On L- fuzzy generalized topology

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ABSTRACT

In this paper, we introduce the concepts of L-fuzzy generalized neighborhood system(f-gns for short) and L-fuzzy generalized topology (fgt, for short)(where L is a fuzzy lattice) which are generalizations of generalized topology and neighborhood systems defined by Csaszar[5]. We also introduce and investigate with the help of these new concepts the concepts of L-(ψ_1, ψ_2) continuity and L-fuzzy generalized continuity on f-gns. The relations between these concepts are investigated and several examples are presented. ■ifx

Keywords: Fuzzy lattice, L-fuzzy generalized topology, L-fuzzy generalized neighborhood systems, L-fuzzy generalized continuity

1.. INTRODUCTION

The usual notion of a set was generalized by Zadeh [21] when he introduced the notion of a fuzzy set which has useful and increasing applications in various fields.
Since many classes of information granules are lattice ordered [2, 14], lattice theory [10] has found renewed interesting applications in diverse areas such as mathematical morphology [12], fuzzy set theory [10, 13, 14], computational intelligence [17], automated decision making [15], and formal concept analysis [8].
Generalizing the concept of a fuzzy set, Goguen [10] in 1967 introduced the concept of L-fuzzy sets, where L is a fuzzy lattice. Recall that an L-fuzzy set is given by a mapping from a universe X to a set L. In this setting, L has a mathematical structure that is at least a partially ordered set. Special emphasis was given to the case where L is a complete lattice. In 1973 Goguen[11] introduced the concept of L-fuzzy topological space as a generalization of a fuzzy topology introduced by Chang [3].
In [4, 5] Csaszar introduced the notion of generalized neighborhood systems and generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Moreover, he studied the simplest separation axioms for generalized topologies in [6, 7]. In [1, 16, 20,] G. Xun, J. Thomas and G. Abbaspour shall examine some topological properties such as, μ-Compactness, the usual continuity, the net-continuity and the net-closure property for generalized topologies.
In this paper, we introduce the notion of generalized open L-fuzzy sets, called gamma open L-fuzzy subsets, by using monotonic mappings defined on the family of L-fuzzy subsets on a set X. We also, introduce the notions of L-fuzzy generalized neighborhood systems(fgns, for short) and L-fuzzy generalized topology(fgt, for short) which can be a generalization of neighborhood systems and generalized topology. L-fuzzy generalized system induces an L-fuzzy generalized neighborhood space. We introduce the new concepts of interior and closure on fgns and fgt on a set X and investigate some of their properties. Weaker forms of continuity are introduced by using these notions. We introduce the concept of L-fuzzy generalized continuity and L-(ψ_1, ψ_2) continuity and we characterize some properties by the new interior and closure operators defined on fgns. We show that every L-(ψ_1, ψ_2) continuous function is L-fuzzy generalized continuous but the converse is not always true.

2.. PRELIMINARIES

Throughout this paper X and Y are non empty ordinary sets and $L = L(\leq, \vee, \wedge, ')$ denotes a fuzzy lattice, i.e, a complete completely distributive lattice with a smallest element 0 and a largest element 1($0 \neq 1$)and with an order reversing involution $x \longrightarrow x'(x \in L)$. We say that $\acute{x}$ is the complement of a in L. L is therefore continuous and spatial[9].
We denote by L^X the lattice of L-fuzzy subsets on X.

Definition 2.1[9] An element p of L is called prime iff $p \neq 1$ and whenever $a, b \in L$ with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$. The set of all prime elements of L will be denoted by pr(L).

Warner[18]determined the prime elements of the fuzzy lattice L^X.We have pr(L^X)=$\{x_p : x \in X \ and \ p \in pr(L)\}$ where, for each $x \in X$ and each $p \in pr(L)$, $x_p : X \longrightarrow L$ is the L-fuzzy subset defined by

$$x_p(y) = \begin{cases} p & y = x \\ 0 & y \neq x \end{cases}$$

These x_p are called the L-fuzzy points of X.The set of all L-fuzzy points of X will be denoted by pt(L^X).

Definition 2.2[19] For an L-fuzzy subset $\mu \in L^X$ and an L-fuzzy point x_p of X , we say that x_p belongs to μ, written $x_p \in \mu$ if $\mu(x) \not\leq p$.

Proposition 2.1[19] Let λ,$\mu \in L^X$ and $x_p \in pt(L^X)$. Then .

(a) If $\lambda \leq \mu$, iff ($x_p \in \lambda \Longrightarrow x_p \in \mu$).
(b) $x_p \in \lambda \wedge \mu$ iff $x_p \in \lambda$ and $x_p \in \mu$
(c) $x_p \in \bigvee_{i \in I} \lambda_i$ iff $x_p \in \lambda_i$ for some $i \in I$

Proposition 2.2[18] Every L-fuzzy subset $\mu \in L^X$ is the meet of L-fuzzy points of X.

Proposition 2.3[19] Let$f : X \longrightarrow Y$,$\lambda \in L^X$ and $\mu \in L^X$. Then the L-fuzzy subset $f(\lambda) \in L^Y$ is defined by $f(\lambda)(y) = \bigvee\{\lambda(x) : x \in f^{-1}(y)\}$. and the L-fuzzy subset $f^{-1}(\mu) \in L^X$ is defined by $f^{-1}(\mu)(x) = (\mu \circ f)(x)$ for $x \in X$. The L-fuzzy set $f(\lambda)$ (resp.$f^{-1}(\mu)$) is called the image(resp., inverse image) of λ.

3.. L-FUZZY GENERALIZED TOPOLOGY

Definition 3.1 A mapping $\gamma : L^X \longrightarrow L^Y$ is a monotonic operator on X if it satisfies: For $\lambda, \mu \in L^X, \lambda \leq \mu \Longrightarrow \gamma(\lambda) \leq \gamma(\mu)$.
In what follows $\Gamma(X)$ denotes the family of all monotonic operators on X .

Definition 3.2 Let γ be a monotonic operator on a set X. An L- fuzzy subset $\lambda \in L^X$ is said to be γ open if $\lambda \leq \gamma(\lambda)$. One may notice that the empty L-fuzzy subset $\underline{0}$ is γ-open. Also the union of γ-open L-fuzzy subsets isγ-open L-fuzzy subset.

Definition 3.3 A family τ of L-fuzzy subset on X is said to be L-fuzzy generalized topology on X (fgt, for short) if $\underline{0} \in \tau$ and τ is closed under arbitrary union of L-fuzzy sets.

Proposition 3.1 Let γ be a monotonic operator on a set X.Then the family τ_γ of all γ-open L-fuzzy subsets is an fgt on X. We say that τ_γ is the fgt on X induced by γ.

Proof: obvious.

In what follow any function $\gamma : L^X \longrightarrow L^X$ is assumed (otherwise stated) to be a monotonic operator on X. Also, by $\Im(X)$, we denote the collection of all fgt on a set X. .

Proposition 3.2 Let $\tau \in \Im(X)$. Then there exists a monotonic operator $\gamma_\tau : L^X \longrightarrow L^X$ such that τ is the family of all γ_τ-open L-fuzzy subsets. Furthermore, we have for $\lambda \in L^X$ (a) $\gamma_\tau(\underline{0}) = 0$ $\quad (b)\gamma_\tau(\lambda) \leq \lambda \quad (c)\gamma_\tau(\gamma_\tau(\lambda)) = \gamma_\tau(\lambda)$. We say that γ_τ is the monotonic operator on X induced by γ.

Proof: For $\lambda \in L^X$, define $\gamma_\tau(\lambda) = \bigvee\{\mu \in \tau : \mu \leq \lambda\}$. It is clear that $\gamma_\tau : L^X \longrightarrow L^X$ is a monotonic operator. Also $\gamma_\tau(\lambda) \in \tau$ and $\gamma_\tau(\lambda) \leq \lambda$ .We show that τ is the family of all γ_τ-open L-fuzzy subsets. Let$\lambda \in \tau$ .Then $\gamma_\tau(\lambda) = \bigvee\{\mu \in \tau : \mu \leq \lambda\} = \lambda$. Hence λ is γ_τ-open. Let λ be γ_τ-open L-fuzzy set . Then $\lambda \leq \gamma_\tau(\lambda) = \bigvee\{\mu \in \tau : \mu \leq \lambda\} \leq \lambda$.Hence $\lambda = \gamma_\tau(\lambda)$ and $\lambda \in \tau$; since τ is a fgt on X.Thus τ is the family of all γ_τ-open L-fuzzy subsets.
One may notice that for $\lambda \in L^X$, we have

$$\lambda \in \tau \quad iff \quad \lambda = \gamma_\tau(\lambda)$$

This proves (a) and (c) since $\underline{0}, \gamma_\tau(\lambda) \in \tau$.Also (b) follows from the definition of γ_τ .

Proposition 3.3 Let $\tau \in \Im(X)$. Then $(a)\gamma_{\tau_\gamma} = \gamma$ and $(b)\tau_{\gamma_\tau} = \tau$.

Proof: This is a direct consequence of Proposition3.1.

Let us consider another way for obtaining a fgt on X.

Definition 3.4 Let $\psi : pt(L^X) \longrightarrow P(L^X)$.Then ψ is called a fuzzy generalized neighborhood operator and $\psi(x_p)$(for $x \in X$ and $p \in pr(L)$) is called a fuzzy generalized neighborhood system(fgns for short) for x_p if it satisfies

$$\mu \in \psi(x_p) \Longrightarrow x_p \in \mu.$$

In this case,we say that $\mu \in \psi(x_p)$ is a fuzzy generalized neighborhood of x_p(fgn for short). By $\Psi(X)$ we denote the collection of all fuzzy generalized neighborhood operators on X.

Proposition 3.4 Let $\psi \in \Psi(X)$. Then there exists a fgt τ_ψ, on X, such that

$$\mu \in \tau_\psi \ \ iff \ \ \forall x_p \in \mu \ \exists \ \lambda \in \psi(x_p) \ \ such \ \ that \ \ \lambda \leq \mu$$

Proof: It is clear that $\underline{0} \in \tau_\psi$. Let $\mu_i \in \tau_\psi$ and let $x_p \in \bigvee \mu_i$. Then $x_p \in \mu_i$ for some i. Hence $\exists \ \lambda_i \in \psi(x_p) \ s.t \ \lambda_i \leq \mu_i$.Thus $\exists \ \lambda_i \in \psi(x_p) \ s.t \ \lambda_i \leq \bigvee \mu_i$. In otherwords $\bigvee \mu_i \in \tau_\psi$. Consequently,$\tau_\psi$ is a fgt on X.

Proposition 3.5 Let $\tau \in \Im(X)$. Then there exists an L-fuzzy generalized neighborhood operator ψ_τ , on X, such that

$$\mu \in \tau \ \ iff \ \ \forall x_p \in \mu \ \exists \ \lambda \in \psi_\tau(x_p) \ \ such \ \ that \ \ \lambda \leq \mu$$

We say that ψ_τ is the L-fuzzy generalized operator induced by τ.

Proof: Define $\lambda \in \psi_\tau(x_p) \iff x_p \in \lambda \in \tau$. It is clear that ψ_τ is a fgn operator on X. Now, let $\mu \in \tau$ and let $x_p \in \mu$. Then $\exists \, \mu \in \psi_\tau(x_p)$ such that $\mu \leq \mu$.

Proposition 3.6 Let $\tau \in \Im(X)$. Then $\tau_{\psi_\tau} = \tau$.

Proof: This is a direct consequence of Propositions 3.4 and 3.5.

Definition 3.5 Let $\tau \in \Im(X)$ and $\lambda \in L^X$. The τ-interior and τ-closure of λ are defined and denoted respectively,by

$$i_\tau(\lambda) = \vee\{\mu \in \tau : \mu \leq \lambda\} \ \ and \ \ c_\tau(\lambda) = \wedge\{\grave{\mu} \in \tau : \mu \geq \lambda\}$$

If $\tau = \tau_\gamma$, we write i_γ for i_{τ_γ} and c_γ for c_{τ_γ}. For a fuzzy generalized neighborhood operator ψ, we write i_ψ(resp. c_ψ) instead of i_{τ_ψ}(resp. c_{τ_ψ}).

Definition 3.6 Let $\psi \in \Psi(X)$ and $\lambda \in L^X$.The two operators $I_\psi, I_\psi : L^X \longrightarrow L^X$ are defined by

$$(i)I_\psi(\lambda) = \bigvee\{x'_p \in L^X : \ \exists \mu \in \psi(x_p) \ \ s.t \ \ \mu \leq \lambda\}$$

$$(ii)\gamma_\psi(\lambda) = \bigwedge\{x_p \in L^X : \ \exists \mu \in \psi(x_p) \ \ s.t \ \ \mu \leq \lambda'\}$$

Proposition 3.7 Let $\psi \in \Psi(X)$ and $\lambda \in L^X$. Then

(a) I_ψ and $\gamma_\psi \in \Gamma(X)$,i.e.$I_\psi$ and γ_ψ are monotonic operators.
(b) $I_\psi(\lambda) \leq \lambda$ and $\gamma_\psi(\lambda) \geq \lambda$
(c) $\gamma_\psi(\lambda) = (I_\psi(\lambda'))'$
(d) $i_\psi(\lambda) \leq I_\psi(\lambda)$ and $c_\psi(\lambda) \geq \gamma_\psi(\lambda)$

Proof:(a) Obvious. (b) Let $y_{p_1} \in I_\psi(\lambda)$, then there exists $x_p \in L^X$ and $\mu \in \psi(x_p)$ such that $y_{p_1} \in x'_p$, where $x'_p(y) = \{^{p'}_{0} \ {}^{if \ y=x}_{if \ y\neq x}$ and $\mu \leq \lambda$. Hence $x'_p(y) \not\leq p_1$, i.e $x'_p(y) = p' \not\leq p_1$. Thus $y = x$ and $y_{p_1} = x_{p_1} \in x'_p$. Since $x_p \in \mu$, then $\mu(x) \not\leq p$ and hence $\mu'(x) \leq p$. In fact, assume that $\mu'(x) \not\leq p$. Since $\mu(x) \wedge \mu'(x) = 0 \leq p$ and $p \in pr(L)$, then $\mu(x) \leq p$ or $\mu'(x) \leq p$, a contradiction. Therefore $\mu'(x) \leq p \not\geq p_1'$ and hence $\mu(x) \not\leq p_1$. So $x_{p_1} \in \mu \leq \lambda$ and $x_{p_1} \in \lambda$. Consequently, $I_\psi(\lambda) \leq \lambda$. Similarly, we can show that $\gamma_\psi(\lambda) \geq \lambda$.

(c)$(I_\psi(\lambda'))' = (\bigvee\{x'_p \in L^X : \ \exists \, \mu \in \psi(x_p) \ s.t \ \mu \leq \lambda'\}'$ $=\bigwedge\{x_p \in L^X : \ \exists \, \mu \in \psi(x_p) \ s.t \ \mu \leq \lambda'\} = \gamma_\psi(\lambda)$.

(d)Let $y_{p_1} \in i_\psi(\lambda) = \bigvee\{\mu \in \tau_\psi : \mu \leq \lambda\}$. Hence $\exists \, \mu \in \tau_\psi$ s.t $y_{p_1} \in \mu \leq \lambda$. So, $\exists \, \delta \in \psi(x_p)$ s.t $y_{p_1} \in \delta \leq \mu \leq \lambda$. Since $y'_{p_1}(y) = p'_1 \not\leq p_1$, then $\exists \, y'_{p_1} \in L^X$ s.t $y_{p_1} \in y'_{p_1}$ and $y_{p_1} \in \delta \leq \mu$. Consequently, $y_{p_1} \in I_\psi(\lambda)$. Hence $i_\psi(\lambda) \leq I_\psi(\lambda)$.The other part follows directly from (c).

Remark 3.1 In general, $I_\psi(\lambda) \neq i_\psi(\lambda)$ and $\gamma_\psi(\lambda) \neq c_\psi(\lambda)$. The following example illustrates this fact.

Example 3.1 Let $X = \{x, y\}$ and $L = \{0, a, b, c, d, e, f, 1\}$ be a fuzzy lattice described as Figure 1

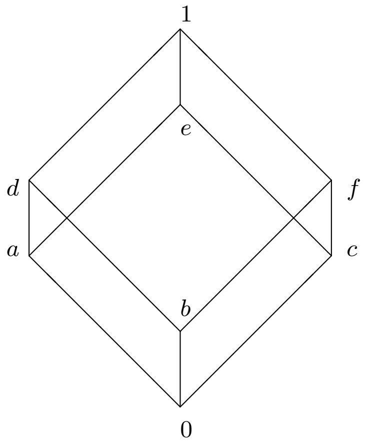

figure 1

It is clear that $pr(L) = \{d, e, f\}$. So $pt(L^X) = \{x_d, x_e, x_f, y_d, y_e, y_f\}$. Let us define $\psi : pt(L^X) \longrightarrow P(L^X)$ by $\psi(x_d) = \{\mu_6, \mu_7\}$,$\psi(x_e) = \{\mu_7\}$, $\psi(x_f) = \{\mu_8, \mu_5, \mu_6\}$ and $\psi(y_d) = \psi(y_e) = \psi(y_f) = 0$. Where $\mu_1(x) = 0$, $\mu_2(x) = a$, $\mu_3(x) = b$, $\mu_4(x) = c$, $\mu_5(x) = d$, $\mu_6(x) = e$, $\mu_7(x) = f$, $\mu_8(x) = 1$ and $\mu_i(y) = 0$, i=1,2,.....,8.
Then $\tau_\psi = \{\mu_1, \mu_6, \mu_7, \mu_8\}$. It is clear that $\mu_1 \in \tau_\psi$. Also $\mu_2 \notin \tau_\psi$. In fact, $\mu_2(x) = a \not\leq f$ and hence $x_f \in \mu_2$. So, $\not\exists$ $\lambda \in \psi(x_f) \leq x_f$. Similarly, $\mu_3, \mu_4, \mu_5 \notin \tau_\psi$. Now, $\mu_6 \in \tau_\psi$. In fact, $x_d, x_f \in \mu_6$ and $\exists$ $\lambda = \mu_6 \in \psi(x_d)$ and $\psi(x_f)$ s.t $\lambda \leq \mu_6$. Similarly, $\mu_7, \mu_8 \in \tau_\psi$.
Now, we show that $i_\psi(\mu_5) \neq I_\psi(\mu_5)$. In fact, $i_\psi(\mu_5) = \bigvee\{\lambda \in \tau_\psi : \lambda \leq \mu_5\} = \mu_1$. Also $I_\psi(\mu_5) = \bigvee\{\acute{x_p} \in L^X : \exists\lambda \in \psi(x_p)\ s.t\ \lambda \leq \mu_5\} = \mu_2$, since $\exists$ $\lambda = \mu_5 \in \psi(x_f)$ s.t $\lambda \leq \mu_5$. Hence $x'_f \in I_\psi(\mu_5)$ and thus $\mu_2 = x'_f = I_\psi(\mu_5)$. Consequently, $I_\psi(\mu_5) = \mu_2 \neq \mu_1 = i_\psi(\mu_5)$.
We show that $\gamma_\psi(\mu_4) \neq c_\psi(\mu_4)$. Since $\tau'_\psi = \{\mu_1, \mu_2, \mu_3, \mu_8\}$, then $c_\psi(\mu_4) = \bigwedge\{\lambda \in \tau'_\psi : \mu_4 \leq \lambda'\}$ =μ_8. Also,$\gamma_\psi(\mu_4) = \acute{(}I_\psi(\mu'_4)') = (I_\psi(\mu_5))' = \mu'_2 = \mu_7 \neq \mu_8 = c_\psi(\mu_4)$.

Proposition 3.8 Let $\tau \in \Im(X)$. Then $I_{\psi_\tau} = i_{\psi_\tau}$ and $\gamma_{\psi_\tau} = c_{\psi_\tau}$

Proof: Let $y_{p_1} \in I_{\psi_\tau}(\lambda)$. Then $\exists$ $x_p \in L^X$ and $\mu \in \psi_\tau(x_p)$ s.t $y_{p_1} \in x'_p$ and $\mu \leq \lambda$. Hence $y = x$ and $x'_p(x) = p' \not\leq p_1$. Therefore, $\mu \in \tau_{\psi_\tau} = \tau$. Since $x_p \in \mu$, then $\mu(x) \not\leq p$ and hence $\mu'(x) \leq p \not\geq p'_1$. So $\mu(x) \not\leq p_1$ and therefore $x_{p_1} \in \mu \leq \lambda$. Consequently, $x_{p_1} \in i_{\psi_\tau}(\lambda)$.The inclusion $i_{\psi_\tau} \subset I_{\psi_\tau}$ follows from Proposition 3.6. The other equality results by considering the complements.

4.. L-FUZZY GENERALIZED CONTINUITY

Definition 4.1 Let $\tau_1 \in \Im(X_1)$ and $\tau_2 \in \Im(X_2)$. Let $f : X_1 \longrightarrow X_2$. Then f is called (τ_1, τ_2) continuous if

$$\mu \in \tau_2 \Longrightarrow f^{-1}(\mu) \in \tau_1$$

We obtain another (more general) kind of L-fuzzy generalized continuity.

Definition 4.2 Let X_1 and X_2 be non empty sets and $f : X_1 \longrightarrow X_2$. Let $\psi_1 \in \Psi(X_1)$ and $\psi_2 \in \Psi(X_2)$. Then f is called (ψ_1, ψ_2) continuous if

$$\forall\ x_p \in pt(L^X)\ and\ \mu \in \psi_2(f(x))_p\ \exists\ \lambda \in \psi_1(x_p)\ s.t\ f(\lambda) \leq \mu$$

The following propositions give the relation between the above two kinds of L-fuzzy generalized continuity.
Proposition 4.1 Let X_1 and X_2 be non empty sets and $f : X_1 \longrightarrow X_2$. Let $\psi_1 \in \Psi(X_1)$ and $\psi_2 \in \Psi(X_2)$. If f is (ψ_1, ψ_2) continuous , then f is $(\tau_{\psi_1}, \tau_{\psi_2})$ continuous.

Proof: Let $\mu \in \tau_{\psi_2}$ and $x_p \in f^{-1}(\mu)$. Then $\mu(f(x)) = (f^{-1}(\mu))(x) \not\leq p$. Hence, $(f(x))_p \in \mu$. Since $\mu \in \tau_{\psi_2}$, then $\exists$ $\lambda \in \psi_2(f(x))_p$ s.t $\lambda \leq \mu$. Since f is (ψ_1, ψ_2) continuous , then $\exists$ $\delta \in \psi_1(x_p)$ s.t $f(\delta) \leq \lambda \leq \mu$. Hence $\delta \leq f^{-1}(\mu)$ and consequently $f^{-1}(\mu) \in \tau_{\psi_1}$.

Remark 3.1 The converse of the previous proposition is not generally true. The following example illustrates this idea.

Example 4.1 Let $X = \{x, y\}$ and $L = \{0, a, b, c, d, e, f, 1\}$ be a fuzzy lattice described as Figure 1. Let $\psi_1, \psi_2 \in \Psi(X)$ be defined as follows
$\psi_1(x_d) = \psi_2(x_d) = \{\mu_6, \mu_7\}$,$\psi_1(x_e) = \psi_2(x_e) = \{\mu_7\}$, $\psi_1(x_f) = \{\mu_8, \mu_6\}$, $\psi_2(x_f) = \{\mu_8, \mu_5, \mu_6\}$ and $\psi_i(y_d) = \psi_i(y_e) = \psi_i(y_f) = 0$ (i=1,2).
It is clear that $\tau_{\psi_1} = \tau_{\psi_2} = \{\mu_1, \mu_8, \mu_6, \mu_7\}$. Let $f = id_X$ i.e, $f : X \longrightarrow X$ such that $f(x) = x$ for all $x \in X$. Hence $f(\mu) = \mu$ for all $\mu \in L^X$ and so $f(x_f) = (f(x))_f = x_f$. It is clear that f is $(\tau_{\psi_1}, \tau_{\psi_2})$ continuous. Now, $\mu_5 \in \psi_2(f(x_f)) = \psi_2(x_f)$, but $\not\exists$ $\lambda \in \psi_1(x_f)$ s.t $f(\lambda) = \lambda \leq \mu_5$. Consequently, f is not (ψ_1, ψ_2) continuous.

Proposition 4.2 Let X_1 and X_2 be non empty sets and $f : X_1 \longrightarrow X_2$. Let $\psi_1 \in \Psi(X_1)$ and $\psi_2 \in \Psi(X_2)$. If f is $(\tau_{\psi_1}, \tau_{\psi_2})$ continuous and $\psi_2 = \psi_{\tau_1}$ for some fgt τ_1 on X_2. then f is (ψ_1, ψ_2) continuous.

Proof: Let $\mu \in \psi_2(f(x)_p) = \psi_{\tau_1}(f(x)_p$. Hence $f(x_p) \in \mu$ and $\mu \in \tau_1$. By Proposition 3.6, $\tau_{\psi_2} = \tau_{\psi_{\tau_1}} = \tau_1$ and thus $\mu \in \tau_{\psi_2}$. So, $x_p \in f^{-1}(\mu) \in \tau_{\psi_1}$ because f is $(\tau_{\psi_1}, \tau_{\psi_2})$ continuous. Therefore $\exists$ $\lambda \in \psi_1(x_p)\ \ s.t\ \ \lambda \leq f^{-1}(\mu)$. Then $f(\lambda) \leq \mu$ and consequently, f is (ψ_1, ψ_2) continuous.

Proposition 4.3 Let X_1 and X_2 be non empty sets and $f : X_1 \longrightarrow X_2$. Let $\psi_1 \in \Psi(X_1)$ and $\psi_2 \in \Psi(X_2)$. The following statements are equivalent

(a) f is (ψ_1, ψ_2) continuous
(b) $f(\gamma_{\psi_1}(\mu)) \leq \gamma_{\psi_2}(f(\mu))\ \forall\ \ \mu \in L^{X_1}$
(c) $\gamma_{\psi_1}(f^{-1}(\lambda)) \leq f^{-1}(\gamma_{\psi_2}(\lambda))\ \forall\ \ \lambda \in L^{X_2}$

Proof: $(a) \Longrightarrow (b)$ Let $x_p \in \gamma_{\psi_1}(\mu)$ and assume that $f(x_p) \notin \gamma_{\psi_2}(f(\mu)) = (I_{\psi_2}(f(\acute{\mu})))'$. Hence $f(x_p) \in I_{\psi_2}(f(\mu'))$ and so $\exists$ $\lambda \in \psi_2(f(x_p))\ s.t\ \lambda \leq f(\mu') = (f(\mu))'$. Therefore $\lambda \wedge f(\mu) = 0$. Since f is (ψ_1, ψ_2) continuous, then $\exists$ $\delta \in \psi_1(x_p)\ s.t\ f(\delta) \leq \mu$. Hence $f(\delta) \wedge f(\mu) \leq \lambda \wedge f(\mu) = 0$ and therefore $f(\delta \wedge \mu) = 0$. Thus $\delta \wedge \mu = 0$ i.e, $\delta \leq \acute{\mu}$. Then $\exists$ $x'_p \in L^X\ and\ \delta \in \psi_1(x_p)\ s.t\ x_p \in x'_p\ and\ \delta \leq \mu'$. So $x_p \in I_{\psi_1}(\mu')$ and thus $x_p \notin (I_{\psi_1}(\mu'))' = \gamma_{\psi_1}(\mu)$. Consequently, $f(x_p) \notin f(\gamma_{\psi_1}(\mu))$.

$(b) \Longrightarrow (c)$ Let $\mu = f^{-1}(\lambda)$. Then by (b) $f(\gamma_{\psi_1}(\mu)) \leq \gamma_{\psi_2}(f(\mu)) = \gamma_{\psi_2}(f(f^{-1}(\lambda))) \leq \gamma_{\psi_2}(\lambda)$. Hence $\gamma_{\psi_1}(\mu) = \gamma_{\psi_1}(f^{-1}(\lambda)) \leq f^{-1}(\gamma_{\psi_2}(\lambda))$.

$(c) \Longrightarrow (a)$ Let $\lambda \in \psi_2(f(x_p)$, then $f(x_p) \in \lambda$. Let $\mu = \lambda'$, then $f(x_p) \notin \gamma_{\psi_2}(\mu)$. In fact, $\exists$ $\lambda \in \psi_2(f(x_p))\ s.t\ \lambda = \mu' \leq \mu'\ but\ f(x_p) \notin f(x_p)$. So, $x_p \notin f^{-1}(\gamma_{\psi_2}(\mu))$. By (c), $x_p \notin \gamma_{\psi_1}(f^{-1}(\mu))$ and hence, $\exists$ $\delta \in \psi_1(x_P)\ \ s.t\ \ \delta \leq (f^{-1}(\mu))'$. Thus $\delta \wedge f^{-1}(\mu) = 0$ and then $f(\delta) \wedge \mu = 0$. Hence $f(\delta) \leq \mu' = \lambda$ and consequently, f is (ψ_1, ψ_2) continuous.

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