PTS and Companding Transform for PAPR Reduction in OFDM

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ABSTRACT
A combined model of Partial Transmit Sequence (PTS) and a Companding Transform is proposed to reduce Peak-to-Average Power Ratio (PAPR) in Orthogonal Frequency Division Multiplexing (OFDM). Simulation results demonstrate that the proposed scheme can substantially offer better PAPR reduction and BER performance.

Keywords
Complementary Cumulative Distribution Function (CCDF), Nonlinear Companding Transform (NCT), Orthogonal Frequency Division Multiplexing (OFDM), Partial Transmit Sequence (PTS), Peak-to-Average Power Ratio (PAPR).

1. INTRODUCTION
ORTHOGONAL frequency division multiplexing (OFDM) has been attracting substantial attention due to its excellent performance under severe channel condition [1]. The rapidly growing application of OFDM includes WiMAX, DAB/DVB, and 4G wireless systems. However, one critical problem in OFDM is its high peak-to-average power ratio (PAPR) [1]. High PAPR increases the complexity of analog-to-digital and digital-to-analog converters, and lowers the efficiency of power amplifiers. Over the past decade various PAPR reduction techniques have been proposed [2]. Among all these techniques the simplest solution is to clip the transmitted signal when its amplitude exceeds a desired threshold. Clipping is a highly nonlinear process it produces significant out-of-band interference (OBI).

A good remedy for the OBI is the so-called companding. The companding transform may be the most significant and attractive method due to its effectiveness and simplicity. It can be employed straightforwardly without any restriction on the OFDM system parameters, i.e. the number of subcarriers, frame format and constellation type, but at the cost of the limited in-band distortion and out-of-band interference. The method was first proposed in [3], which employed the classical μ-law transform and showed to be rather effective. Nevertheless, its average power of output signals is increased after such compression. Since then many different companding transforms with better performances have been published [4]-[7].

In order to achieve more efficient reduction in PAPR and improved BER perfomance, the paper introduced a new model using combination of two different PAPR reduction techniques one is among one of the very popular distortionless PAPR reduction technique i.e. Partial Transmit Sequence (PTS) which effectively reduces PAPR by selecting least peak power sample among all the scrambled samples [8]-[9]. Second is is a Nonlinear Companding Transform (NCT) which uses a smooth function, namely the airy special function [10].

The new proposed model exhibits advantages of both the techniques PTS which is distortionless and NCT, so improving overall system performance better than other PAPR reduction techniques.

The paper is organized as follows. In Section 2 the PAPR problem in OFDM is formulated briefly. Section 3 presents the proposed model comprises of PTS and NCT. Section 4 discusses the simulation performance. Section 5 draws the conclusion.

2. PAPR IN OFDM
Let *X(0), X(1),….., X(N-1)* represent the data sequence to be transmitted in an OFDM symbols with N subcarriers. The baseband representation of the OFDM symbol is given by (1)

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) \exp \frac{j2\pi nt}{N} \qquad 0 \le t \le T \qquad (1)$$

Where T is the duration if the OFDM symbol. According to the central limit theorem, when N is large, both the real and imaginary parts of $x(t)$ become Gaussian distributed, each with zero mean and variance of $E[|x(t)|^2]/2$, and the amplitude of the OFDM symbol follows a Rayleigh distribution. Consequently it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. Practical hardware (e.g. A/D and D/A converters, power amplifiers) has finite dynamic range; therefore the peak amplitude of OFDM signal must be limited.

PAPR is mathematically defined as:

$$PAPR = 10\log_{10} \frac{\max\left[|x(t)|^2\right]}{\frac{1}{T}\int_0^T |x(t)|^2 \, dt} \quad \text{(dB)} \qquad (2)$$

It is easy to see from (2) that PAPR reduction may be achieved by decreasing the numerator $\max[|x(t)^2|]$, increasing the denominator $(1/T)$, $\int_0^T |x(t)|^2 dt$, or both.

The effectiveness of a PAPR reduction technique is measured by the Complementary Cumulative Distribution Function (CCDF), which is the probability that PAPR exceeds some threshold, i.e.

$$\text{CCDF} = \text{Probability}(\text{PAPR} > p_0), \qquad (3)$$

where, p_0 is the threshold.

3. PROPOSED MODEL

In the proposed model as shown in Fig. 1, the IFFT output is sent to PTS block where the input data block is partitioned into disjoint subblocks or clusters which are combined to minimize the PAPR as shown in Fig. 2.

First, we define the data block as a vector, $X= [X_0\ X_1\ ....\ X_{N-1}]^T$. Then, partition X into M disjoint sets, represented by the vectors $\{X_m,\ m = 1, 2, ..., M\}$. Here, we assume that the clusters consist of a contiguous set of subcarriers and are of equal size. The objective is to optimally combine the M clusters

$$X' = \sum_{m=1}^{M} b_m X_m \tag{4}$$

Where $\{b_m,\ m = 1, 2, ..., M\}$ are weighting factors W and are assumed to be pure rotations. In the time domain

$$x' = \sum_{m=1}^{M} b_m x_m \tag{5}$$

Where x_m, the IFFT of X_m, is called the partial transmit sequence. The phase factors are chosen to minimize the PAPR of x'. Each sample is multiplies with the phase factor then find the sample which having least PAPR depends on the optimal phase factor.

Now the selected sample having least PAPR is further sent to NCT block, where companding function given in [10] will compress the signal amplitude by setting all the parameters given in [10].

The companding function *f(x)*of the transform is given in (6)

$$f(x) = \beta.sign(x).[airy(0) - airy(\alpha.|x|)] \tag{6}$$

Where, *airy(.)* is the airy function of the first kind. α is the parameter that controls the degree of companding and finally PAPR. β is the factor adjusting the average output power of the compander to the same level as the average input power as given in (7)

$$\beta = \sqrt{\frac{E\left[|x|^2\right]}{E\left[\left(airy(0) - airy(\alpha|x|)\right)^2\right]}} \tag{7}$$

Where, *E[.]* denotes the expectation.
The decompanding function is the inverse of *f(x)* as given in (8)

$$f^{-1}(x) = \frac{1}{\alpha}.sign(x).airy^{-1}\left[airy(0) - \frac{|x|}{\beta}\right] \tag{8}$$

Notice that the input to the decompander is a quantized signal with finite set of values. We can therefore numerically pre-compute inverse of *f(x)* and use table look-up to perform the decompanding in practice. In Fig. 3 Companding and decompanding profile is given. From simulation result in Section 4, we can verify the PAPR reduction and improved BER performance.

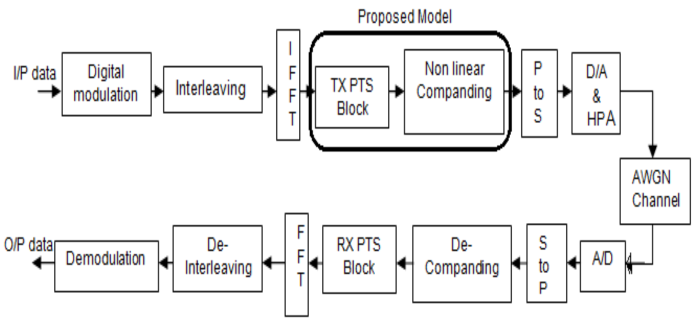

Fig. 1 Block diagram of OFDM system with proposed model

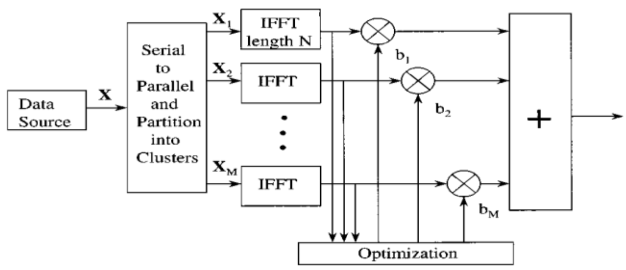

Fig.2 Block diagram of PTS approach

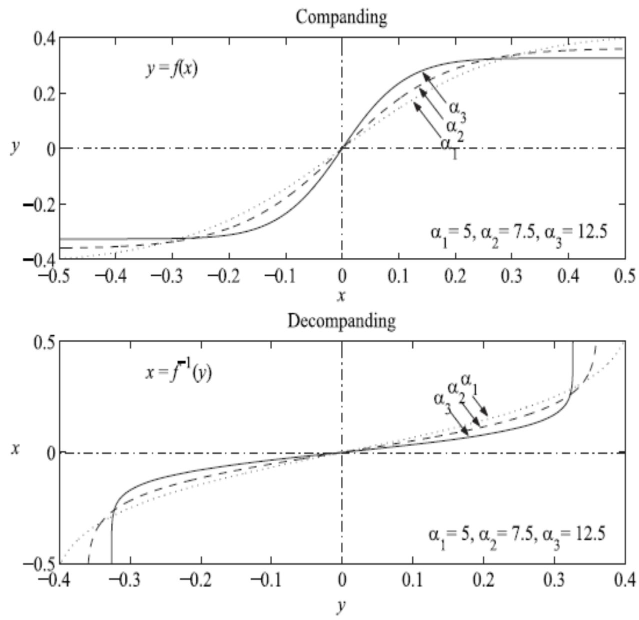

Fig.3 Companding and decompanding profile

4. SIMULATION PERFORMANCE

We consider an OFDM system with number of subcarriers $N = 256$, QPSK Modulation, number of sub-blocks $M = 64$, over-sampling factor $l = 4$, weighting factor $W = 4$, the parameter α in the companding function is chosen to be 30. Consecutively about 19.6 percent of companded signal is within the noise-suppression range of the decompanding function and compander input power is 3dBm. It is assumed that symbol timing is perfect and the frequency offset is absent at the receiver.

As shown in Fig.4, the proposed model can significantly reduce PAPR. The PAPR of the proposed model is approximately 2.9dB which is 10.2dB, 3dB and 1dB better than original signal, μ-law and NCT techniques respectively at CCDF = 10^{-3}. The comparison of CCDF of PAPR of proposed model with various companding schemes is given in Table 1.

Fig. 5 shows that the proposed scheme provides better BER as compared to other companding schemes. The E_b/N_0 of proposed scheme is approximately 11.2dB which is 3dB higher than original signal and 2.8dB better than μ-law companding scheme at BER = 10^{-4}. Comparison of BER performance of various companding scheme with proposed model is given in Table 2.

Table 1. Comparison of PAPR of proposed model with various companding schemes

COMPANDING SCHEMES	PAPR (dB) at CCDF= 10^{-3}
Original Signal	13.1
μ-law in [11]	5.9
NCT in [10]	3.9
Proposed (PTS+NCT)	2.9

Table 2. Comparison of BER performance of proposed model with various companding schemes

COMPANDING SCHEMES	E_b/N_0 at BER = 10^{-4}
Original Signal	8.2
μ-law in [11]	~14
Proposed (PTS+NCT)	11.2

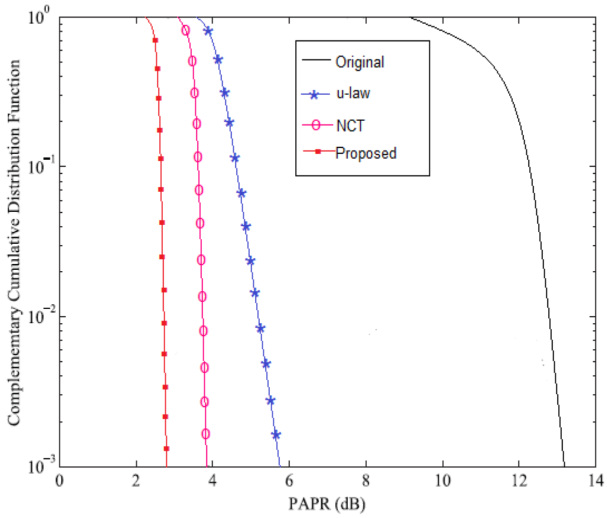

Fig. 4 CCDF statistics of comparison of proposed model (PTS+NCT) with various Companding schemes and original signal

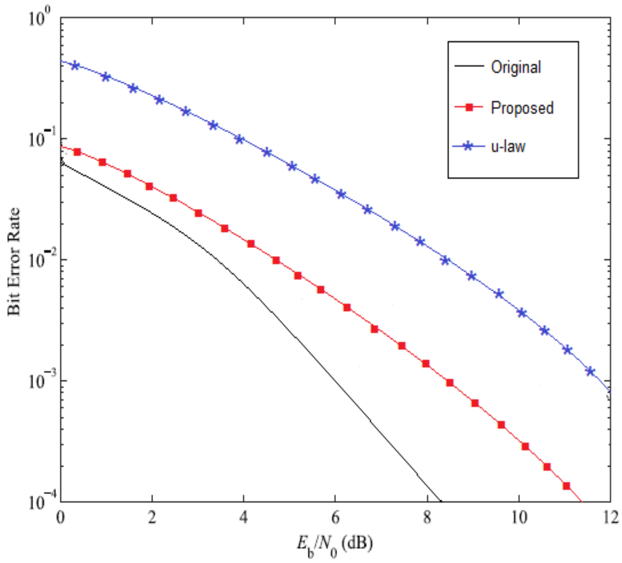

Fig. 5 BER versus E_b/N_0 comparison of proposed model (PTS+NCT) with μ-law companding and original signal

CONCLUSION

A combined model of two different PAPR reduction techniques i.e. Partial Transmit Sequence (PTS) and Nonlinear Companding Transform (NCT) is proposed and simulated. The model reduces PAPR better than other techniques and improves BER performance as shown in Fig. 4 and Fig. 5 respectively. Depends on the value of α for NCT and the value of M for PTS, overall syatem performance can be improved remarkbly. Thus, this is in line with the preceding analyses. The proposed model can be used where the overall system performance is essential along with the PAPR reduction.

5. ACKNOWLEDGMENTS

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