

On Tiling Patterns Involving Islamic Rosettes with an Odd Number of Vertices

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ABSTRACT

Islamic rosette patterns have been extensively studied for their symmetry and aesthetic appeal. This paper presents a simple construction method for generating these rosettes on computers and constructing new tiling patterns that involve rosettes with an odd number of vertices

General Terms

Islamic, Art, Pattern, Algorithm, Turbo C++, Program.

Keywords

Polygon, Rosette, Star, Symmetry, Trigonometric,

1. INTRODUCTION

Owing to their symmetry and aesthetic appeal, Islamic star and rosette patterns have been studied extensively by several researchers (Grünbaum and Shephard [8], Abas and Salman [1], Bourgoïn [2], Dewdney [5], Castera [3], Dunham [6]). The innate symmetry of these structures have led to geometrical constructions of these as well as tiling designs devised through putting such structures in contact (Castera [4], Grünbaum and Shephard [7], Kaplan [12]). Kaplan [11] has devised an elaborate construction method for stars based on methods described originally by Henkin [9] and Lee [13]. In addition Kaplan [12] has also devised a number of interesting tiling methods by putting stars in contact. Most of these tiling styles use stars and rosettes with an even number of vertices. Tiling with stars having an odd number of vertices have been studied by Gangopadhyay [9], [10]. In this paper, a simpler method to construct a rosette is described along with two new generalized tiling methods which use rosettes with both even and odd number of vertices. In addition a threefold symmetric tiling is constructed for all rosettes whose number of vertices is a multiple of three. Using rosettes with odd number of vertices to create new tiling patterns with them are the main distinguishing features of this paper.

2. HOW TO DRAW A ROSETTE

Let n be an integer. Every rosette contains a star, so to start with, a star with n vertices is drawn. The construction is the one described in Gangopadhyay [10]. Let $A[i]$ be the i th vertex of the star, and let $x[i]$ and $y[i]$ denote respectively the abscissa and ordinate of $A[i]$, where $0 \leq i \leq n-1$. Let x_0 and y_0 respectively denote the abscissa and ordinate of the centre of the star and let its radius be k . Then

$$x[i] = k \cos(2\pi i / n) + x_0 \text{ and } y[i] = k \sin(2\pi i / n) + y_0.$$

For each i , $0 \leq i \leq n-1$, The points $A[0]$ to $A[n-1]$ constitute the vertices of a regular n -gon. For $0 \leq i \leq n-1$ and $k_0 \leq k$, let $B[i]$ be a point with abscissa $xx[i]$ and ordinate $yy[i]$, where,

$$xx[i] = k_0 \cos(2\pi i / n) + x_0 \text{ and } yy[i] = k_0 \sin(2\pi i / n) + y_0.$$

By joining $B[i]$ to both $A[(i-1) \bmod n]$ and $A[(i+1) \bmod n]$, for all i , $0 \leq i \leq n-1$, one obtains $\text{star}(n, k, k_0)$.

Figure 1 below illustrates the construction of $\text{star}(11, 120, 30)$ $A[3]$ and $B[3]$ being respectively highlighted by a green and a blue circle.

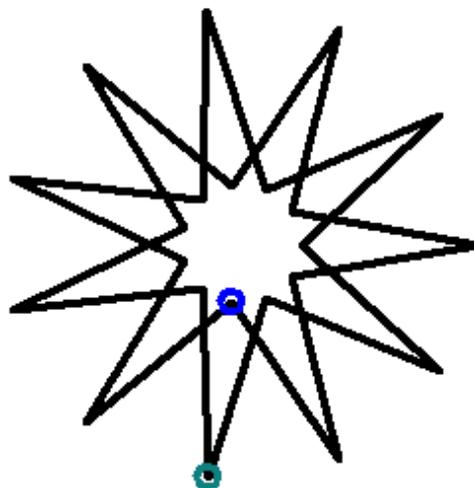


Fig 1 : $\text{star}(11, 120, 30)$

Next for $0 \leq i \leq n-1$ and $k_1 \geq k$, let $C[i]$ be a point with abscissa $xx_0[i]$ and ordinate $yy_0[i]$, where,

$$xx_0[i] = k_1 \cos(2\pi(i+.5) / n) + x_0$$

and

$$yy_0[i] = k_1 \sin(2\pi(i+.5) / n) + y_0.$$

Also for $0 \leq i \leq n-1$ and $k_2 \geq k_1$, let $D[i]$ be a point with abscissa $xx_1[i]$ and ordinate $yy_1[i]$, where,

$$xx_1[i] = k_2 \cos(2\pi i / n) + x_0$$

and

$$yy_1[i] = k_2 \sin(2\pi i / n) + y_0.$$

Now, for each i , $0 \leq i \leq n-1$, let $E[i]$ be the point of intersection of the lines $A[i]B[i+1]$ and $C[i-1]D[i-1]$ and let $F[i]$ be the point of intersection of the lines $A[i]B[i-1]$ and $C[i]D[i+1]$. Then the rosette (n, k, k_0, k_1, k_2) is drawn by joining $E[i]$ to both $A[i]$ and $D[i-1]$ and also $F[i]$ to both $A[i]$ and $D[i+1]$ for each i , $0 \leq i \leq n-1$.

Figure 2 below illustrates the construction of rosette(11,75,30,105,150). The embedded star(11,75,30) is presented in cyan color.

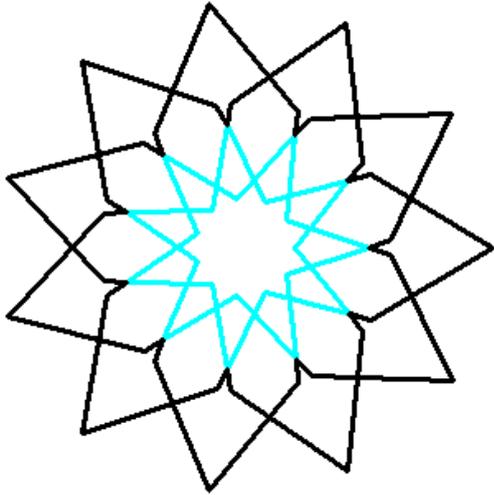


Fig 2 : rosette(11,75,30,105,150).

In the code in section 3, the functions interx and intery respectively obtains the abscissa and ordinate of two lines joining two pairs of points. The function rose draws rosette(11,75,30,105,150) when flag variable fl is set to zero and its mirror image when it is set to 1.

3. TILING WITH ROSETTES

The process of tiling is illustrated for rosette(7,75,30,105,150).. The basic prototype is a double rosette which is shown in Figure 2 and is drawn as follows. First rosette(11,75,30,105,150). is drawn with its center coordinates at i and j. Then its mirror image is drawn adjacent to it so that their corresponding vertices touch each other. This is achieved by setting the abscissa of the center of the mirror image as i+5k, while keeping the ordinate unchanged. In Figure 3, rosette(11,75,30,105,150). and its mirror image are drawn in the manner described above.

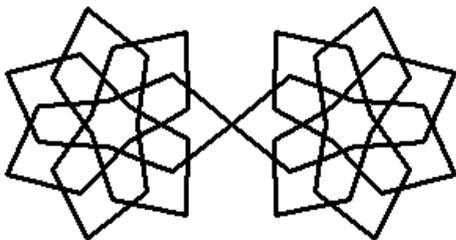


Fig. 3 : The Prototype - double rosette

The image in Figure 2 is then repeated by increasing the center coordinates of rosette(11,75,30,105,150) at an interval

of $5k(1+\cos((\text{ang}/2 \pi /180.))$ for the abscissa i and $5k \sin(((n+1) / 2)\text{ang}/2) \pi / 180)$ for the ordinate j. where angle = $360 / n$.

The final output is shown in Figure 4.

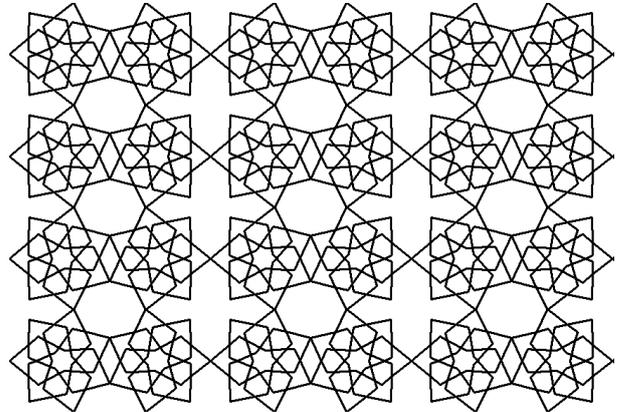


Fig 4 : The final output of tiling

The algorithm described in sections 2 and 3 is presented in the form of the C++ code in section 4.

4. THE CODE

The code uses three functions interx, intery and rose which are declared first..

```
float interx(float x1,float y1,float x2,float y2,float x3,float
y3,float x4,float y4)
{return (y3-y1-x3*(y4-y3)/(x4-x3)
+x1*(y2-y1)/(x2-x1))/((y2-y1)/(x2-x1)-(y4-y3)/(x4-x3));}
```

```
float intery(float x1,float y1,float x2,float y2,float x3,float
y3,float x4,float y4)
{return (x3-x1-y3*(x4-x3)/(y4-y3)
+y1*(x2-x1)/(y2-y1))/((x2-x1)/(y2-y1)-(x4-x3)/(y4-y3));}
```

```
void rose(int fl,float k,float k0,float k1,float k2,int n,int x0,int
y0)
```

```
{float
x[100],y[100],xx[100],yy[100],xz[100],yz[100],xx0[100],
yy0[100],xx1[100],yy1[100],u[100],v[100],u0[100],v0[100];
```

```
for(int i=0;i<n;i++)
```

```
{ x[i]=k*cos(2*3.14*(i+fl*.5)/n)+x0;
```

```
y[i]=k*sin(2*3.14*(i+fl*.5)/n)+y0;
```

```
}
```

```
for(int i=0;i<n;i++)
```

```
{ xx[i]=k0*cos(2*3.14*(i+fl*.5)/n)+x0;
```

```
yy[i]= k0*sin(2*3.14*(i+fl*.5)/n)+y0;}
```

```
for(int i=0;i<n;i++)
```

```

{xx0[i]=k1*cos(2*3.14*((i+fl*.5)+.5)/n)+x0;
yy0[i]=k1*sin(2*3.14*((i+fl*.5)+.5)/n)+y0;
}
for(int i=0;i<n;i++)
{xx1[i]=k2*cos(2*3.14*(i+fl*.5)/n)+x0;
yy1[i]=k2*sin(2*3.14*(i+fl*.5)/n)+y0;
}
for(int i=0;i<n;i++)
{
line(xx[i],yy[i],x[(i-1+n)%n],y[(i-1+n)%n]);
line(xx[i],yy[i],x[(i+1)%n],y[(i+1)%n]);
}
for(int i=0;i<n;i++)
{
u[i]=interx(x[i],y[i],xx[(i+1)%n],yy[(i+1)%n],xx0[(i-1+n)%n],yy0[(i-1+n)%n],xx1[(i-1+n)%n],yy1[(i-1+n)%n]);
v[i]=intery(x[i],y[i],xx[(i+1)%n],yy[(i+1)%n],xx0[(i-1+n)%n],yy0[(i-1+n)%n],xx1[(i-1+n)%n],yy1[(i-1+n)%n]);
u0[i]=interx(x[i],y[i],xx[(i-1+n)%n],yy[(i-1+n)%n],xx0[(i)%n],yy0[(i)%n],xx1[(i+1)%n],yy1[(i+1)%n]);
;
v0[i]=intery(x[i],y[i],xx[(i-1+n)%n],yy[(i-1+n)%n],xx0[(i)%n],yy0[(i)%n],xx1[(i+1)%n],yy1[(i+1)%n]);
;

line(u[i],v[i],x[(i)%n],y[(i)%n]);line(u[i],v[i],xx1[(i-1+n)%n],yy1[(i-1+n)%n]);
line(u0[i],v0[i],x[(i)%n],y[(i)%n]);line(u0[i],v0[i],xx1[(i+1)%n],yy1[(i+1)%n]);
}
}

void main()
{
initwindow(1040,760);
int l=15;int k0=1;
int k=2*1;int n=11;float ang=360./n;

for(int k=1.5*1;k<=5*1;k+=1.5*1){
for(int i=-60;i<1140;i+=10*1*(1+cos((ang/2)*3.14/180.)))
for(int j=0;j<800;j+=10*1*(sin(((n-1)/2*ang/2)*3.14/180.)))
{float k1=3.5*k0,k2=5*k0;
rose(0,k,k0,k1,k2,n,i,j);r
ose(1,k,k0,k1,k2,n,i+10*1,j);}
}
}

```

```

getch();
closegraph();
}

```

4. VARIATIONS ON THE SAME THEME

By changing the values of n and d in the main program we may generate a number of interesting star patterns.,

VARIATION 1.

Let n=11. Then the resulting output is illustrated in Figure 3.

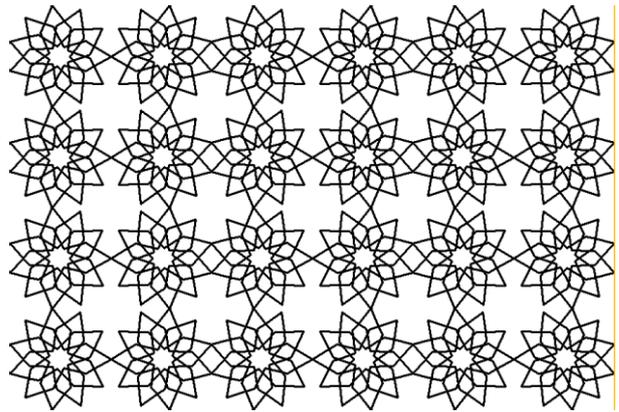


Fig. 5 : Variation 1 -

VARIATION 2.

Let n=11 and k=4 x l. Then the resulting output is illustrated in Figure 6.

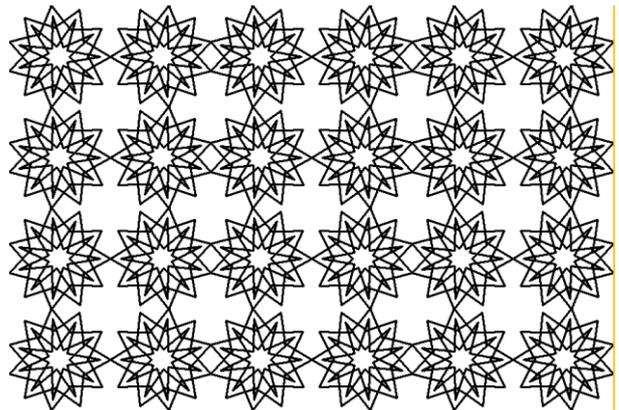


Fig. 6 : Variation 2

VARIATION 3.

Let n=11 and k=1.5x1; Then the resulting output is illustrated in Figure 7.

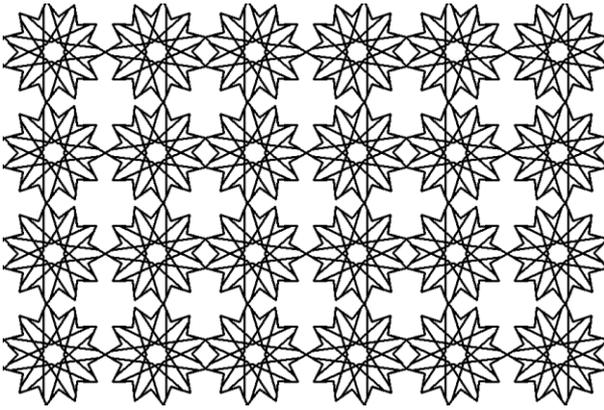


Fig. 7 : Variation 3

VARIATION 4.

Let $n=11$. By juxtaposing $k=1.5 \times 1$, 3×1 and 4.5×1 we obtain the output illustrated in Figure 8.

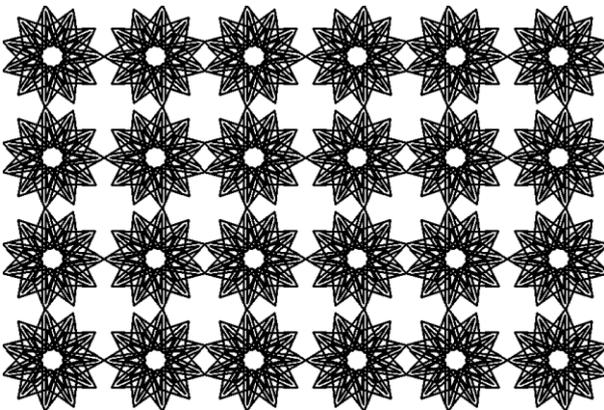


Fig. 8 : Variation 4

VARIATION 5.

Let $n=7$. if k_0 is allowed to vary in a range and each corresponding realization is colored by the color k_0 , then one obtains the output illustrated in Figure 9.

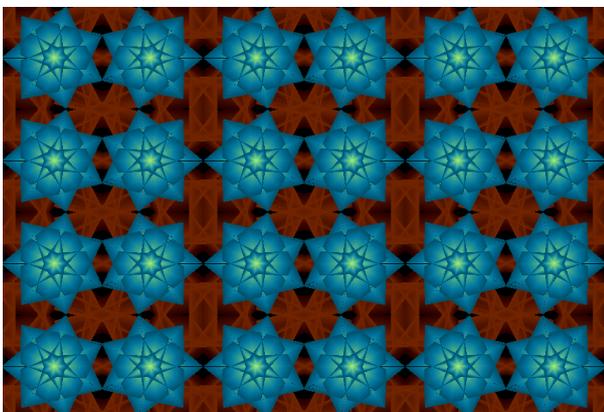


Fig. 9 : Variation 5

5. AN ALTERNATE DESIGN

A more compact style of tiling is achieved by first drawing two copies of $rosette(n,k,k_0,k_1,k_2)$ - one with its center coordinates as (i, j) and the other with its center coordinates at $(i-a+5k\cos((ang/2) \pi /180.),j-b)$ where

$$a=k2\cos(2\pi(2.5+(n-7)/2)/n)-k2\cos(2\pi(n)/n)$$

and

$$b=k2\sin(2\pi(2.5+(n-7)/2)/n)-k2\sin(2\pi(n)/n)..$$

Next their mirror images are drawn with center coordinats at $(i-5k\cos((ang/2) \pi /180.),j)$ and $(i-a,j-b)$. This results in two double stars as shown in figure 10.

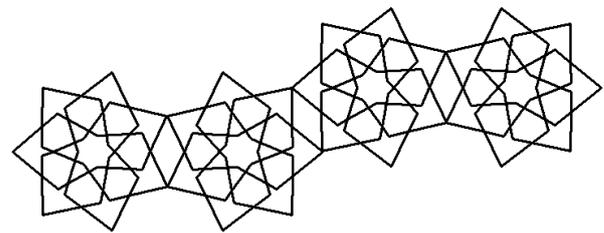


Fig. 10 : Two double rosettes

The image in Figure 10 is then repeated by increasing the center coordinates (i, j) of $rosette(n,k,k_0,k_1,k_2)$ in the first double star at an interval of $-2a+10k\cos((ang/2) \pi /180.)$ for the abscissa i and $2k2\sin(((n+1.)/4.ang) \pi /180.)$ for the ordinate j .

The main program in the code in section 3 above could be changed as follows to accommodate this new style of tiling

```
void main()
{
initwindow(1040,760);

int l=15;

int k=2.5*1;int n=11;float ang=360./n;float k0=1,i=300,j=400;

float a,b;float k1=3.5*k0,k2=5*k0;

a=k2*cos(2*3.14*(2+(n-7)/2+.5)/n)-k2*cos(2*3.14*(n)/n);

b=k2*sin(2*3.14*(2+(n-7)/2+.5)/n)-k2*sin(2*3.14*(n)/n);

for(float i=-50;i<1201;i+=-2*a+20*1*cos((ang/2)*3.14/180.))

for(float j=70;j<900;j+=2*k2*sin(((n+1.)/4.*ang)*3.14/180.))
{
rose(0,k,k0,k1,k2,n,i,j);
```

```

rose(1,k,k0,k1,k2,n,i-10*1*cos((ang/2)*3.14/180.),j);

rose(1,k,k0,k1,k2,n,i-a,j-b);

rose(0,k,k0,k1,k2,n,i-a+10*1*cos((ang/2)*3.14/180.),j-b);
}

getch();

closegraph();

}

```

The output of this is illustrated in Figure 11. One may compare Figure 11 to Figure 4.

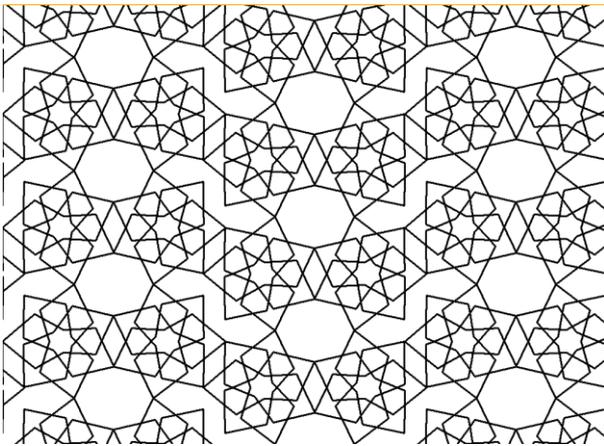


Fig. 11 : An alternate style of tiling

If n is changed to 9, k_0 is allowed to vary in a range and each corresponding realization is colored by the color k_0 , then this new style of tiling yields the output given in Figure 12. One may compare this to Figure 9.

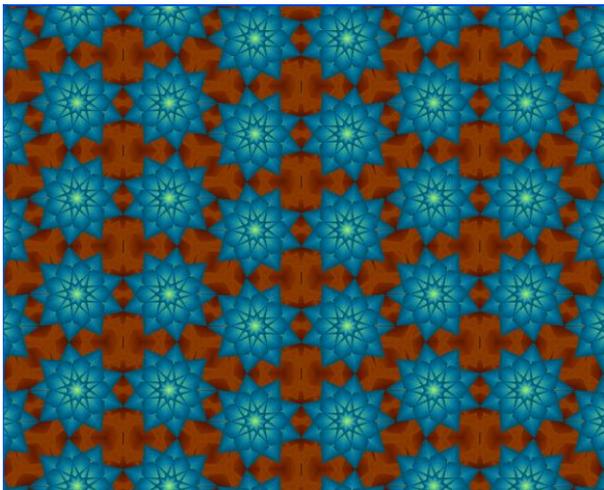


Fig. 12 : The new tiling with juxtaposition and colors

6 . A SPECIAL CASE

When n is a multiple of three, a three-fold symmetric tiling can be constructed as follows: first rosette(n,k,k_0,k_1,k_2) is constructed with its centre coordinates at (i, j) . Then another rosette(n,k,k_0,k_1,k_2) is drawn with its centre coordinates at $(i+\sqrt{3}.k_2\sin(((n+1)/4(\text{ang}))\pi /180.),j+k_2\sin(((n+1)/4(\text{ang}))\pi /180.))$.

This creates two touching rosettes which will be used as a prototype for the tiling. This prototype is displayed in figure 13, for $n=9$.

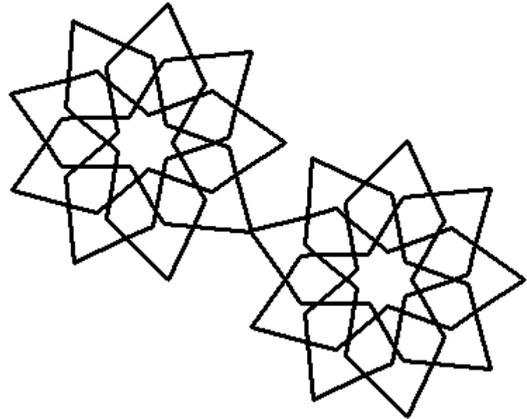


Fig. 13: The prototype for $n=9$ and $d=4$

This prototype is then repeated by increasing the center coordinates (i, j) of the first star(n,d) in the prototype by an interval of $2\sqrt{3}.k_2\sin(((n+1)/4(\text{ang}))\pi /180.)$ for the abscissa i and an interval of $2k_2\sin(((n+1)/4(\text{ang}))\pi /180.)$ for the ordinate j . The main program in the code in section 3 above could be changed as follows to accommodate this new style of tiling

```

void main()

{

initwindow(1040,760);

int l=20;

int k=2.5*1;int n=9;float ang=360./n;float k0=1,i=300,j=400;

float k1=3.5*k0,k2=5*k0;

for(float i=-70;

i<1001;i+=2*sqrt(3.)*k2*sin(((n+1)/4*ang)*3.14/180.))

for(float j=-50;

j<900;j+=2*k2*sin(((n+1.)/4.*ang)*3.14/180.))

{

rose(0,k,k0,k1,k2,n,i,j);

```

```

rose(0,k,k0,k1,k2,n,i+sqrt(3.)*k2*sin(((n+1)/4*ang)*3.14/180
.),j+k2*sin(((n+1)/4*ang)*3.14/180.));
}

getch();

closegraph();

}

```

The output of this is illustrated in Figure 14. One may compare Figure 14 to Figures 4 and 11

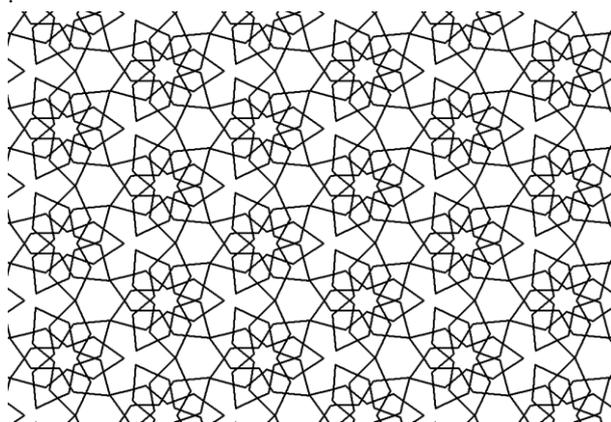


Fig. 14: The tiling for the special case

If k_0 is allowed to vary in a range and each corresponding realization is colored by the color k_0 , then this special style of tiling yields the output given in Figure 15. One may compare this to Figures 9 and 12.

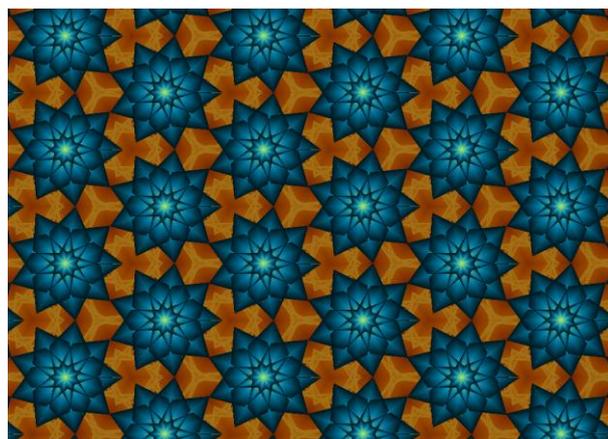


Fig. 15 : The special tiling with juxtaposition and colors

8. CONCLUSION

The tiling with with rosettes having a nonprime odd number of vertices may yield manifold symmetry.. This, as well as various new styles of tiling with even-vertexed stars would be explored in detail in future work.

9. ACKNOWLEDGMENTS

The author wishes to acknowledge his debt to the referee(s) for their constructive suggestions and encouragement

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