we get

$$
\begin{aligned}
Av(t) &= \frac{-\alpha\mu\mu_1\mu_2}{(\lambda_1+\mu_1)z_1z_2z_3} \\
&+\left\{\frac{(\lambda_1+\mu_1)^4 - b_3(\lambda_1+\mu_1)^3 + b_2(\lambda_1+\mu_1)^2 - b_1(\lambda_1+\mu_1) + b_0}{(\lambda_1+\mu_1)(\lambda_1+\mu_1+z_1)(\lambda_1+\mu_1+z_2)(\lambda_1+\mu_1+z_3)}\right\} \\
&\times e^{-(\lambda_1+\mu_1)t} \\
&+\left\{\frac{\left(z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0\right)}{z_1(\lambda_1+\mu_1+z_1)(z_1-z_2)(z_1-z_3)}\right\}e^{z_1t} \\
&-\left\{\frac{\left(z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0\right)}{z_2(\lambda_1+\mu_1+z_2)(z_1-z_2)(z_2-z_3)}\right\}e^{z_2t} \\
&+\left\{\frac{\left(z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0\right)}{z_3(\lambda_1+\mu_1+z_3)(z_1-z_3)(z_2-z_3)}\right\}e^{z_3t}
\end{aligned} \tag{47}
$$

z_1, z_2 and z_3 are three roots of the equation $\left(s^3 + s^2a_2 + sa_1 + a_0\right)$

6.2. Cost-benefit analysis of the user

If K_1 is revenue cost per unit time and expected profit H (t) during the interval (0, t] is given by

$$
H(t) = K_1\int_0^t Av(t)dt - C_B
$$

$$
= K_1\left\{
\begin{aligned}
&\frac{-\alpha\mu\mu_1\mu_2 t}{(\lambda_1+\mu_1)z_1z_2z_3} \\
&+\left\{\frac{(\lambda_1+\mu_1)^4 - b_3(\lambda_1+\mu_1)^3 + b_2(\lambda_1+\mu_1)^2 - b_1(\lambda_1+\mu_1) + b_0}{(\lambda_1+\mu_1)^2(\lambda_1+\mu_1+z_1)(\lambda_1+\mu_1+z_2)(\lambda_1+\mu_1+z_3)}\right\} \\
&\times\left(1-e^{-(\lambda_1+\mu_1)t}\right) \\
&+\left\{\frac{\left(z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0\right)}{z_1^2(\lambda_1+\mu_1+z_1)(z_1-z_2)(z_1-z_3)}\right\}\left(e^{z_1t}-1\right) \\
&-\left\{\frac{\left(z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0\right)}{z_2^2(\lambda_1+\mu_1+z_2)(z_1-z_2)(z_2-z_3)}\right\}\left(e^{z_2t}-1\right) \\
&+\left\{\frac{\left(z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0\right)}{z_3^2(\lambda_1+\mu_1+z_3)(z_1-z_3)(z_2-z_3)}\right\}\left(e^{z_3t}-1\right)
\end{aligned}
\right\}
-\left[C_R\lambda_1(L-W)\right] \tag{48}
$$

7. NUMARICAL ANALYSIS

Table-1: Effect of failure rate (λ) on Reliability (R(t))

Time in days	λ_1	λ_m	α	R(t) for λ=0.01	R(t) for λ=0.02	R(t) for λ=0.03
1	0.02	0.04	0.003	0.951273	0.941822	0.932465
2	0.02	0.04	0.003	0.905003	0.887139	0.869627
3	0.02	0.04	0.003	0.861066	0.835738	0.811158
4	0.02	0.04	0.003	0.819342	0.787422	0.75675
5	0.02	0.04	0.003	0.779718	0.742002	0.70612
6	0.02	0.04	0.003	0.742086	0.699302	0.659001
7	0.02	0.04	0.003	0.706345	0.659158	0.615148
8	0.02	0.04	0.003	0.672398	0.621414	0.574332

Table-2: Effect of failure rate (λ_1) on Reliability (R(t))

Time in days	λ	λ_m	α	R(t) for λ_1 =0.01	R(t) for λ_1 =0.03	R(t) for λ_1 =0.05
1	0.01	0.04	0.003	0.951287	0.951258	0.951229
2	0.01	0.04	0.003	0.90506	0.904947	0.904837
3	0.01	0.04	0.003	0.86119	0.860944	0.860708
4	0.01	0.04	0.003	0.819557	0.819133	0.818731
5	0.01	0.04	0.003	0.780045	0.779402	0.778801
6	0.01	0.04	0.003	0.742544	0.741646	0.740818
7	0.01	0.04	0.003	0.706952	0.705767	0.704688
8	0.01	0.04	0.003	0.67317	0.671668	0.67032

Table-3: Effect of transition rate (λ_m) on Reliability (R(t))

Time in days	λ	λ_1	α	R(t) for λ_m =0.04	R(t) for λ_m =0.05	R(t) for λ_m =0.06
1	0.01	0.02	0.003	0.951273	0.941822	0.932465
2	0.01	0.02	0.003	0.905003	0.887139	0.869627
3	0.01	0.02	0.003	0.861066	0.835738	0.811158
4	0.01	0.02	0.003	0.819342	0.787422	0.75675
5	0.01	0.02	0.003	0.779718	0.742002	0.70612
6	0.01	0.02	0.003	0.742086	0.699302	0.659001
7	0.01	0.02	0.003	0.706345	0.659158	0.615148
8	0.01	0.02	0.003	0.672398	0.621414	0.574332

Table-4: Effect of rate (α) of completion of warranty period on Reliability(R(t))

Time in days	λ	λ_1	λ_m	R(t) for α=0.007	R(t) for α =0.005	R(t) for α =0.003
1	0.01	0.02	0.04	0.95133	0.951301	0.951273
2	0.01	0.02	0.04	0.905223	0.905113	0.905003
3	0.01	0.02	0.04	0.861541	0.861304	0.861066
4	0.01	0.02	0.04	0.82015	0.819747	0.819342
5	0.01	0.02	0.04	0.780927	0.780324	0.779718
6	0.01	0.02	0.04	0.743754	0.742923	0.742086
7	0.01	0.02	0.04	0.70852	0.707437	0.706345
8	0.01	0.02	0.04	0.67512	0.673766	0.672398

Table-5: Effect of repair cost (C_R) on Profit of the user (H(t))

Time in days	λ =0.01, λ_1=0.02, λ_m=0.04,α=0.003,μ=0.1, μ_1 =0.1, μ_2 =0.3					
	K_1	L	W	H(t) For C_R=150	H(t) For C_R=100	H(t) For C_R=50
1	500	10	3	467.7027	474.7027	481.7027
2	500	10	3	937.9105	944.9105	951.9105
3	500	10	3	1394.481	1401.481	1408.481
4	500	10	3	1840.975	1847.975	1854.975
5	500	10	3	2280.044	2287.044	2294.044
6	500	10	3	2713.703	2720.703	2727.703
7	500	10	3	3143.52	3150.52	3157.52
8	500	10	3	3570.747	3577.747	3584.747

8. INTERPRETATION OF THE RESULTS

Tables 1, 2, 3 and 4 show the behavior of system reliability. Tables 1, 2 and 3 indicate that the reliability of the system decreases with the increase of failure rates (λ), (λ_1) and transition rate (λ_m) with respect to (w.r.t.) time and for fixed values of other parameters. From table 4 it is analyzed that the reliability of the system increases with the decrease of rate of completion of warranty (α) w.r.t. time. Table-5 shows that expected profit H (t) during the interval (0, t] increase with the decrease of repair cost (C_R) from 150 to 50.

9. CONCLUSION

From tables 1, 2, 3 and 4, it is concluded that a single unit system with PM under warranty can be made more reliable and profitable to use by the following ways:

1) By decreasing the rate of completion of warranty.
2) By decreasing the repair cost.

Also, PM during the warranty may provide the consumer better product service in the postwarranty period and reduce the cost of repairing the deteriorated product.

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