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δg^*-Continuous Functions in Topological Spaces

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ABSTRACT
The Aim of this paper is to introduce the new class of function called δg^*-continuous function by using δg^*-closed set and study their basic properties in topological spaces. We also investigate its relationship with other types of functions.

Keywords and phrases
Generalized closed sets, δ-closure, δg-closed sets and g-open sets.

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1. INTRODUCTION
Different types of generalizations of continuous maps were introduced and studied by various authors in the recent development of topology. In 2011, Lellis Thivagar et. al., [10] introduced the notion of $\delta\hat{g}$-continuity in topological spaces. A new weaker form of the closed sets called δg^*-closed sets is introduced and investigated by Sudha et. al., [17]. In this paper, we study δg^*-continuous maps as a new generalization of δg-continuous and obtain their characterizations, properties and discuss the relationships with some other functions.

2. PRELIMINARIES
Throughout this paper (X,τ) (or simple X) represents topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

2.1 Definition
A subset A of a space (X,τ) is called a
(i) semi-open set [11] if $A \subseteq cl(int(A))$.
(ii) α-open set [16] if $A \subseteq int(cl(int(A)))$.
(iii) regular open set [17] if $A = int\ (cl(A))$.

The complement of a semi open (resp. α-open, regular open) set is called semi-closed (resp. α-closed, regular closed). The semi-closure [5] (resp.α-closure [16]) of a subset A of (X,τ), denoted by scl(A) (resp. αcl(A)) is defined to be the intersection of all semi-closed (resp.α-closed) sets containing A. It is known that scl(A) (resp. αcl(A)) is a semi-closed (resp.α-closed) set.

2.2 Definition [19]
The δ-interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $int_\delta(A)$. The subset A is called δ-open if $A = int_\delta(A)$. i.e., a set is δ-open if it is the union of regular open set. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subseteq X$ is called δ-closed if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X;\ int\ (cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

2.3 Definition
A subset A of (X, τ) is called
1) generalized closed (briefly g-closed) [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) δ-generalized closed (briefly δg-closed) [7] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3) δg^* - closed [18] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a g-open set in (X, τ).
4) $\delta\hat{g}$-closed [8] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).
5) α-generalized closed (briefly αg-closed) [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
6) generalized α closed (briefly gα-closed) [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
7) generalized closed (briefly g-closed) [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
8) semi-generalized closed (briefly sg-closed) [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
9) generalized semi-closed (briefly gs-closed) [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
10) $\alpha\hat{g}$-closed [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ - open in (X, τ).

The complements of the above mentioned sets are called their respective open sets.

2.4 Definition
A map $f: (X, \tau) \quad (Y, \sigma)$ is called
1) $\delta\hat{g}$-continuous [8] if $f^{-1}(V)$ is $\delta\hat{g}$-closed in (X, τ) for every closed set V of (Y, σ).
2) $\alpha\hat{g}$-continuous [9] if $f^{-1}(V)$ is $\alpha\hat{g}$-closed in (X, τ) for every closed set V of (Y, σ).
3) sg-continuous [12] if $f^{-1}(V)$ is sg-closed in (X, τ) for every closed set V of (Y, σ).
4) gs-continuous [3] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).
5) αg-continuous [6] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ).

6) gα-continuous [6] if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V of (Y, σ).
7) α-continuous [15] if $f^{-1}(V)$ is α-closed in (X, τ) for every closed set V of (Y, σ).
8) g-continuous [12] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ).

2.5 Definition [18]

A space X is called

1) $_{\delta g^*}T_{\delta}$ -space if every δg^* -closed set in it is δ-closed.
2) $_{\alpha g}T_{\delta g^*}$ -space if every αg-closed set in it is δg^* -closed.

3. δg^* -CONTINUOUS FUNCTIONS

In this section we introduce δg^* -continuous map in topological spaces and study some of their properties. We prove that the composition of two δg^* -continuous maps need not be δg^* -continuous.

3.1 Definition

A map f : (X, τ) $\longrightarrow$ (Y, σ) is said to be δg^* -continuous if the inverse image of every closed set in (Y, σ) is δg^* -closed in (X, τ).

3.2 Theorem

A map f : (X, τ) $\longrightarrow$(Y, σ) is δg^* -continuous if and only if the inverse image of every open set in (Y, σ) is δg^* -open in (X, τ).

Proof: (Necessity) Let f : (X, τ) $\longrightarrow$ (Y, σ) be δg^* -continuous and U be an open set in (Y,σ). Then Y – U is closed in (Y,σ). Since f is δg^* -continuous, $f^{-1}(Y-U) = X - f^{-1}(U)$ is δg^* -closed in (X,τ) and hence $f^{-1}(U)$ is δg^* -open in (X, τ).

(Sufficiency) Assume that $f^{-1}(V)$ is δg^* -open in (X, τ) for each open set V in (Y, σ). Let V be a closed set in (Y, σ). Then Y – V is open in (Y, σ). By assumption, $f^{-1}(Y-V) = X - f^{-1}(V)$ is δg^* -open in (X,τ), which implies that $f^{-1}(V)$ is δg^* -close in (X,τ). Hence f is δg^* -continuous.

3.3 Proposition

Every δ-continuous map is δg^* -continuous

Proof: Let f : (X, τ) $\longrightarrow$(Y, σ) be a δ-continuous map. Let V be any closed set in (Y,σ). Since f is a δ-continuous map, $f^{-1}(V)$ is δ-closed in (X,τ). By Proposition 3.2.[18], every δ-closed set is δg^* -closed, $f^{-1}(V)$ is δg^* -closed in (X,τ). Therefore f is δg^* -continuous.

The converse of the above Proposition need not be true as seen from the following example.

3.4 Example

Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {a, b}}. Let f : (X, τ) (Y, σ) be the identity map. Then f is δg^* -continuous but not δ-continuous, since for the closed set {c} in (Y,σ), $f^{-1}(\{c\}) = \{c\}$ is δg^* -closed but not δ-closed in (X,τ).

3.5 Proposition

Let f : (X, τ) $\longrightarrow$(Y, σ) be a δg^* -continuous map and (X, τ) be a $_{\delta g^*}T_{\delta}$ -space. The f is δ-continuous.

Proof: Let V be a closed set in (Y, σ). Since f is δg^* -continuous, $f^{-1}(V)$ is δg^* -closed in (X, τ). Since (X, τ) is a $_{\delta g^*}T_{\delta}$ -space, $f^{-1}(V)$ is δ-closed in (X, τ). Hence f is δ-continuous.

3.6 Corollary

Every δg^* -continuous map is δg-continuous.

Proof: Follows from the fact that every δg^* -closed set is δg-closed.

The converse of the above Proposition need not be true as seen from the following example.

3.7 Example

Let X = Y = {a, b, c, d} with τ = {X, ϕ, {a, b}} and σ = {Y, ϕ, {a, b, c}}. Let f : (X, τ) $\longrightarrow$(Y, σ) be the identity map. Then f is δg-continuous but not δg^* -continuous, since for the closed set {d} in (Y, σ), $f^{-1}(\{d\}) = \{d\}$ is δg^* -closed in (X, τ).

3.8 Proposition

Every δg^* -continuous map is $\delta\hat{g}$ -continuous

Proof: Let f : (X, τ) $\longrightarrow$(Y, σ) be a δg^* -continuous map. Let V be any closed set in (Y,σ). Since f is a δg^* continuous map, $f^{-1}(V)$ is δg^* -closed in (X,τ). By Proposition 3.20. [18], every δg^* -closed set is $\delta\hat{g}$ -closed, $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X,τ). Therefore f is $\delta\hat{g}$ -continuous.

The converse of the above Proposition need not be true as seen from the following example.

3.9 Example

Let X = Y = {a, b, c} with τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a}, {a, b}}. Let f : (X, τ) $\longrightarrow$ (Y, σ) be the identity map, then f is $\delta\hat{g}$ -continuous but not δg^* -continuous, since for the closed set {b, c} in (Y, σ), $f^{-1}(\{b,c\}) = \{b,c\}$ is $\delta\hat{g}$ -closed but not δg^* -closed in (X, τ).

3.10 Proposition

Every δg^* -continuous map is αg-continuous

Proof: Let f : (X, τ) $\longrightarrow$ (Y, σ) be a δg^* -continuous map. Let V be any closed set in (Y,σ). Since f is a δg^* -continuous map, $f^{-1}(V)$ is αg-closed in (X,τ), as every δg^* -closed set is αg-closed [Proposition 3.14, [18]]. Therefore f is αg-continuous.

The converse of the above Proposition need not be true as seen from the following example.

3.11 Example

Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {b, c}}. Let f : (X, τ) $\longrightarrow$(Y, σ) be defined by f(a) = b , f(b) = a, f(c) = c. Then f is αg-continuous but not δg^* -

continuous, since for the closed set {a} in (Y, σ), $f^{-1}(\{a\}) = \{b\}$ is αg-closed but not δg^*-closed in (X, τ).

3.12 Proposition
Let f : (X, τ) ⟶ (Y, σ) be a αg-continuous map and (X, τ) be a ${}_{\alpha g}T_{\delta g^*}$-space. Then f is δg^*-continuous.

Proof: Let V be a closed set in (Y, σ). Since f : (X, τ) ⟶ (Y, σ) is αg-continuous, $f^{-1}(V)$ is αg-closed in (X, τ). Since (X, τ) is ${}_{\alpha g}T_{\delta g^*}$-space, $f^{-1}(V)$ is δg^*-closed in (X, τ). Hence f is δg^*-continuous.

3.13 Proposition
Every δg^*-continuous map is gs-continuous.

Proof: The proof follows from the fact that every δg^*-closed set is gs-closed.

The converse of the above Proposition need not be true as seen from the following example.

3.14 Example
Let X = Y = {a, b, c, d} with τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {c, d}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = a , f(b) = c, f(c) = d, f(d) = b. Then f is gs-continuous but not δg^*-continuous, since for the closed set {a, b} in (Y, σ), $f^{-1}(\{a,b\}) = \{a,d\}$ is gs-closed but not δg^*-closed in (X, τ).

3.15 Proposition
Every δg^*-continuous map is $\alpha\hat{g}$-continuous.

Proof: The proof follows from the fact that every δg^*-closed set is $\alpha\hat{g}$-closed

The converse of the above Proposition need not be true as seen from the following example.

3.16 Example
Let X = Y = {a, b, c, d} with τ = {X, ϕ, {a, b}} and σ = {Y, ϕ, {b}, {b, a}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = a , f(b) = d, f(c) = b, f(d) = c. Then f is $\alpha\hat{g}$-continuous but not δg^*-continuous, since for the closed set {c, d} in (Y, σ), $f^{-1}(\{c, d\}) = \{b, d\}$ is $\alpha\hat{g}$-closed but not δg^*-closed in (X, τ).

3.17 Remark
The following examples show that the notions of δg^*-continuity and gα-continuity are independent.

3.18 Example
Let X = Y = {a, b, c} with τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b, c}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = b , f(b) = a, f(c) = c. Then f is gα-continuous but not δg^*-continuous, since for the closed set {a} in (Y, σ), $f^{-1}(\{a\}) = \{b\}$ is gα-closed but not δg^*-closed in (X, τ).

3.19 Example
Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {c}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = b , f(b) = c, f(c) = a. Then f is δg^*-continuous but not gα-continuous, since for the closed set {a, b} in (Y, σ), $f^{-1}(\{a, b\}) = \{a, c\}$ is δg^*-closed but not gα-closed in (X, τ).

3.20 Remark
The following examples show that the notions of δg^*-continuity and sg-continuity are independent.

3.21 Example
Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y, ϕ, {a}, {a, b}}. Let f : (X, τ) ⟶ (Y, σ) be the identity map. Then f is sg-continuous but not δg^*-continuous, since for the closed set {b} in (Y, σ), $f^{-1}(\{b\}) = \{b\}$ is sg-closed but not δg^*-closed in (X, τ).

3.22 Example
Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {c}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = b , f(b) = c, f(c) = a. Then f is δg^*-continuous but not sg-continuous, since for the closed set {a, b} in (Y, σ), $f^{-1}(\{a, b\}) = \{a, c\}$ is δg^*-closed but not sg-closed in (X, τ).

3.23 Remark
The following examples show that the notions of δg^*-continuity and α-continuity are independent.

3.24 Example
Let X = Y = {a, b, c, d} with τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a, b, d}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = d , f(b) = c, f(c) = b, f(d) = a. Then f is α-continuous but not δg^*-continuous, since for the closed set {c} in (Y, σ), $f^{-1}(\{c\}) = \{b\}$ is α-closed but not δg^*-closed in (X, τ).

3.25 Example
Let X = Y = {a, b, c, d} with τ = {X, ϕ, {a, b}} and σ = {Y, ϕ, {c}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = c , f(b) = d, f(c) = a, f(d) = b. Then f is δg^*-continuous but not α-continuous, since for the closed set {a, b, d} in (Y, σ), $f^{-1}(\{a, b, d\}) = \{b, c, d\}$ is δg^*-closed but not α-closed in (X, τ).

3.26 Remark
The composition of two δg^*-continuous maps need not be δg^*-continuous as seen from the following example.

3.27 Example
Let X = Y = {a, b, c} with τ = {X, ϕ, {a}, {b, c}}, σ = {Y, ϕ, {a, b}} and η = {Z, ϕ, {c}}. Let f : (X, τ) ⟶ (Y, σ) be defined by f(a) = c , f(b) = b, f(c) = a and g :(Y, σ) ⟶ (Z, η) be defined by g(a) = a , g(b) = c, g(c) = b. Then the maps f and g are δg^*-continuous but their composition $g \circ f$: (X, τ) ⟶ (Z, η) is not δg^*-continuous, since for the closed set {a, b} in (Z, η), $(g \circ f)^{-1}\{a, b\} = \{a, c\}$ is not δg^*-closed in (X, τ).

3.28 Proposition
Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be any topological space and (Y, σ) be a ${}_{\delta g^*}T_\delta$-space. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ and $g : (Y, \sigma) \longrightarrow (Z,\eta)$ are δg^*-continuous maps, then their composition $g \circ f : (X, \tau) \longrightarrow (Z, \eta)$ is a δg^*-continuous maps.

Proof: Let V be a closed set in (Z, η). Since g: (Y, σ) (Z,η) is δg^*-continuous, $g^{-1}(V)$ is δg^*-closed in (Y, σ). Since (Y, σ) is a ${}_{\delta g^*}T_\delta$-space, $g^{-1}(V)$ is δ-closed in (Y, σ). Now $g^{-1}(V)$ is closed in (Y, σ). Since $f : (X, \tau)$ (Y, σ) is δg^*-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is δg^*-closed in (X, τ). Hence $g \circ f : (X, \tau) \longrightarrow (Z, \eta)$ is δg^*-continuous.

3.29 Proposition
Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a δg^*-continuous map. Then for every subset A of (X, τ) $f(\delta g^* cl(A)) \subseteq \delta cl(f(A))$.

Proof: Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a δg^*-continuous map and A be any subset of (X,τ). Then $\delta cl(f(A))$ is a closed set in (Y, σ). Since f is δg^*-continuous, we have $f^{-1}(\delta cl(f(A)))$ is δg^*-closed in (X,τ). Hence $\delta g^* cl(f^{-1}(\delta cl(f(A)))) = f^{-1}(\delta cl(f(A)))$ by Proposition 3.3.. Since $f(A) \subseteq cl(f(A))$, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\delta(f(A)))$. Hence $f^{-1}(\delta cl(f(A)))$ is δg^*-closed set containing A. By definition of δg^*-closure, we have $\delta g^* cl(A) \subseteq f^{-1}(\delta cl(f(A)))$ which implies that $f(\delta g^* cl(A)) \subseteq \delta cl(f(A))$.

3.30 Proposition
Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a map. If for each point $x \in X$ and each open set V in (Y, σ) containing f(x), there exists a δg^* open set U in (X, τ) containing x such that $f(U) \subseteq V$, then for every subset A of (X, τ), $f(\delta g^* cl(A)) \subseteq cl(f(A))$.

Proof: Let $y \in f(\delta g^* cl(A))$. Therefore y = f(x) for some $x \in \delta g^* cl(A) \subseteq X$. Let V be any open set in (Y, σ) containing f(x). Then by hypothesis, there exists a δg^* open set U in (X, τ) containing x such that $f(U) \subseteq V$. This implies that $U \cap A \neq \phi$. Then $f(U \cap A) \neq \phi$ which implies that $V \cap f(A) \neq \phi$. Hence $y \in cl(f(A))$.

4. CONCLUSION
We have introduced and studied the concept of δg^*-continuous functions using δg^*-closed sets. The same can be extended to Bitopological spaces, Fuzzy topological spaces in the Future.

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