A Novel in painting Algorithm based on Sparse Representation

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ABSTRACT
Recent studies in sparse representations show a variety of applications in the field of image processing. Such as in image denoising, inpainting, compression and more. But always the size of the dictionary has a trade off between the approximation speed and accuracy. In this paper, a moving k-means based dictionary pruning algorithm is applied to the patches of the dictionary to discover an optimum number of dictionary elements for a given data set. This optimized dictionary size feature, will improve the convergence speed of the decomposition algorithm without compromising its approximation accuracy. It also increases the performance of the decomposition algorithm. Simulation results show that the proposed optimized dictionary selection with KSVD (K-means singular value decomposition) yield better image inpainting than traditional KSVD.

Keywords
Image restoration, image inpainting, OMP, KSVD

1. INTRODUCTION
Inpainting is an art of modification of the images in an undetectable form. Image and video inpainting is still a challenging problem in computer graphics and computer vision. It has various applications include object removal, image restoration as removal of scratches or text, recovery of missing blocks in image transmission and more.

Generally the image inpainting approaches are divided into two categories as partial differential equation (PDE) based method and exemplar based method. In this, former method adapts diffusion approach [1-5]. Although the PDE based method is excellent in filling the non-textured or relatively smaller missing region, they tend to introduce blurry effect in texture region and large missing region. The latter method is exemplar based inpainting which fills the missing region by copying the content from the existing part of the image [6-10]. Hence the greedy way of filling the images in exemplar based methods lead to visual inconsistencies. However for inpainting large missing region, exemplar based method is best compared to diffusion based methods but with more processing time.

Several recent works suggested that inpainting is one of the inverse problem [11]. Thus the sparse representation of signal is introduced in image inpainting which utilizes composite textures and structures to fill the missing blocks in an image. In paper [12] the author puts forward ways for handling non homogeneous noise and missing information. Also the authors show the state-of-the-art results in applications such as color image denoising, demosaicing, and inpainting. Another solution is based on EM and bound optimization approach [13].In that the authors refer the algorithm as an easy and efficient sparse representation based iterative algorithm for image inpainting. A hybrid image inpainting method based on sparse and redundant representation is addressed in [14] which remove both homogeneous and non homogeneous noise.

This paper utilizes sparse representation with proposed dictionary pruning algorithm for image and video inpainting. By this proposed dictionary pruning algorithm, the dictionary learning algorithms finds an optimum number of dictionary elements by reducing redundancies in the learned dictionary. Hence the decomposition algorithm yields better convergence speed and performance. Thus the processing time for the algorithm is also reduced. In this paper, the proposed dictionary pruning algorithm is also examine with KSVD, it leads a substantial benefit in inpainting than it with fixed dictionary.

The rest of the paper is organized as follows: Section 2 discusses the inpainting based on sparse representation. Section 3 describes the proposed dictionary pruning algorithm. Section 4 discusses inpainting using sparse coding. Simulation results are reported in section 5 and section 6 concludes the paper.

2. INPAINTING BASED ON SPARSE REPRESENTATION
Sparse representation for signals make use an overcomplete dictionary matrix $\Phi \in R^{nxK}$ that contains K prototype signals, hence referred as atoms. A given set of signal $y \in R^n$ can be represented by sparse linear combination of these dictionary atoms. Thus the signal Y is such that $y_i \sim \Phi x_i$, where x_i is a vector which contains the coefficients for the linear combination and $y_i \in Y$. Removal of non homogenous noise is formulated as sparse problem as

$$\min_{\Phi, x_i} \|x_i\|_0 \;\; s.t. \;\; \|y_i - \Phi x_i\|_2 < \varepsilon \quad (1)$$

where $\|.\|_0$ denotes the ℓ_0 norm which counts the number of nonzero element of the vector and ε is the tolerable limit of error in reconstruction. It is realized as an NP hard problem. For that several pursuit algorithms are available to find the sparse representation of a signal in a dictionary [15-18]. They include some greedy algorithms that iteratively build up the signal approximation as one coefficient at a time, e.g. matching pursuit algorithms and those that process all coefficients simultaneously, e.g. Basic pursuit and the Focal Undetermined System Solver Family of Algorithms.

An over complete dictionary Φ is used to find the sparse representation for a given signal. It can be a prespecified dictionary such as wavelets, curvelets, contourlets, streerable wavelet filters, short time fourier transform basis[19] or learned dictionary e.g. KSVD algorithm. In this prespecified dictionary is not always suit for all kinds of signals. So a learned dictionary is the best option. But inpainting based on sparse representation using learned dictionary would take more computation time. In order to minimize the computation time optimum dictionary designing is vital. For this a novel dictionary pruning algorithm based on moving k-means is proposed in this paper. By this proposed algorithm, a more manageable sized dictionary is achieved. It increases the convergence speed of the decomposition algorithm by reduces the total number of iterations.

3. PROPOSED DICTIONARY PRUNING ALGORITHM

From the previous section, it is clear that learned dictionary is a best choice for all type of signals. But the size of the dictionary is usually a tradeoff between the approximation speed and accuracy. Solution to it is, optimized dictionary selection. In this paper a moving k-means based dictionary pruning algorithm is adapted which finds an optimized dictionary size for a given data set. The proposed method prunes the large number of dictionary elements to produce a well trained dictionary with minimum redundant elements. Such as every patches in the image are classified into different classes and the moving K-Means algorithm attempts to identify a dictionary which has a close representation of it. Such optimized dictionary selection will increase the speed and performance of the decomposition algorithm and the same is essential for an efficient dictionary learning algorithm.

In order to design an optimized dictionary, a dictionary is formed using the source image of size nxn by overlapping sliding window which samples the image region into patches. To optimize the size of the dictionary, the moving K means based pruning algorithm is applied to every patches of the dictionary. Consider the dictionary of $\{x_1,\ldots,x_n\}$ patches of data is to be grouped into few clusters of n_c regions. Here $x_{(i)} \in R^n$ as usual .The optimizing algorithm is as follows:

Initialize dictionary matrix as cluster centers $\{\mu_1, \mu_2, \mu_3\ldots \mu_{n_c}\} \in R^n$ randomly and each data is assigned to the nearest center. The search for the final clusters starts from the initial clusters. The position of the center pc_j is calculated as

$$pc_j = \frac{1}{n_j}\sum_{i\in pc_j} x_i\ ,\ \ i = 1,\ldots., N \qquad (2)$$

where, x_i is data sample belonging to the centre pc_j. After all data are assigned to the nearest centers, the fitness of the centers is verified by distance function. The total Euclidean distance between the centers and all the data assigned to the center is as follows

$$D_e = \sum_{j=1}^{n_c}\sum_{i=1}^{N}\left(\| x_i - pc_j \|\right)^2,\ j = 1,\ldots., n_c \quad (3)$$

where, D_e is the Euclidean distance function, N is the number of data and n_c is the number of cluster centers. Hence the process is repeated and the center values are updated till a minimum distance is obtained. The proposed dictionary pruning algorithm is summarized in Figure 1. By fitness verification, the proposed dictionary pruning algorithm reduces the center redundancy and dead centers. Thus it optimizes the dictionary which in turn improves the convergence speed of the decomposition algorithms. In this paper, image sequence inpainting is reported for the proposed dictionary pruning algorithm with the hybrid method [14].

4. INPAINTING USING SPARSE CODING

Image inpainting is a technique of modifying an image into an undetectable form [12, 20]. This paper mainly focuses on the removal of scratches and text in an image sequences.

Step 1: Proper initialization of center pc_j (not too far from data), by taking random values from within the range.

Step 2: Assign all data to nearest centers.

Step 3: Calculate the centre position pc_j as

$$pc_j = \frac{1}{n_j} \sum_{i \in pc_j} x_i \,, \qquad i = 1, \ldots, N$$

Step 4: In each iteration, check the fitness of the centers.

$$D_e = \sum_{j=1}^{n_c} \sum_{i=1}^{N} \left(\| x_i - pc_j \| \right)^2, \qquad j = 1, \ldots, n_c$$

Step 5: Find the value of the centers having the largest and the smallest values of D_e.

Low value of D_e: pc_j is less suitable for initialization.

Pcj = 0: it has no members (outside data-range).

Step 6: If D_e between the data and the centers becomes minimized then stop else goto step 3.

Figure 1: Proposed dictionary Pruning algorithm

In the proposed inpainting algorithm, texture distribution analysis is used to separate the missing area into homogenous and non homogenous textures and separate methods are adapted for the removal [14]. According to the classification map, the proposed method inpaint the missing region using fast inpainting method and sparse representation approach. For non homogenous noise removal, the proposed pruning algorithm is examined with weighted KSVD algorithm. Let P_0 be the clean image as a column vector of N. Consider σ_l be the deviation of the noise at the pixel l. In order to use OMP, noises in the patches are approximated to sphere structure by using a vector α composed of pixel weights.

$$\alpha_l = \frac{\min_{l' \in image} \sigma_{l'}}{\sigma_l} \qquad (4)$$

Addressing the inpainting problem as a sparse decomposition technique per each patch leads to the following optimization problem.

$$\left\{ \hat{x}_{ij}, \hat{\Phi}, \hat{P} \right\} = \underset{\Phi, x_{ij}, p}{\arg\min} \, \lambda \, \| \alpha \otimes (p - y) \|_2^2 + \sum_{ij} \mu_{ij} \| x_{ij} \|_0 + \sum_{ij} \| (R_{ij} \alpha) \otimes (\Phi x_{ij} - R_{ij} p) \|_2^2 \quad (5)$$

In the above equation, $\hat{P}$ is the estimator of P_0 and the optimized dictionary estimator $\hat{\Phi} \in R^{n \times k}$ which leads to the sparsest representation of the patches in the restored image. The vectors $\hat{x}_{ij} \in R^k$ are the sparse representations for the $(i,j)^{th}$ patch in $\hat{P}$ using the optimized dictionary $\hat{\Phi}$. The operator R_{ij} is a binary $n \times N$ matrix which extracts the square $\sqrt{n} \times \sqrt{n}$ patch of coordinates (i,j) from the image as a column vector of size N . The symbol $\otimes$ denotes element-wise multiplication between the two vectors. The sparse coding step adapt a patch based process as

$$\forall ij, \hat{x}_{ij} = \arg\min_{x_{ij}} \| x_{ij} \|_0 \; \textit{subject to}$$
$$\| (R_{ij}\alpha) \otimes (R_{ij} \hat{x} - \hat{\Phi} x_{ij}) \|_2^2 \le$$
$$\| R_{ij}\alpha \|_0 \, (C \min_l \sigma_l)^2 \qquad (6)$$

The term $\| R_{ij}\alpha \|_0$ counts the number of pixels in the ijth patch. This step minimizes each atom s as follows

$$(\hat{d}_s, \hat{x}^s) = \arg\min_{x^s, \|d_s\|_2 = 1} \| \alpha^s \otimes (M_s - d_s \alpha^s) \|_F^2 \quad (7)$$

where α^s is the matrix whose size is same as M_s and where each column corresponding to (i,j) is $R_{ij}\alpha$. This is weighted rank one approximation matrix. Finally it performs the weighted averaging of overlapping patches after each patch has been sparse coded as

$$\hat{P} = \frac{(\lambda y + \sum_{ij} R_{ij}^T \hat{\Phi} \hat{\alpha}_{ij})}{(\lambda I + \sum_{ij} R_{ij}^T R_{ij})} \qquad (8)$$

Each pixel in a patch is a weighted linear combination of different pixels and their weights being derived from the sparse coding. Hence the patches are overlapping the final value of each pixel is thus an average of all representations obtained from sparse coding stage. Thus KSVD algorithm makes use its in build averaging capability to inpaints the missing region.

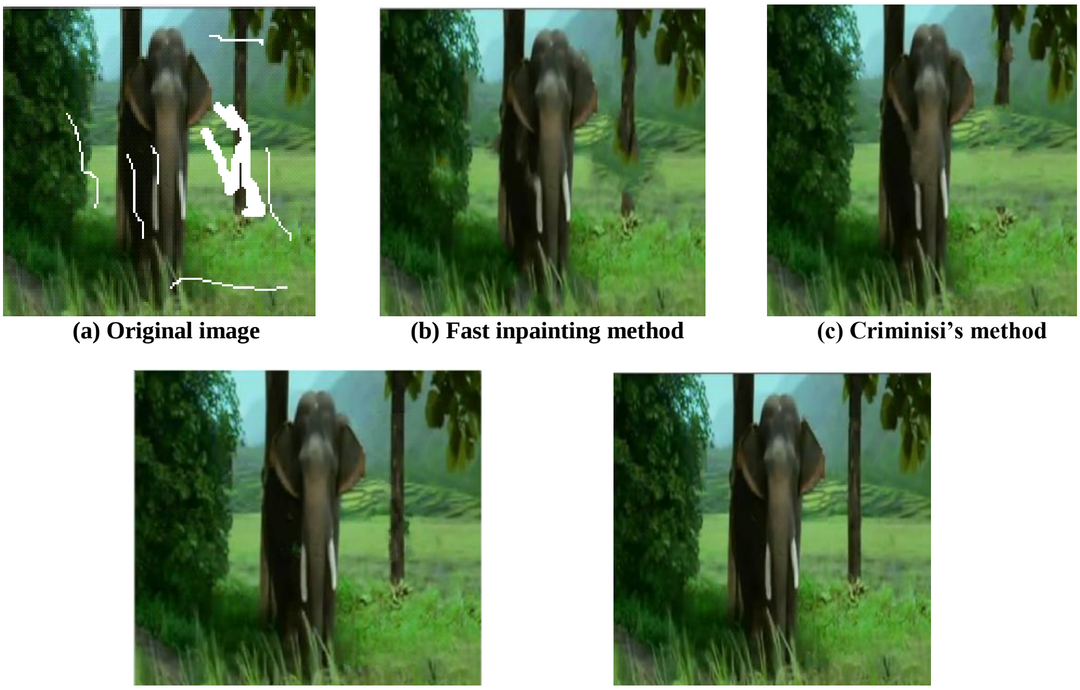
5. SIMULATION RESULTS

In this section, the proposed dictionary pruning algorithm is tested on a number of standard gray, color images and image sequences. The proposed algorithm is applied to the applications of scratch and text removal. This framework is successful for filling smaller holes of size upto16x16 pixels. For filling larger holes, it would require too much computational time and memory to use the K-SVD with a highly redundant dictionary. The proposed method is also evaluated by comparing it with other well known inpainting algorithm such as fast inpainting [20], criminisi's method [7] and Hybrid method [14]. The results for scratch removal obtainded for the gray and color images are shown in Figure 2 and 3. From the results, it is clear that the proposed inpainting method not only preserves more edge information but also reconstructs the texture information. The results for text removal for the image new orleans is shown in Figure 4. The proposed method obtains a better convergence speed.

(a) Original image **(b) Fast inpainting method** **(c) Criminisi's method**

(d) Hybrid method **(e) Proposed method**

Figure 2: Inpainting results of image "Barbara"



(a) Original image (b) Fast inpainting method (c) Criminisi's method

(d) Hybrid method (e) Proposed method

Figure 3: Inpainting results of video frame "Rainy frame 117"

(a) Original image (b) Fast inpainting method (c) Criminisi's method

(d) Hybrid method (e) Proposed method

Figure 4: Inpainting results of image "New Orleans"

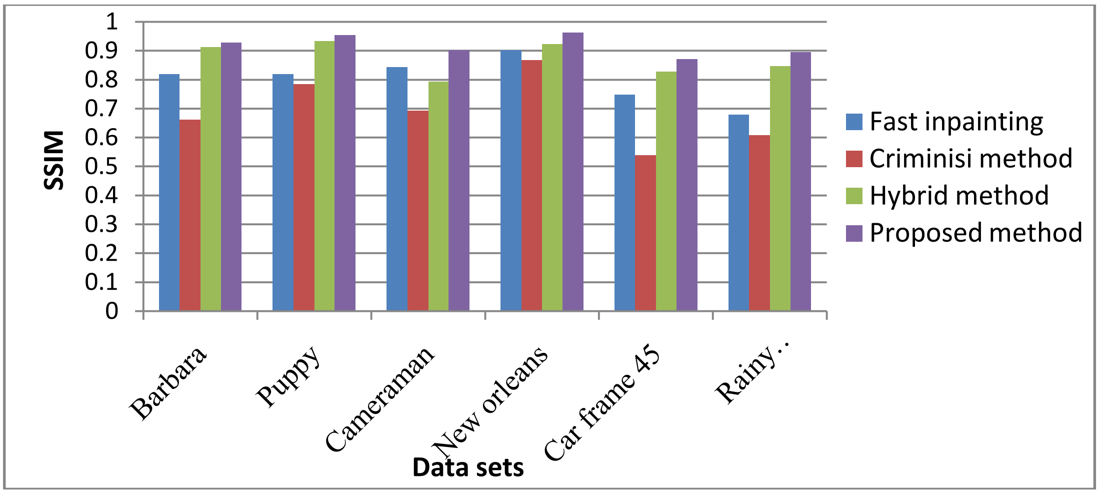


Figure 5: Comparison of SSIM values of the proposed method with other methods

Table 1. Comparison of PSNR values of the proposed method with other methods

Image name	Fast inpainting	Criminisi method	Hybrid method	Proposed method
Barbara	30.67	24.39	32.07	33.35
Puppy	32.03	28.48	33.53	34.71
Cameraman	30.45	25.63	27.65	32.84
New orleans	33.76	30.43	33.25	35.69
Car frame 45	26.46	23.66	29.54	31.47
Rainy frame 117	28.49	25.09	30.75	33.85

5.1. Performance Analysis

5.1.1 Comparison of PSNR values

The proposed method is evaluated objectively using peak signal to noise ratio (PSNR) and structural similarity (SSIM) index. The PSNR values are calculated for both scratch and text degraded test image and image sequences. It is observed that the PSNR values of the proposed algorithm sounds high compared to the existing algorithm by 1 dB to 2 dB. The results are reported in Table 1.

5.1.2 Structural Similarity (SSIM) Index

The structural similarity (SSIM) index is a recent popular image quality assessment algorithm. The SSIM index is a full reference metrics [21]. It is designed to improve on traditional methods like mean squared error and peak signal-to-noise ratio, which have proved to be inconsistent with human eye perception. The SSIM index expresses quality by comparing local correlations in contrast, luminance, and structure between reference and distorted images. The results are reported in Figure 5.The SSIM metric is calculated on various windows of an image. The measure between two windows X and Y of common size NxN is

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (9)$$

μ_x and μ_y are the average of x and y; σ_x^2 and σ_y^2 are the variance of x and y; σ_{xy} the covariance of x and y; $C_1 = (k_1L)^2$, $C_2 = (k_2L)^2$ are constant: L dynamic range of pixel values

6. CONCULSION

In this paper, a novel dictionary pruning algorithm is presented for sparse representation inpainting which is used to obtain a better convergence speed and performance for the decomposition algorithms. By optimizing the dictionary with Moving K means clustering algorithm, better image inpainting results were obtained than with fixed dictionary learning algorithms. The proposed dictionary pruning algorithm reduces the size of the dictionary and also the total number of OMP iterations to minimum. By this, the dictionary learning technique like KSVD finds an optimum number of dictionary coefficients by reducing redundancies in the learned dictionary. All these extensions are tested on an extensive set of image sequences and are found that the proposed dictionary pruning algorithm dramatically improves the speed and performance of the decomposition algorithms.

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