Further Tiling Patterns Involving Islamic Stars with an Odd Number of Vertices

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ABSTRACT

Islamic star patterns have been extensively studied for their symmetry and aesthetic appeal. This paper presents a new construction method for generating these stars on computers and for constructing new tiling patterns that involve stars with an odd number of vertices.

General Terms

Islamic, Art, Pattern, Algorithm, Turbo C++, Program.

Keywords

Polygon, Rosette, Star, Symmetry, Trigonometric,

1. INTRODUCTION

Owing to their symmetry and aesthetic appeal, Islamic star and rosette patterns have been studied extensively by several researchers(Gr¨unbaum and Shephard [8], Abas and Salman [1], Bourgoin[ ],Dewdney [5], Castera [3], Dunham[6]). The innate symmetry of these structures have led to geometrical constructions of these as well as tiling designs devised through putting such structures in contact(Castera [4], Gr¨unbaum and Shephard[7], Kaplan[1]. Kaplan[11] has devised an elaborate construction method for stars based on methods described originally by Henkin [10] and Lee [13]. In addition Kaplan[12] has also devised a number of interesting tiling methods by putting stars in contact. Most of these styling styles use stars with an even number of vertices. In an earlier paper(Gangopadhyay[9]), a simpler method to construct a star has been described along with two new generalized tiling methods which use stars with both even and odd number of vertices. In this paper a completely new method of constructing the Islamic star is presented. In addition, two new generalized tiling methods are devised which use stars with both even and odd number of vertices. These are the main distinguishing features of this paper.

2. HOW TO DRAW A STAR

Let n be an integer. Let A[i] be the ith vertex of the star, and let x[i] and y[i] denote respectively the abscissa and ordinate of A[i], where $0 \leq i \leq n-1$. Let x0 and y0 respectively denote the abscissa and ordinate of the centre of the star and let its radius be k. Then

x[i] = k cos(2π i / n)+x0 and y[i]=k sin(2π i / n)+y0.

For each i, $0 \leq i \leq n-1$, The points A[0] to A[n-1] constitute the vertices of a regular n-gon. For $0\leq i \leq n-1$ and $k0 \leq k$, we also define B[i] as a point with abscissa xx[i] and ordinate yy[i], where,

xx[i] = k0 cos(2π i / n)+x0 and yy[i]=k0 sin(2π i / n)+y0.

By joining B[i] to both A[(i-1)mod n] and A[(i+1)mod n], for all i, $0 \leq i \leq n-1$, we obtain star(n,k,k0).

In Figure 1 below, we illustrate the construction of star(11, 120,30) in which A[3] and B[3] are respectively highlighted by a green and a blue circle.

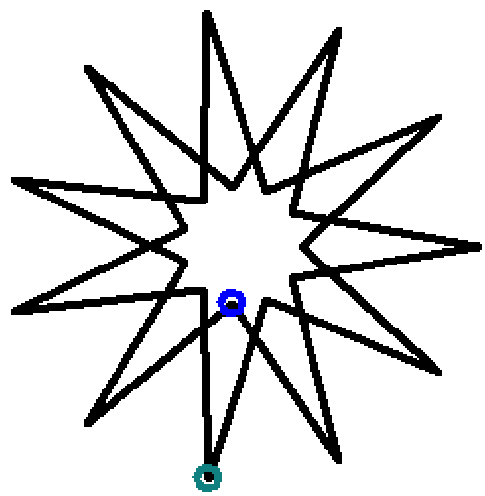

Fig 1 : star(11, 120, 30)

In the code in section 3 the function star draws star(n ,k, k0) .

3. TILING WITH STARS

Let n=11, k=120 and k0=30.Then the process of tiling for star(n,k,k0) is as follows. First star(n,k,k0) is drawn with its center coordinates at i and j and the length of its radius as k. Then another star(n,k,k0) is drawn by aligning the qth vertex of the old star with the pth vertex of the new one in such a manner that the line joining the center of the star with its pth vertex in the new star is parallel to the similar line in the old one, where the integer p = n+1- n(mod 3) and q = (n+1)/2- (n-7)/4%2. This basic prototype of two aligned stars is displayed in Figure 2.

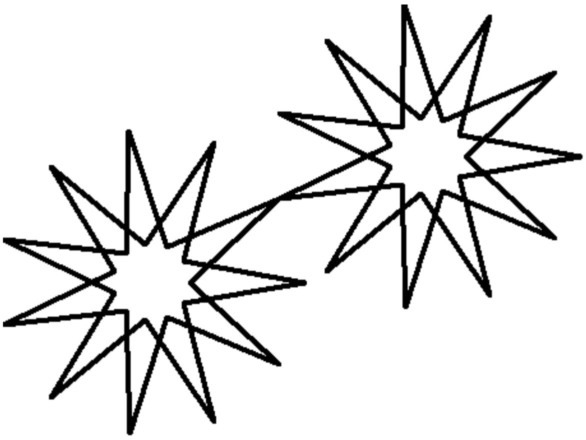

Fig. 2 : The Prototype – two aligned stars

The image in Figure 2 is then repeated by increasing the center coordinates of star(n,k,k0) at an interval of 2k cos(2 π p / n)-2k*cos(2 π q / n) for the abscissa i and 2k sin((((n+1) / 4)angle) π / 180) for the ordinate j.where angle = 360 / n. In figure 3 we show the output after one vertical and one horizontal rotation. The line in pink indicates the distance 2k cos(2 π p / n)-2k*cos(2 π q / n) between the center of the first star in the prototype and that of the corresponding star after one horizontal(abscissa-wise) iteration. Similarly the line in green indicates the distance 2k sin((((n+1) / 4)ang) π / 180) between the center of the first star in the prototype and that of the corresponding star after one vertical(ordinate-wise) iteration. The final output is shown in Figure 4.

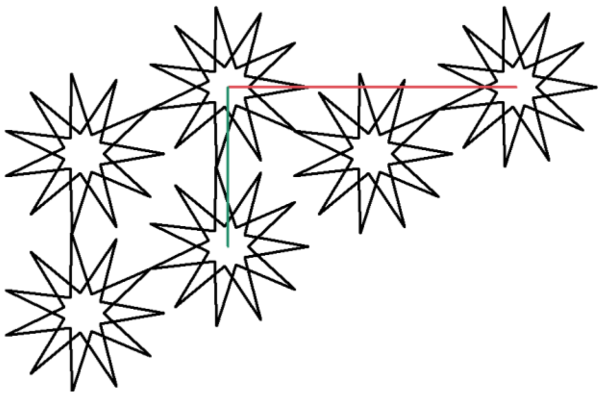

Fig. 3 : The double star after iteration

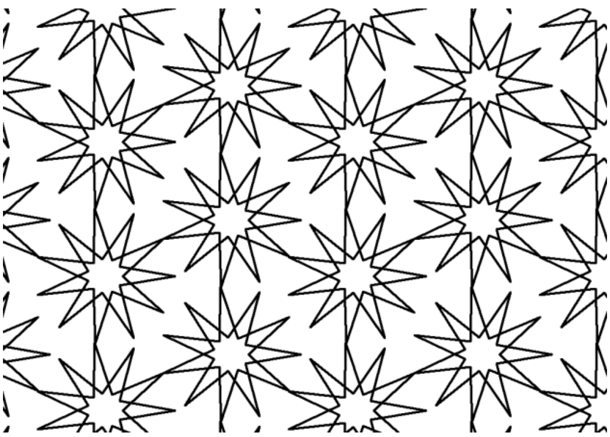

Fig 4 : The final output of tiling

The algorithm described in sections 2 and 3 is presented in the form of the C++ code in section 4.

4. THE CODE

The code uses a function star which is declared first..

```
void star(float k,float k0,int n, int x0,int y0)
{float x[100],y[100],xx[100],yy[100];
for(int i=0;i<n;i++)
{
x[i]=k*cos(2*3.14*i/n)+x0;
y[i]=k*sin(2*3.14*i/n)+y0;
}
for(int i=0;i<n;i++)
{
xx[i]=k0*cos(2*3.14*i/n)+x0;
yy[i]= k0*sin(2*3.14*i/n)+y0;
}
for(int i=0;i<n;i++)
{
line(xx[i],yy[i],x[(i-1+n)%n],y[(i-1+n)%n]);
line(xx[i],yy[i],x[(i+1)%n],y[(i+1)%n]);
}
}
void main()
{
initwindow(1040,760);
int n=11;int k=120;float ang=360./n;float k0=30;
int p,q;
p=n+1-n%3;q=(n+1)/2-(n-7)/4%2;
float xa=k*cos(2*3.14*(p)/n)-k*cos(2*3.14*(q)/n);
float ya=k*sin(2*3.14*(q)/n)-k*sin(2*3.14*(p)/n);
r=k*cos(2*3.14*(q)/n)-xa-k*cos(2*3.14*(p)/n);
setlinestyle(SOLID_LINE,0,4);
 for(int i=-40;i<1400;i+=2*xa)
for(int j=-100;j<900;j+=2*k*sin(((n+1)/4*ang)*3.14/180.))
{
star(k,k0,n,i,j);
star(k,k0,n,i-xa,j+ya);
}
getch();
closegraph();
}
```

5. VARIATIONS ON THE SAME THEME

By changing the values of n and k0 in the main program we may generate a number of interesting star patterns.,

VARIATION 1.

Let n=11 and k0=70.Then the resulting output is illustrated in Figure 3.

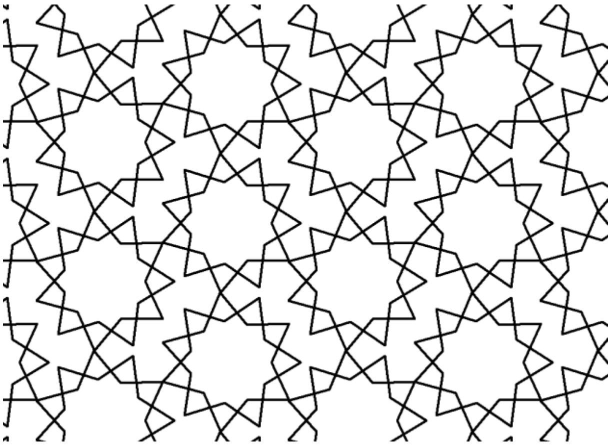

Fig. 5 : Variation 1 -

VARIATION 2.

Let n=7 and k0=50. Then the resulting output is illustrated in Figure 6.

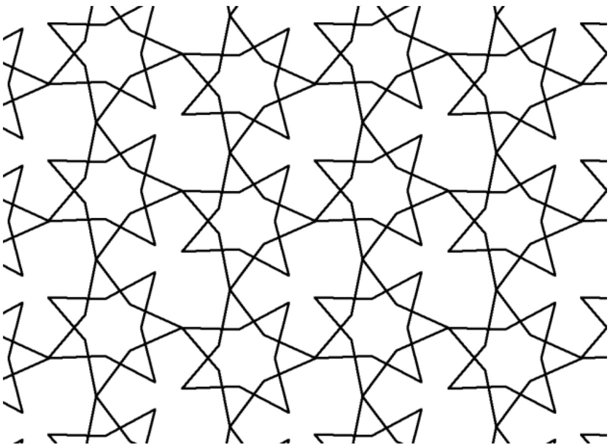

Fig. 6 : Variation 2

VARIATION 3.

Let n=11 and k0=15; Then the resulting output is illustrated in Figure 7.

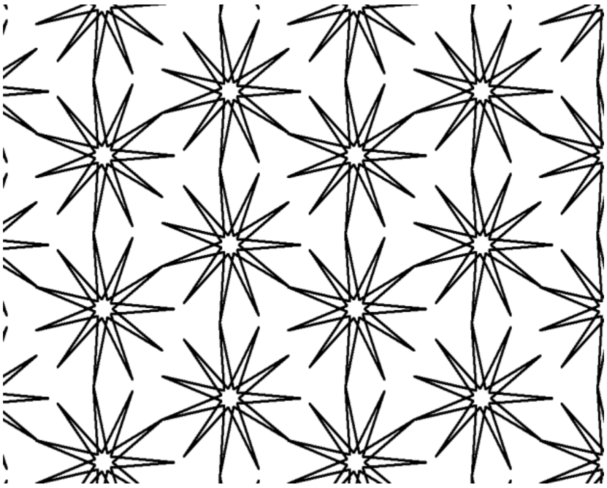

Fig. 7 : Variation 3

VARIATION 4.

Let n=11. By juxtaposing k0=15, 40 and 65 we obtain the output illustrated in Figure 8.

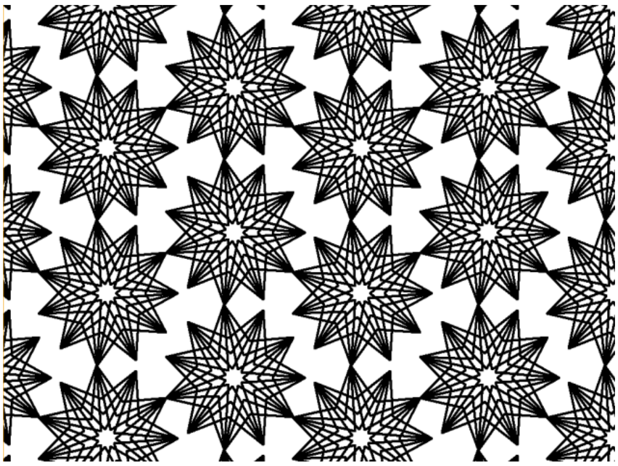

Fig. 8 : Variation 4

VARIATION 5.

Let n=11. In addition to juxtaposing k0=15, 40 and 65 if k is allowed to vary in a range and each cooresponding realization is coloured by the colour k, then one obtains the output illustrated in Figure 9.

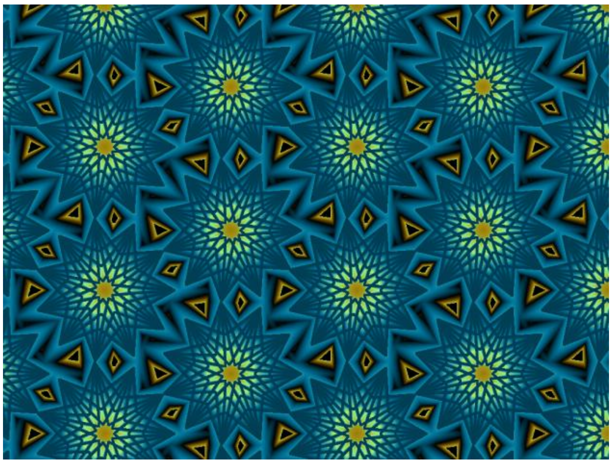

Fig. 9 : Variation 5

6. AN ALTERNATE DESIGN

An alternating diagonalized design is obtained by continuing the process of aligning the qth vertex of the old star with the pth vertex of the new one as described in section 3 for several iterations where p and q are as defined in section 3. For instance, after four iterations we obtain the output illustrated in figure 10.

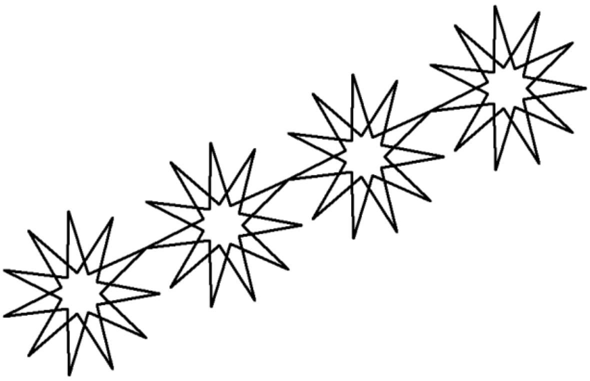

Fig. 10 : Two double stars

The image in Figure 10 is then repeated by increasing the ordinate of the centre of the first star(n, k, k0) by 2k sin((((n+1) / 4)angle) π / 180). The main program in the code in section 4 above could be changed as follows to accommodate this new style of tiling

```
void main()
{
initwindow(1040,760);
int n=11;int k=120;float ang=360./n;float k0=30;
int p,q;
p=n+1-n%3;q=(n+1)/2-(n-7)/4%2;
float xa=k*cos(2*3.14*(p)/n)-k*cos(2*3.14*(q)/n);
float ya=k*sin(2*3.14*(q)/n)-k*sin(2*3.14*(p)/n);
r=k*cos(2*3.14*(q)/n)-xa-k*cos(2*3.14*(p)/n);
setlinestyle(SOLID_LINE,0,4);
for(int j=0;j<1200;j+=2*120*sin(((n+1)/4*ang)*3.14/180.))
for(int gn=0;gn<9;gn++)
{
star(k,30,n,gn*(xa),j-gn*(ya));
}
getch();
closegraph();
}
```

The output of this is illustrated in Figure 11. One may compare Figure 11 to Figure 4.

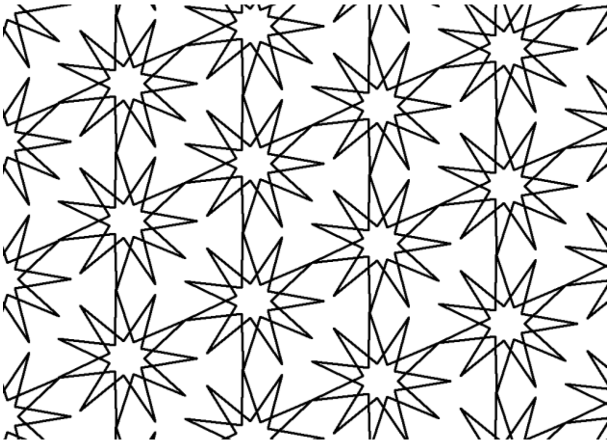

Fig. 11 : An alternate style of tiling

If n is 11, the values of k0=15, 40 and 65 are juxtaposed and k is varied in an appropriate range, then this new style of tiling yields the output given in Figure 12. One may compare this to Figure 9.

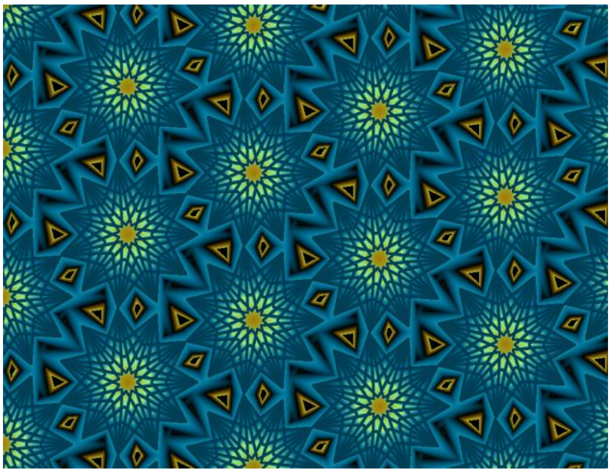

Fig. 12 : The new tiling with juxtaposition and colors

7 . ALIGNING OTHER VERTICES

If, in section 3, instead of aligning the qth vertex of the first star with the pth vertex of the new one, we align the (q+1)st vertex of the old star with the (p+1)st vertex of the new one, then the tiling yields a different pattern for the original style and its alternate versions. The final output of these two altered styles are displayed respectively in figures 13 and 14. They may be respectively compared to figures 9 and 12 and to each other.

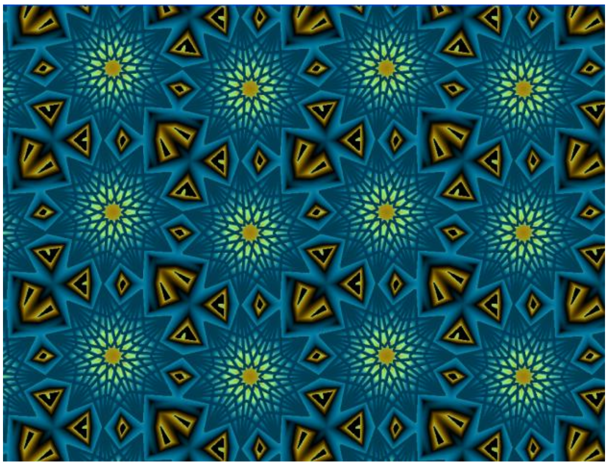

Fig. 13: The newly aligned original tiling

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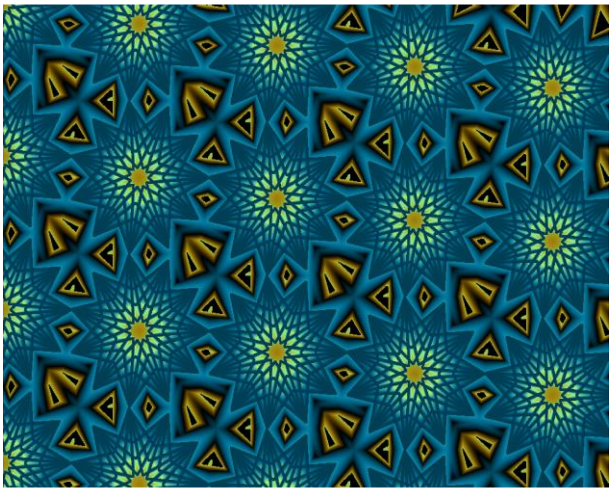

Fig. 14: The newly aligned alternate tiling

8. CONCLUSION

The tiling with with stars having a nonprime odd number of vertices may yield manifold symmetry.. This, as well as further new styles of tiling with even-vertexed stars would be explored in detail in future work.

9. ACKNOWLEDGMENTS

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