

# Parameter Estimation and Control of Induction Machine using a new Recursive Algorithm

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## ABSTRACT

Two schemes control for induction machine are proposed and compared. A new off-line recursive algorithm is used to estimate the parameters of a commercial 1 kW induction motor, which is described by a multivariable state space mathematical model. The self-tuning control scheme and the control scheme with disturbance compensation are proposed. Those techniques are based on the concept of quadratic optimal control. The schemes control performance is tested using stator current, voltage and speed measurements. The obtained results demonstrate the proposed schemes's effectiveness.

## Keywords:

Induction machine, Recursive algorithm, Off-line parameter estimation, Control schemes

## 1. INTRODUCTION

The low maintenance costs, the robustness, the usability are advantages that characterize the induction machine. This latter is widely used in many industrial applications and currently there is an significant effort being done toward obtaining advanced induction machines [5], [6]. Control schemes of such motors require an accurate knowledge of at least some of the induction machine parameters. Any gap between the parameter values used within the controller and actual parameter values in the motor leads to a worsening in the drive performance.

A wide variety of modeling, signal processing, and control schemes have been developed to ensure that induction motors function reliably. Investigations that describe the scope of this existing research are presented in [7], [8]. The parametric estimation of the electric motors have widely integrated the optimization methods. Most of the estimation procedures use complex algorithms for the parametric estimation. The major difficulty behind the optimization techniques is the choice of the motor physical model and of the optimization algorithm.

The paper is organized in four sections. Next section presents the recursive parameter estimation algorithm. Section three was reserved to testing the proposed algorithm by application for an induction machine. Section four includes the self-adjusting control of induction machine. Section five presents the control scheme with disturbance compensation. Finally a brief conclusion is presented in section six.

## 2. RECURSIVE PARAMETER ESTIMATION ALGORITHM

Let us consider a linear multivariable system described by the following discrete-time state-space mathematical model:

$$\begin{aligned} x(k+1) &= A(k+1)x(k) + B(k+1)u(k) + v(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \quad (1)$$

where  $u(k) \in R^r$ ,  $y(k) \in R^z$  and  $x(k) \in R^n$  represent the input vector, the output vector and the state vector of the system at the discrete time  $k$ . The matrices  $A(k+1)$ ,  $B(k+1)$  and  $C$  represent the state matrix, the command matrix and the measurement matrix, respectively,  $v(k) \in R^n$  is the noise vector which acts on the system,  $e(k)$  indicates the noise which affects the measurement of the output  $y(k)$ .

The parameter estimation problem of the dynamic multivariable system described as a state mathematical model is composed of an estimation procedural development that allows us to estimate the unknown parameters of the  $A(k+1)$  and  $B(k+1)$  matrices.

The formulation of this problem is based on experimental measurements (input and state sequences) originated in the considered system and minimizing a quadratic criterion comprising the estimation error (also called prediction error), which is defined as:

$$\delta(k) = x(k) - x_p(k) \quad (2)$$

with  $x_p(k)$  the state vector of the adjustable model, as follows:

$$x_p(k+1) = \hat{A}(k+1)x(k) + \hat{B}(k+1)u(k) \quad (3)$$

where  $\hat{A}(k+1)$  and  $\hat{B}(k+1)$  are the estimated matrices of  $A(k+1)$  and  $B(k+1)$  at the discrete-time  $k+1$ , respectively.

For the estimation of the parameters of the two matrices  $A(k+1)$  and  $B(k+1)$  of the state-space mathematical model, This recursive parameter estimation algorithm, which is developed for multivariable systems, is used:

$$\begin{aligned} \hat{A}(k+1) &= \hat{A}(k) + \xi(k)R\delta(k)x^T(k-1) \\ \hat{B}(k+1) &= \hat{B}(k) + \xi(k)R\delta(k)u^T(k-1) \\ \delta(k) &= x(k) - \hat{A}(k)x(k-1) - \hat{B}(k)u(k-1) \end{aligned} \quad (4)$$

$$\xi(k) = \frac{l(k)}{\lambda_R[u^T(k-1)u(k-1) + x^T(k-1)x(k-1)]}$$

where  $l(k)$  is a positive parameter gain and  $\lambda_R$  is the maximum eigenvalue of the matrix  $R$ .

The analysis of the stability conditions leads to the following condition, which is related to the choice of the gain intervening in the algorithm (4):

$$1 < l(k) < 2 \quad (5)$$

### 3. OFF-LINE PARAMETER ESTIMATION OF INDUCTION MACHINE

In this section, the recursive algorithm which is presented in the previous section is applied to the estimation of the induction machine's parameters.

The induction machine is described by the following discrete-time state-space mathematical model:

$$\begin{aligned} x(k+1) &= A(k+1)x(k) + Bu(k) + v(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \quad (6)$$

where  $u(k)$ ,  $y(k)$  and  $x(k)$  represent the input vector, the output vector and the state vector of the induction machine. These are defined by:

$$u^T(k) = [u_{ds}(k) \ u_{qs}(k)] \quad (7)$$

$$y^T(k) = [i_{ds}(k) \ i_{qs}(k)] \quad (8)$$

and

$$x^T(k) = [i_{ds}(k) \ i_{qs}(k) \ \phi_{dr}(k) \ \phi_{qr}(k)] \quad (9)$$

The state vector  $x(k)$  contains the stator currents  $i_{ds}(k)$  and  $i_{qs}(k)$  and the rotor flux  $\phi_{dr}(k)$  and  $\phi_{qr}(k)$ . The input and the output of the induction machine correspond to the stator voltage  $u_{ds}(k)$  and  $u_{qs}(k)$  and to the stator currents  $i_{ds}(k)$  and  $i_{qs}(k)$ , respectively.

The matrices  $A(k)$ ,  $B$  and  $C$  are defined, respectively, by the following expressions:

$$A(k) = \begin{pmatrix} a_{11} & a_{12} & a_{13}(k) & a_{14}(k) \\ a_{21} & a_{22} & a_{23}(k) & a_{24}(k) \\ a_{31}(k) & a_{32}(k) & a_{33} & a_{34} \\ a_{41}(k) & a_{42}(k) & a_{43} & a_{44} \end{pmatrix} =$$

$$\begin{pmatrix} -\frac{R_s}{L_s - M^2 L_r} & \frac{R_r M}{L_r(L_s - M^2 L_r)} & \frac{M^2 L_r \omega_m(k)}{L_s - M^2 L_r} & \frac{M \omega_m(k)}{L_s - M^2 L_r} \\ \frac{R_s M}{L_r(L_s - M^2 L_r)} & -\frac{R_r L_s}{L_r(L_s - M^2 L_r)} & -\frac{M L_s \omega_m(k)}{L_r(L_s - M^2 L_r)} & -\frac{L_s \omega_m(k)}{L_s - M^2 L_r} \\ \frac{M^2 L_r \omega_m(k)}{L_s - M^2 L_r} & -\frac{M \omega_m(k)}{L_s - M^2 L_r} & -\frac{R_s}{L_s - M^2 L_r} & \frac{R_r M}{L_r(L_s - M^2 L_r)} \\ \frac{M L_s \omega_m(k)}{L_r(L_s - M^2 L_r)} & \frac{L_s \omega_m(k)}{L_s - M^2 L_r} & \frac{R_s M}{L_r(L_s - M^2 L_r)} & -\frac{R_r L_s}{L_r(L_s - M^2 L_r)} \end{pmatrix} \quad (10)$$

$$\begin{aligned} B &= \begin{pmatrix} b_{11} & 0 \\ b_{21} & 0 \\ 0 & b_{33} \\ 0 & b_{43} \end{pmatrix} \\ &= \begin{pmatrix} \frac{L_s^2}{L_s - M^2 L_r} & 0 \\ -\frac{M L_s}{L_r(L_s - M^2 L_r)} & 0 \\ 0 & \frac{L_s^2}{L_s - M^2 L_r} \\ 0 & -\frac{M L_s}{L_r(L_s - M^2 L_r)} \end{pmatrix} \end{aligned} \quad (11)$$

and

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (12)$$

The parameters  $L_s$ ,  $L_r$ ,  $M$ ,  $R_s$  and  $R_r$  represent the stator cyclic inductance, the rotor cyclic inductance, the stator-rotor mutual cyclic inductance, the stator resistance, and the rotor resistance, respectively. The time-varying parameters of the matrix  $A(k+1)$  depend on the mechanical speed  $\omega_m(k)$ .

The parameter estimation problem of the induction machine, described as a state-space mathematical model, is composed of an estimation procedural development that allows us to estimate the unknown constant parameters and the unknown varying-time parameters, which are depending on the mechanical speed  $\omega_m(k)$ , of the matrices  $A(k+1)$  and  $B$ .

The evolution curves of the estimated parameters  $\hat{a}_{11}(k)$ ,  $\hat{a}_{12}(k)$ ,  $\hat{a}_{13}(k)$  and  $\hat{a}_{14}(k)$  are shown in Figures 1 and 2.

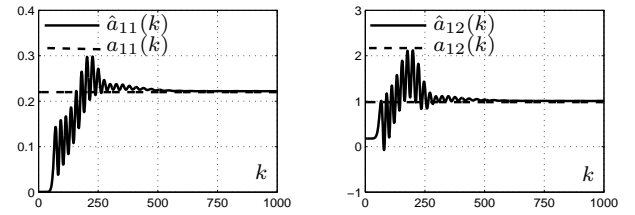


Fig. 1. Evolution curves of the estimated parameters  $\hat{a}_{11}(k)$  and  $\hat{a}_{12}(k)$ .

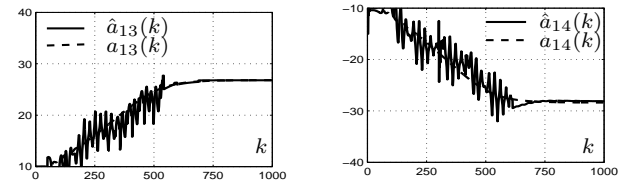


Fig. 2. Evolution curves of the estimated parameters  $\hat{a}_{13}(k)$  and  $\hat{a}_{14}(k)$ .

#### 4. SELF-TUNING CONTROL SCHEME

In this section, two control schemes, which can be applied to the considered induction machine, are developed. For the synthesis of an explicit self-adjusting control scheme of induction machine, the recursive parameter estimation algorithm (4) is used. To calculate the control, the following quadratic criterion is minimized:

$$J(k) = \frac{1}{2} \sum_0^{\infty} x^T(k)Qx(k) + u^T(k)Gu(k) \quad (13)$$

where  $Q$  and  $G$  represent positive definite matrices.

The control which minimizes  $J(k)$  is the following:

$$u(k) = -Kx(k) \quad (14)$$

where  $K$  is the matrix, which is given by:

$$K = [R + B^T P B]^{-1} B^T P A \quad (15)$$

The positive definite matrix  $P$  is the solution of the following Ricatti equation:

$$A^T P A - P - A^T P B [R + B^T P B]^{-1} B^T P A + Q = 0 \quad (16)$$

The steps of the self-adjusting control, using an explicit scheme, are the following:

- (1) step 1: estimate the parameters involved in the matrices  $A$  and  $B$  of the state model (6), using the recursive algorithm (4);
- (2) step 2: determine the matrix  $K$ , using the estimated matrices  $\hat{A}$  and  $\hat{B}$ ;
- (3) step 3: determine the control  $u(k)$ , from equation (14).

The evolution curves of the estimated parameters, the components of the control  $u(k)$ , state variables of the induction machine and the chosen speeds as references are given in Figures 3, 4, 5, 6 and 7.

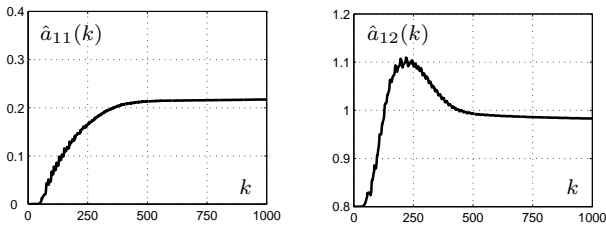


Fig. 3. Evolution curves of the estimated parameters  $\hat{a}_{11}(k)$  and  $\hat{a}_{12}(k)$ .

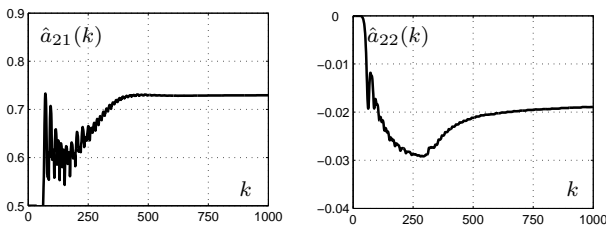


Fig. 4. Evolution curves of the estimated parameters  $\hat{a}_{21}(k)$  and  $\hat{a}_{22}(k)$ .

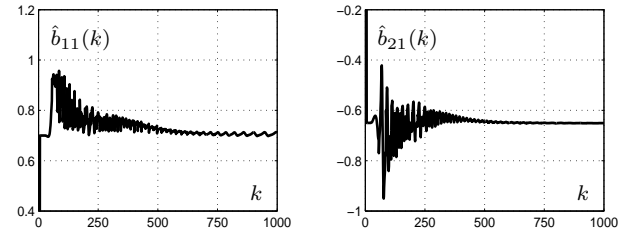


Fig. 5. Evolution curves of the estimated parameters  $\hat{b}_{11}(k)$  and  $\hat{b}_{21}(k)$ .

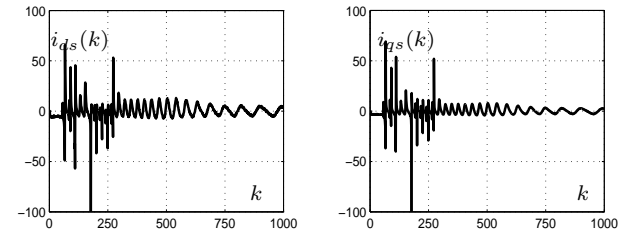


Fig. 6. Evolution curves of the state variables  $i_{ds}(k)$  and  $i_{qs}(k)$ .

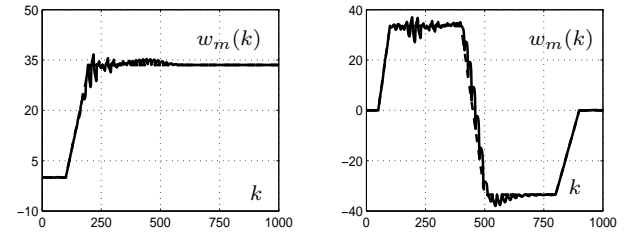


Fig. 7. Evolution curves of the two types of speed reference.

Examining Figures 3, 4 and 5 and based on knowledge of the numerical values of the estimated parameters, the quality of estimation of the induction machine is good. The real values of some simulated parameters are :  $a_{11} = 0.22$ ,  $a_{12} = 0.98$  et  $a_{21} = 0.73$ . Therefore, the estimated values, based on the figures, are close to the real values.

Concerning the speed reference, the control  $u(k)$  allows the estimation algorithm to follow both types of imposed reference signals. So, this control scheme has assured good performance.

#### 5. CONTROL SCHEME WITH DISTURBANCE COMPENSATION

In this section, a control schemes is developed, which compensates disturbance. A dynamic system is considered. This system is affected by a disturbance  $v(k)$ , which is described by the following state space model:

$$x(k+1) = A(k)x(k) + B(k)u(k) + v(k) \quad (17)$$

where  $x(k)$  and  $u(k)$  represent respectively the vector state variable and the control law of the system at discrete time  $k$ .

The control  $u(k)$  is seeked. This control minimizes the following criterion:

$$J(k) = \frac{1}{2} \sum_{k=0}^{k=n} x^T(k)Qx(k) + u^T(k)G(k)u(k) \quad (18)$$

This is a problem in which the unknown is the control law  $u(k)$ . It is a minimization of (18) under the constraint  $x(k+1) =$

$\hat{A}(k)x(k) + \hat{B}(k)u(k) + v(k)$ . The Lagrangien then reads:

$$L(k) = \sum_{k=0}^{k=n} \left( \frac{1}{2} x^T(k) Q(k) x(k) + \frac{1}{2} u^T(k) R(k) u(k) \right. \\ \left. + p^T(k+1)(-x(k+1) + \hat{A}(k)x(k) + \hat{B}(k)u(k) + v(k)) \right) \quad (19)$$

The optimum solution satisfies the following equation:

$$\frac{\partial L}{\partial u(k)} = R(k)u(k) + \hat{B}^T(k)p(k+1) = 0 \quad (20)$$

$$\frac{\partial L}{\partial x(k)} = Q(k)x(k) - p(k) + \hat{A}^T(k)p(k+1) = 0 \quad (21)$$

and

$$\frac{\partial L}{\partial p(k+1)} = -x(k+1) + \hat{A}(k)x(k) + \hat{B}(k)u(k) + v(k) = 0 \quad (22)$$

The equation for the control (20) gives:

$$u(k) = -R^{-1}(k)\hat{B}^T(k)p(k+1) \quad (23)$$

Replacing the control (23) in the (22) and by taking (21), a system of equations is obtained, whose unknown signals are  $x(k)$  and  $p(k)$ :

$$Qx(k) - p(k) + \hat{A}^T(k)p(k+1) = 0 \quad (24)$$

$$-x(k+1) + \hat{A}(k)x(k) - \hat{B}(k)R^{-1}(k)\hat{B}^T(k)p(k+1) + v(k) = 0 \quad (25)$$

Using equations (24) and (25), the adjoint state can be calculated  $p(k)$  in each discrete time  $k$  by the following recurrent equation:

$$p(k+1) = (\hat{A}(k)^T)^{-1} [[\hat{B}(k)R^{-1}(k)\hat{B}^T(k)]^{-1} \\ [-x(k) + \hat{A}(k)x(k-1) + v(k)] - Qx(k)] \quad (26)$$

Using the control (23) in the algorithm (4). The evolution curves of the speeds chosen selected as references are given in Figure 8.

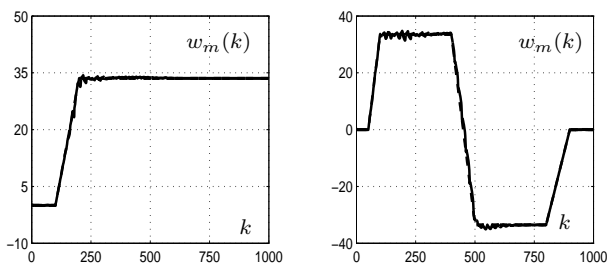


Fig. 8. Evolution curves of the two types of speed reference.

By comparing the curves for speed reference obtained using the command (14) and control with disturbance compensation (23), the quality of estimation is improved.

## 6. CONCLUSION

In this paper, a recursive parameter estimation algorithm and two control schemes, which can be applied to an induction machine, are developed. By means of the stator voltage and currents measurements, the recursive algorithm is applied to estimate the parameters of induction machine which is described by state-space

mathematical model. The obtained results show the effectiveness of this algorithm on the convergence of the estimated parameters towards the real one. The quadratic optimal control is applied to the induction motor. The figures of the estimated parameters show good performance of the self-adjusting control. Tracking speeds reference of the induction motor is improved, using a control law which compensates disturbance.

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