A Hybrid Method for Blur Invariants in Images using Contourlet Transforms

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ABSTRACT
Radiometric degradation is a common problem in the image restoration part of many applications. The degradation may involve blurring, information loss due to sampling, quantization effects and various sources of noise. The purpose of image restoration is to estimate the original image from the degraded image. There is much research carried out in an effort to deblur such images. To tackle this problem, different blur invariants had existed so far. Wavelet domain blurs invariants are used only for discrete 2D signals in spatial domain where it is concentrated in centrally symmetric blurs and also in the wavelet domain, directional prediction is so hard to find smoother contours. In this paper, the Contourlet Transform was proposed to address the lack of geometrical structure in the separable 2D Wavelet Transform. Because of its filter bank structure, the Contourlet Transform is not Shift invariant. Contourlet not only possess the features of wavelet (namely multiscale and time frequency localization), but also offer a high degree of directionality and anisotropy. The Contourlet domain invariant is proposed for both 2D and 3D signals based on frequency domain. By using disc and motion filter, the blur images are produced and are divided into equal pixels and also the dependent terms are discarded in blur invariants which reduce correlation, simplifies computation and also reduces noise using a Wiener filter in order to get the deblurred image. It is also proved that frequency domain blur invariants are a special version of the proposed invariants and numerical experiments on an image deblurring show that the proposed new Contourlet Transform can significantly outperform in terms of PSNR (by several DB's). It is widely used in various fields of applications, such as medical imaging, astronomical imaging, remote sensing, microscopy imaging, photography imaging, photography deblurring and forensic

Science. The software tool used in the proposed work is MATLAB R2009a.

Keywords: Blur moment invariants, Centrally Symmetric blur, Contourlet domain invariants, shift-invariant wavelet transform.

1. INTRODUCTION
Image restoration is an important issue in high-level image processing. Images are often degraded during the data acquisition process. The degradation may involve blurring, information loss due to sampling, quantization effects, and various sources of noise. The purpose of image restoration is to estimate the original image from the degraded data. It is widely used in various fields of applications, such as medical imaging, astronomical imaging, remote sensing, microscopy imaging, photography deblurring, and forensic science, etc. Images are produced to record or display useful information. Due to imperfections in the imaging and capturing process, however, the recorded image invariably represents a degraded version of the original scene. The undoing of these imperfections is crucial to many of the subsequent image processing tasks. There exists a wide range of different degradations, which are to be taken into account, for instance noise, geometrical degradations (pincushion distortion), illumination and color imperfections (under/overexposure, saturation), and blur. Blurring is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process. It can be caused by relative motion between the camera and the original scene, or by an optical system that is out of focus. When aerial photographs are produced for remote sensing purposes, blurs are introduced by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the ground. Such blurring is not confined to optical images, for example electron micrographs are corrupted by spherical aberrations of the electron lenses. In addition to these blurring effects, noise always corrupts any recorded image. Noise may be introduced by the medium through which the image is created (random absorption or scatter effects), by the recording medium (sensor noise), by measurement errors due to the limited accuracy of the recording system, and by quantization of the data for digital storage. The field of image restoration (sometimes referred to as image deblurring or image deconvolution) is concerned with the reconstruction or estimation of the encrypted image from a blurred and noisy one. Essentially, it tries to perform an operation on the image that is the inverse of the imperfections in the image formation system. In the use of image restoration methods, the characteristics of the degrading system and the noise are assumed to be known a priori. In practical situations, however one may not be able to obtain this information directly from the image formation process. The goal of blur identification is to estimate the attributes of the imperfect imaging system from the observed degraded image itself prior to the restoration process.

Deteriorations in images are basically of two types, i.e., geometric distortions and radiometric degradations. In geometric distortion includes distortions such as translation, scaling, and rotation. There are many different approaches in the literature that are proposed for dealing with these problems. Hu, for the first time, proposed descriptors that are invariant to some of basic linear geometric distortions [1]. Recently, a novel approach has been put forward by Flusser *et al.* [3], where implicit moment invariants are introduced for dealing with nonlinear deformations. For surveys on similar invariants refer to [4] –[6]. Unlike geometric distortions, there are fewer research works carried out on radiometric degradations. They are generally introduced to images due to the movement of the subject, unfocused camera, and nonideal image-capturing environment. The general model that is commonly used in the observed image is

$$\boldsymbol{y}(\boldsymbol{u}) = \boldsymbol{B}\boldsymbol{x}(\boldsymbol{n}) + \eta(\boldsymbol{u}) \qquad \textbf{(1)}$$

In this model, y is the observed image; and x and η are the original image and noise, respectively; and B is the degradation operator. If it is assumed that B is linear and space invariant, for 2-D discrete signals, the general model can be simplified to

$$\boldsymbol{y}[\boldsymbol{n_1}, \boldsymbol{n_2}] = \boldsymbol{b} * \boldsymbol{x}[\boldsymbol{n_1}, \boldsymbol{n_2}] \qquad \textbf{(2)}$$

Where '*' denotes the convolution, and b is the point spread function of the system. The proposed approaches for dealing with blur can be categorized into two types: 1) optical blur and 2) motion blur. In blind restoration techniques, the purpose is to identify the blur system model and extract the actual signal [7], [8]. The main application of these methods is in signal restoration. There are also similar techniques in which the effort is to only restore the features of degraded signals [9], with their main application in pattern recognition. Although these techniques have been vastly used, they suffer from major drawbacks: deblurring is an ill-posed problem, and because of the identification part, they are computationally expensive, while it is not required to identify the blur system in many applications. The second type of technique focuses on developing descriptors that are inherently invariant to blur. The main advantage of these methods is that they do not go through the process of identifying the blur system. Flusser *et al.* [10] Established this type for the first time. Their invariant descriptors were developed in the spatial domain and based on geometric moments. Their assumption was that the blur operator is symmetric. Later, they represented a closed format of the invariants [11] and added extra properties to the descriptors in order to make them invariant to geometric distortions while changing their assumption for the blur systems to centrally symmetric [12], [13]. They also developed these invariants for 1-D signals [14]. Utilizing complex moments, Flusser and Zitovà proposed descriptors that are both invariant to centrally symmetric blur and rotation [15]. Liu and Zhang [16] also developed similar blur invariants using complex moments. Metari and Deschenes [17] exploited the Mellin transform properties in order to define a different representation of geometric-moment-based blur-invariant descriptors. They also showed that their proposed descriptors are invariant to a few geometric distortions as well. Instead of using geometric moments, Zhang *et al.* [18] employed Legendre moments in order to define their invariants in the spatial domain. On the other hand, Ji and Zhu [19] proved that Zernikemoments are inherently invariant to Gaussian blur when .Along with the blur-invariant descriptors defined in the spatial domain, there are some other methods that are developed in the Fourier domain. Flusser and Suk [13] proposed their Fourier-based invariants for 1-D signals based on the tangent of the phase of signals and proved that they are invariant to blur. They developed this representation for 2-D signals and showed their relationship with their invariants in the spatial domain [12]. These invariants were later developed for signals and made to be invariant to some of the geometric distortions [20].

Ojansivu and Heikkilä [21], however, showed that Flusser's invariants are sensitive to noise when implemented in the Fourier domain because of their use of a tangent operator. They proposed a different representation of invariants in the Fourier domain. Subsequently, they made them invariant to the affinetransform as well [22]. Blur moment invariants showed their practicality in a vast area of research: image registration [13], [23], remote sensing [24], [25], forgery detection [26], recognition [11], [16], [27], stereo matching [28], and control point extraction [29].

In this paper, Contourlet-domain descriptors are proposed, which are invariant to centrally symmetric Blur systems. The main advantages of these invariants will involve the exploration of a directional extension of the multidimensional wavelet transform called, 'Contourlets'. The idea will be developed for 2D images and the effect of different types of blur will be studied and also how to recover the deblur images using wiener filter using Contourlet Transform.

The rest of the paper is organized as follows: section 2 introduces the Contourlet Transform. Section 3explains proposed method. Simulation results and comparisons with different Blur systems are presented in the section. Finally, Section v concludes the paper.

2. CONTOURLET TRANSFORM

Wavelets are classified as a linear transform that is capable of displaying the transformed output at multiple resolutions depending on the point of time/space and at the desired frequency. In contrast to the Short-Time Fourier Transform (STFT), the resolution changes depending on the frequency that is to be examined and at what time or spatial area is to be examined [30]. In the 1-D case, wavelets are used for signal processing by the virtue that wavelets can store more frequency information with less coefficients and reconstruction is only limited by the coefficients that are available. Wavelets can be naively extended to the 2-D case by means of separable functions, but there is limited directional information stored in a regular 2-D wavelet transform. Because of the Severability limitations, only a horizontal, vertical, and 45 degree component can be easily determined. Incidentally, edges can be seen easily, but directional information about the edge is not known. Because of this, it takes more coefficients to do a proper reconstruction of the edges [31]. Typically, a separable 2-D wavelet transform provides:

- Multiresolution, which is the ability to visualize the transform with varying resolution from coarse to fine.
- Localization, which is the ability of the basis elements to be localized in both the spatial and frequency domains.
- Critical sampling, which is the ability for the basis elements to have little redundancy.

However, it is not capable of providing:

- Directionality, which is having basis elements defined in a variety of directions.
- Anisotropy, which is having basis elements defined in various aspect ratios and shapes [32].

There are many directional extensions of the 2-D wavelet transform that could be potentially examined that also possess directionality and anisotropy. The contourlet transform is a discrete extension of the curvelet transform that aims to capture curves instead of points, and provides for directionality and anisotropy. Figure 1 shows the general concept of capturing curves [33].

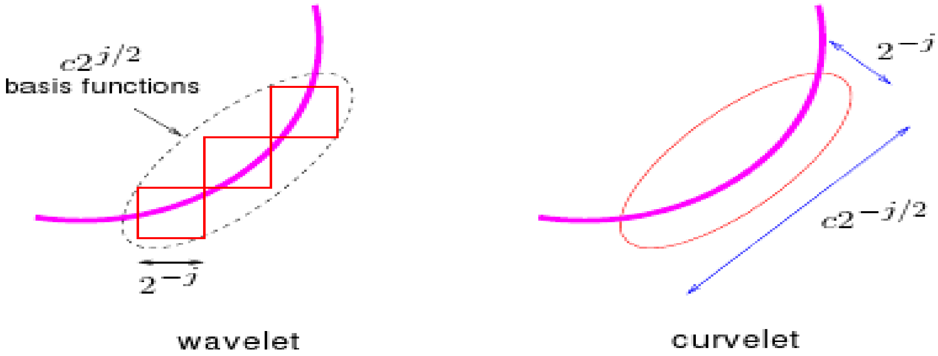

Figure 1: Conceptual visualization of curvelets/contourlets.

Contourlets are implemented by using a filter bank that decouples the multiscale and the directional decompositions. In Figure 2, Do and Vetterli shows a conceptual filter bank setup that shows this decoupling. We can see that a multiscale decomposition is done by a Laplacian pyramid, then a directional decomposition is done using a directional filter bank. This transform is suitable for applications involving edge detection with a high curve content [34].

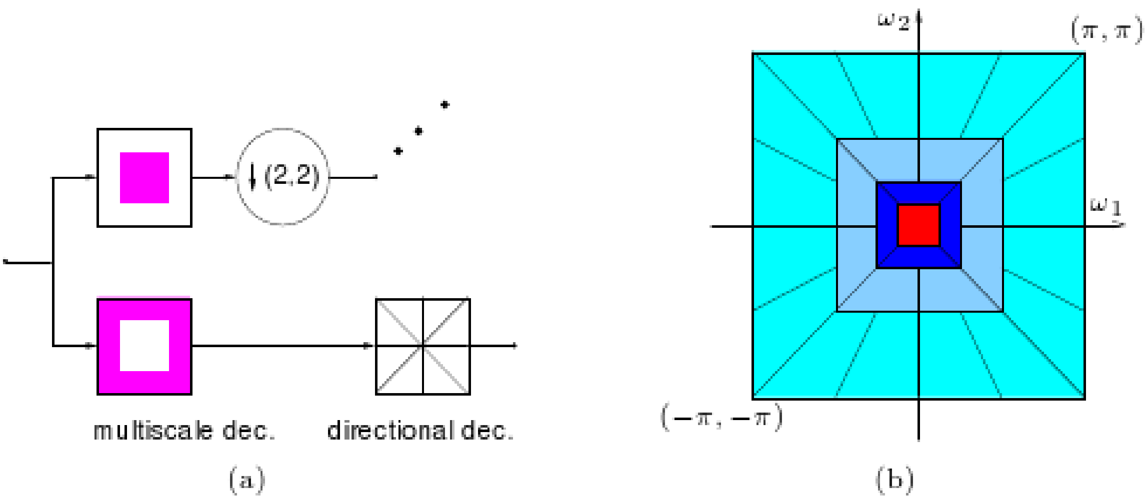

Figure 2: Filter bank for contourlet transform.

Using Contourlets for Edge Detection

Our approach involves taking the contourlet transform of test grayscale images. The code for the Contourlet transform is available through the author's website. [35] The code for the contourlet transform is flexible enough to also do the regular 2-D separable wavelet transform. The edge detection algorithm is as follows:

1. Take the contourlet transform of the image.
2. Choose a scale factor to use, and truncate all other coefficients.
3. Invert the transform.
4. Threshold using the mean of the pixel values of the image.

In addition, there is built in code in MATLAB's Image Processing Toolbox for doing edge detection using the Prewitt gradient operator, the Sobol gradient operator, and Canny's method. These are also run on the test grayscale image for comparison purposes. A MATLAB script is used to automate this process.

3. PROPOSED METHOD

In this paper, the main objective is to deblur the image by using Contourlet transform in order to achieve high PSNR values. Photo deblurring has been a major research topic in now-a-days. So far, proposed methods have focused on removing the blur due to camera shake and object motion. In order, to deblur images, we want to recover an original image from distortion. First want to model the degradation and after that applying inverse process in order to recover the original image. Figure 3 and 4 presents the diagram that shows the sequence we used in the proposed Blur invariant analysis in color images. In order to compute Contourlet transform of blur image we use the Contourlet transform toolbox provided in [35].

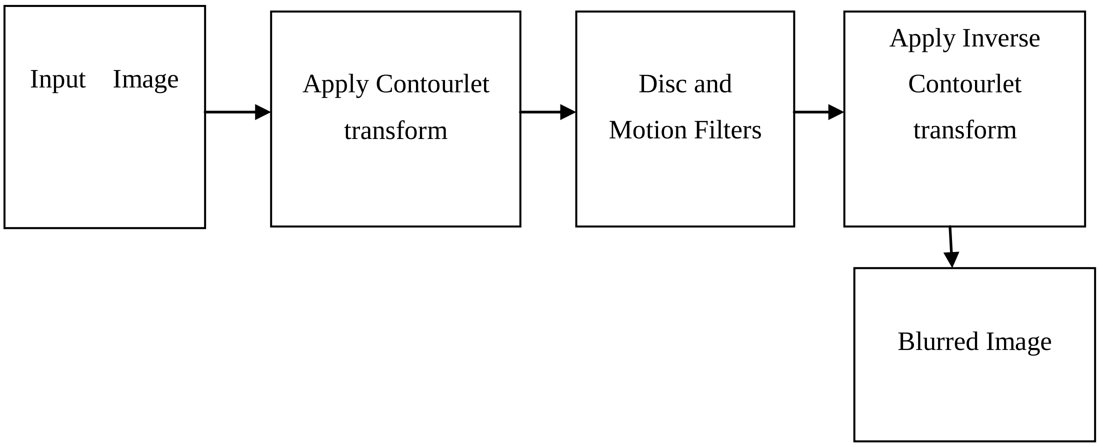

Figure 3 Block Diagram of Blur System

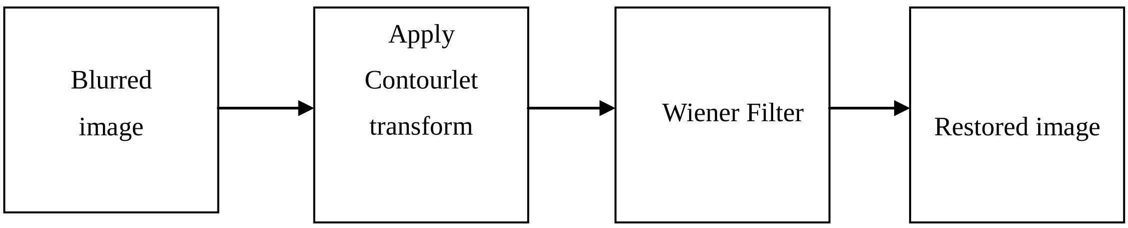

Figure 4 Block Diagram of Deblur Image

In our method, at first color image are converted into blocks and after that extract the edges of the original image in the frequency domain by using Contourlet transform. We use two types of filters namely disk and motion filter which produces optical blur and motion Blur. Finally, want to deblurring an image using wiener filter in Blind Deconvolution technique. Moreover the prefect reconstruction of the Contourlet property gives a fast algorithm to solve the images along edges. It is also provided that frequency domain blur invariants are a special version of the proposed invariants.

4. SIMULATION RESULTS

In this section, the performance of the proposed technique is evaluated on three different problems. The first experiment is designed to study how different types of blur affect the proposed invariants; three benchmark images are artificially degraded, and the changing of the value of the proposed blur invariants with different wavelet functions at different branches is studied. By branch, we mean all the possible combinations of wavelet and scaling filters at a certain level. In the second experiment, a database of different objects is degraded with different blurs and noises in order to evaluate the performance of the invariants in a recognition task. In this experiment, the spatial-domain invariants are also employed for the sake of comparison. In the third experiment, real-world degraded images are acquired, and the invariants are used for registration. The SDBIs are used here as well for a comparison. These have been implemented using MATLAB R2009a. Peak Signal to Noise Ratio (PSNR) is used to evaluate quality of Motion blur and Disk blur images along their length and radius. The performance in terms of capacity and PSNR (in dB) is demonstrated for the method in the following subsections.

PSNR is defined as

$$\mathbf{PSNR = 10log\left(\frac{255^2}{MSE}\right)} \quad \textbf{(3)}$$

Where, MSE=Mean Square Error value. It is given by

$$\mathbf{MSE} = \left(\frac{\mathbf{1}}{\mathbf{M\times N}}\right)\sum_{\mathbf{i=1}}^{\mathbf{M}}\sum_{\mathbf{j=1}}^{\mathbf{N}}\left(\mathbf{X_{ij}} - \mathbf{Y_{ij}}\right)^{\mathbf{2}} \quad \textbf{(4)}$$

Where,

M×N=Size of the Blur Image,

X_{ij}, Y_{ij} = pixel values of the original image and Blur image respectively.

The calculated PSNR usually adopts a dB value for quality judgement, the larger PSNR is, the higher the image quality (which means there is a little difference between the original image and Blur image). On the contrary smaller dB value means there is a more distortion. PSNR values falling below 30dB indicate fairly a low quality. However, high quality strives for 40dB or more.

Table 1 Similarity Blur for Lena Image

Dimensions	Similarity of Disc Blur (at radius)	Similarity of Motion Blur (at length)
2	5.369	2.356
4	9.632	4.783
6	12.90	6.982
8	15.74	8.814
10	18.24	10.42

The above table 1 shows the Similarity blur values obtained from Lena images. It is used to estimate PSNR values. Similarity blurs is estimated at different radii (2,4,6,8,10) in disk filter and at different length (2,4,6,8,10) in motion filter.

Table 2 PSNR values for Blur in Lena Image

Dimensions	PSNR values of Disc Blur (at radius)	PSNR values of Motion Blur (at length)
2	53.65	66.43
4	45.67	55.49
6	40.92	49.98
8	38.25	46.75
10	36.35	44.30

The above table 2 shows the PSNR values obtained from Lena images during blur. PSNR ratio is used to evaluate the quality of the image during Disc and Motion blur. Compared to Disc Blur image where the motion blur image having high PSNR value. So the quality of motion blur is good during quality judgement and PSNR usually adopts a dB value. The larger the PSNR is, the higher the image quality. On the contrary, smaller dB value means there is more distortion. PSNR values falling below 30 dB indicates fairly low quality and above indicates high quality.

.Table 3 PSNR values for deblur in Lena Image

Dimensions	PSNR values of Disc Blur (at radius)	PSNR values of Motion Blur (at length)
2	69.26	78.95
4	61.64	70.86
6	58.48	65.68
8	56.06	62.49
10	54.27	60.63

The above table 3 shows the PSNR values obtained from Lena images during deplore. PSNR ratio is used to evaluate the quality of the image during Disc and Motion blur. Compared to Disc Blur image where the motion blur image having high PSNR value. So the quality of motion blur is good during or quality judgement and PSNR usually adopts a dB value. The larger the PSNR is, the higher the image quality. On the contrary, smaller dB value means there is more distortion. PSNR values falling below 30 dB indicates fairly low quality and above indicates high quality. The below figure shows simulation results for blur invariants using Contourlet transform.

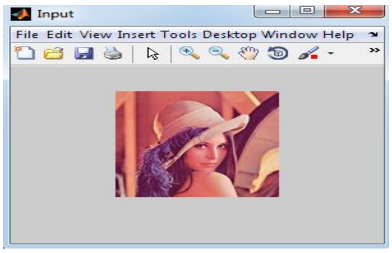

Figure 5 Images of Lena in their original shape

r=2 r=4 r=6 r=8 r=10

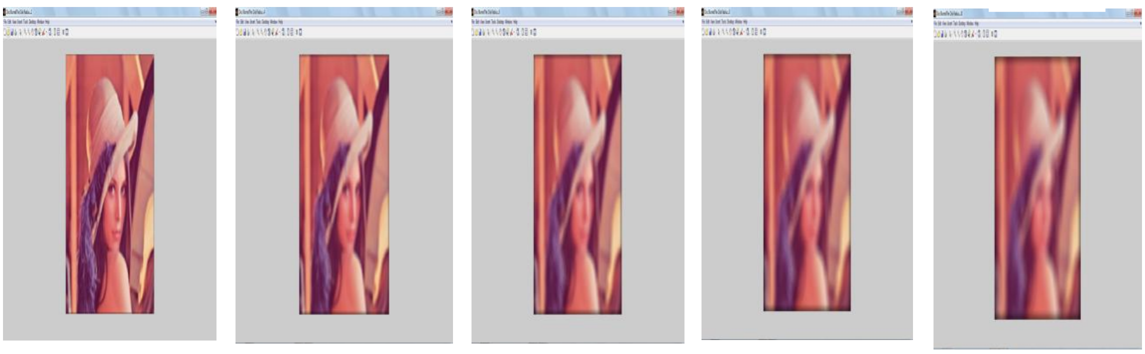

Figure 6 Blurred image of Lena with disk filters of different radii

l=2 l=4 l=6 l=8 l=10

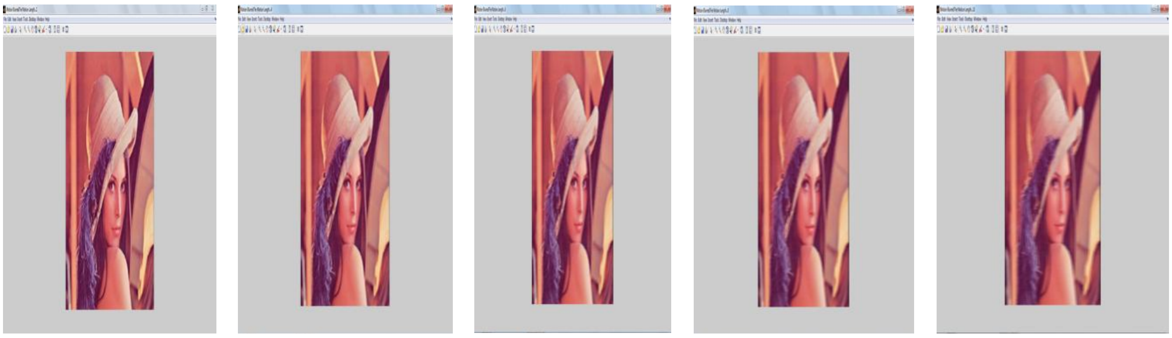

Figure 7 Blurred image of Lena with motion filters of different directions

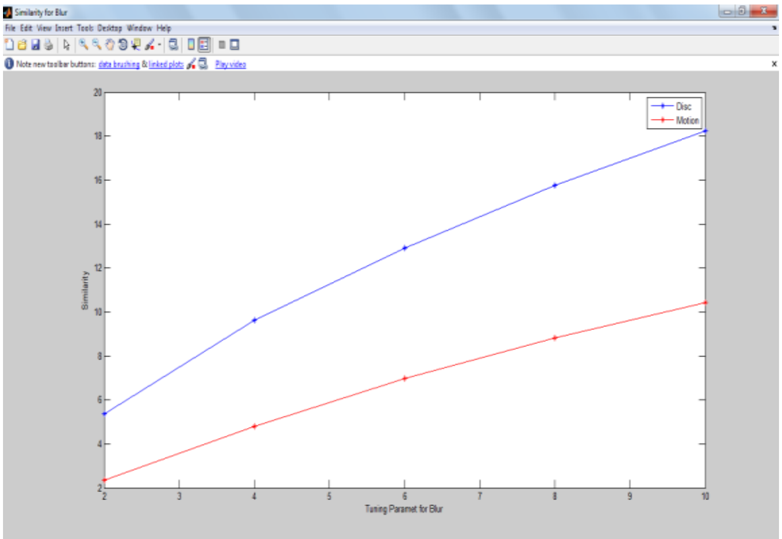

Figure 8 Comparison of Similarity Blur in Disk and Motion

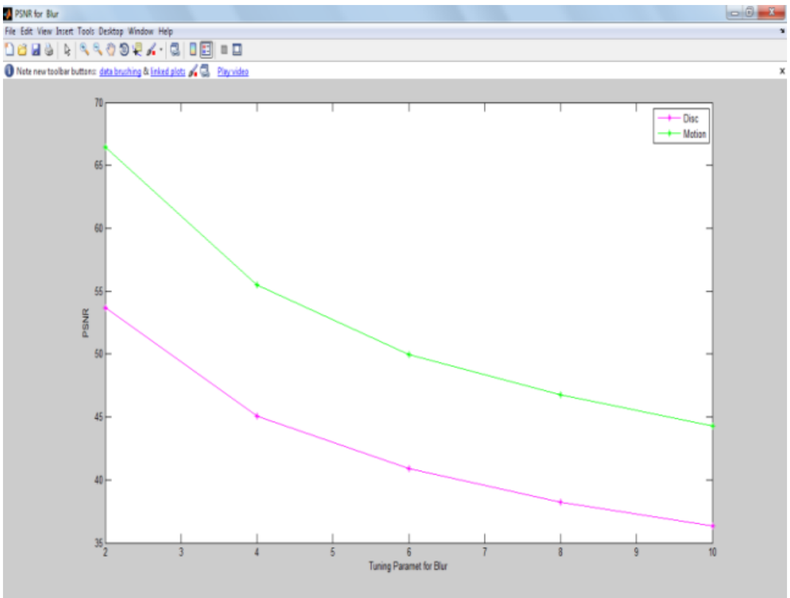

Figure 9 Comparison of PSNR in Disk and Motion

r=2 r=4 r=6 r=8

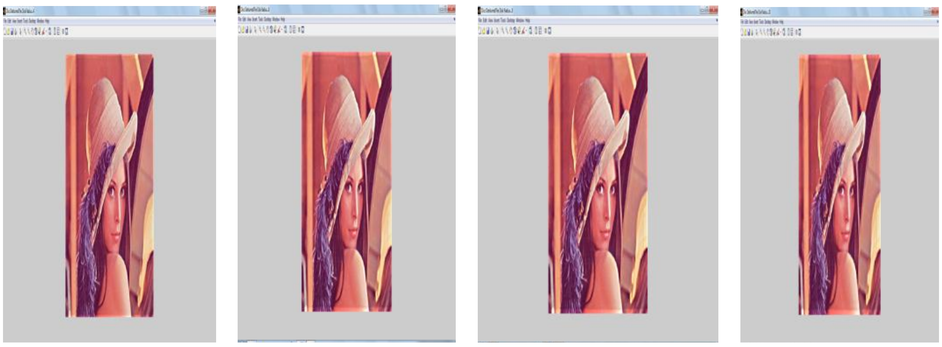

Figure 10 Deblurred images of Lena with disk filters of different radii

l=2 l=4 l=6 l=8 l=10

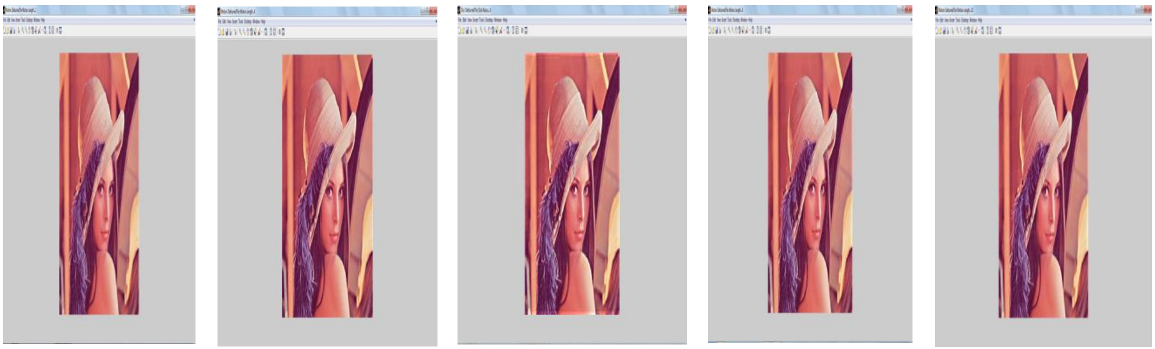

Figure 11 Blurred image of Lena with motion filters of different directions

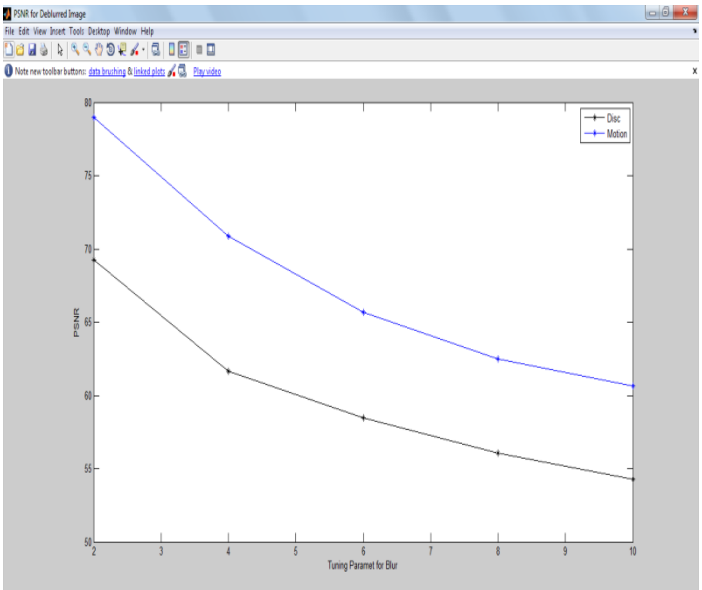

Figure 12 PSNR for Deblurred image

5. CONCLUSION AND FUTURE WORK

In this paper, a new set of blur invariant descriptors has been proposed. These descriptors have been developed in the Contourlet domain for discrete 2-D and 3-D images to be invariant to centrally symmetric blur. Defining them in the Contourlet Transform grants them the advantages that this domain has, i.e., different alternatives of bases, and analysis at different scales. The frequency domain blurs invariants are also proved to be a special case of these invariants. In a discussion of the proposed blur invariants, the Contourlet Transform was proposed to address the lack of geometrical structure in the separable 2D Wavelet Transform. Because of its filter bank structure, the Contourlet Transform is not Shift invariant. Contourlet not only possess the features of wavelet (namely multiscale and time frequency localization), but also offer a high degree of directionality and anisotropy. In order to evaluate the performance of these invariants, three different experiments have been carried out. In the first experiment, Lena images have been employed and artificially degraded with different blurring filters. The invariants successfully put a discrepancy between different subjects while showing the very negligible difference due to the degradations introduced by blurring namely Optical and Motion. The Contourlet-based blur invariants of different branches have been used separately to study the effect of this term as well. The results have shown that the robustness of the Contourlet domain blur invariants considerably changes with respect to the choice of branches in their decomposition. In the experiment, there were photos taken such that the scenes have been deteriorated by defocus blur. Then, a part of the scene, which has been selected from a sharp photo, has been used as templates for image registration. Despite the presence of severe blurs, registration has been perfectly performed. There are numerous papers on Contourlet bases, each of which yields its specific advantages. In future, finding out what type of Contourlet functions is the most beneficial in blur-invariant extraction in the frequency domain is a major direction for further research on the proposed invariants.

6. REFERENCES

[1] Iman Makaremi and Majid Ahmadi, "Wavelet-Domain Blur Invariants For Image Analysis" IEEE Trans. In image processing, Vol.21, No.3, March 2012.

[2] M. -K. Hu, "Visual pattern recognition by moment invariants," *IRE Trans. Inf. Theory*, vol. 8, no. 2, pp. 179–187, Feb. 1962.

[3] J. Flusser, J. Kautsky, and F. Roubek, "Implicitmoment invariants," *Int. J. Comput. Vis.*, vol. 86, no. 1, pp. 72–86, Jan. 2010.

[4] T. H. Reiss, *Recognizing Planar Objects Using Invariant Image Features*. Secaucus, NJ: Springer-Verlag, 1993.

[5] J. Wood, "Invariant pattern recognition: A review," *Pattern Recognit.*, vol. 29, no. 1, pp. 1–17, Jan. 1996.

[6] J. Flusser, T. Suk, and B. Zitov, *Moments and Moment Invariants in Pattern Recognition*. Hoboken, NJ: Wiley, 2009.

[7] M. I. Sezan and A. M. Tekalp, "Survey of recent developments in digital image restoration," *Opt. Eng.*, vol. 29, no. 5, pp. 393–404, May 1990.

[8] D. Kundur and D. Hatzinakos, "Blind image deconvolution," *IEEE Signal Process. Mag.*, vol. 13, no. 3, pp. 43–64, May 1996.

[9] B. Gunturk, A. Batur, Y. Altunbasak, M. H. Hayes, III, and R. Mersereau, "Eigenface-domain super-resolution for face recognition," *IEEE Trans. Image Process.*, vol. 12, no. 5, pp. 597–606, May 2003.

[10] J. Flusser, T. Suk, and S. Saic, "Image features invariant with respect to blur," *Pattern Recognit.*, vol. 28, no. 11, pp. 1723–1732, Nov. 1995.

[11] J. Flusser, T. Suk, and S. Saic, "Recognition of blurred images by the method of moments," *IEEE Trans. Image Process.*, vol. 5, no. 3, pp. 533–538, Mar. 1996.

[12] J. Flusser and T. Suk, "Degraded image analysis: An invariant approach," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 20, no. 6, pp. 90–603, Jun. 1998.

[13] T. Suk and J. Flusser, "Features invariant simultaneously to convolution and affine transformation," in *Proc. CAIP*, W. Skarbek, Ed., 2001, vol. 2124, Lecture Notes in Computer Science, pp. 183–190.

[14] J. Flusser and T. Suk, "Classification of degraded signals by the method of invariants," *Signal Process.*, vol. 60, no. 2, pp. 243–249, Jul. 1997.

[15] J. Flusser and B. Zitová, "Combined invariants to linear filtering and rotation," *Int. J. Pattern Recognit. Artif. Intell.*, vol. 13, no. 8, pp. 1123–1136, Dec. 1999.

[16] J. Liu and T. Zhang, "Recognition of the blurred image by complex moment invariants," *Pattern Recognit. Lett.*, vol. 26, no. 8, pp. 1128–1138, Jun. 2005.

[17] S. Metari and F. Deschenes, "New classes of radiometric and combined radiometric-geometric invariant descriptors," *IEEE Trans. Image Process.*, vol. 17, no. 6, pp. 991–1006, Jun. 2008.

[18] H. Zhang, H. Shu, G. Han, G. Coatrieux, L. Luo, and J. Coatrieux, "Blurred image recognition by Legendre moment invariants," *IEEE Trans. Image Process.*, vol. 19, no. 3, pp. 596–611, Mar. 2010.

[19] H. Ji and H. Zhu, "Degraded image analysis using Zernike moment invariants," in *Proc. IEEE ICASSP*, Apr. 2009, pp. 1941–1944.

[20] J. Flusser, J. Boldys, and B. Zitova, "Moment forms invariant to rotation and blur in arbitrary number of dimensions," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 25, no. 2, pp. 234–246, Feb. 2003.

[21] V. Ojansivu and J. Heikkilä, "Object recognition using frequency domain blur invariant features," in *Proc. SCIA*, B. K. Ersbll and K. S. Pedersen, Eds., 2007, vol. 4522, Lecture Notes in Computer Science, pp. 243–252.

[22] V. Ojansivu and J. Heikkilä, "A method for blur and similarity transform invariant object recognition," in *Proc. ICIAP*, R. Cucchiara, Ed., 2007, pp. 583–588.

[23] V. Ojansivu and J. Heikkila, "Image registration using blur-invariant phase correlation," *IEEE Signal Process. Lett.*, vol. 14, no. 7, pp. 449–452, Jul. 2007.

[24] Y. Bentoutou, N. Taleb, M. C. E. Mezouar, M. Taleb, and L. Jetto, "An invariant approach for image registration in digital subtraction angiography," *Pattern Recognit.*, vol. 35, no. 12, pp. 2853–2865, 2002.

[25] Y. Bentoutou, N. Taleb, K. Kpalma, and J. Ronsin, "An automatic image registration for applications in remote sensing," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, no. 9, pp. 2127–2137, Sep. 2005.

[26] B. Mahdian and S. Saic, "Detection of copy-move forgery using a method based on blur moment invariants," *Forensic Sci. Int.*, vol. 171, no. 2/3, pp. 180–189, Sep. 2007.

[27] X. Dai, H. Zhang, H. Shu, and L. Luo, "Image recognition by combined invariants of legendre moment," in *Proc. IEEE ICIA*, 2010, pp. 1793–1798.

[28] M. Pedone and J. Heikkilä, "Blur and contrast invariant fast stereo matching," in *Advanced Concepts for Intelligent Vision Systems*, J. Blanc-Talon, S. Bourennane, W. Philips, D. C. Popescu, and P. Scheunders, Eds. Berlin, Germany: Springer, 2008.

[29] Y. Bentoutou and N. Taleb, "Automatic extraction of control points for digital subtraction angiography image enhancement," *IEEE Trans. Nucl. Sci.*, vol. 52, no. 1, pp. 238–246, Feb. 2005.

[30] P. A. Mlsna and J. J. Rodriguez, "Gradient and laplacian edge detection," in Handbook of Image and Video Processing, 2nd ed. Elsevier Academic Press, 2005, ch. 4.13.

[31] S. G. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation," IEEE Transations on Pattern Analysis and Machine Intelligence, vol. 11, no. 7, July 1989.

[32] J. Antoine, P. Carrette, R. Murenzi, and B. Piette, "Image analysis with two-dimensional continuous wavelet transform," Signal Processing, vol. 31, pp. 241–272, 1993.

[33] M. N. Do and M. Vetterli, "The contourlet transform: An efficient directional multiresolution image representation," IEEE Transactions on Image Processing, vol. 14, no. 12, pp. 2091–2096, Dec 2005.

[34] J.-L. Starck, E. J. Candes, and D. L. Donoho, "The curvelet transform for image denoising," IEEE Transactions on Image Processing, vol. 11, no. 6, pp. 2091–2096, Jun 2002.

[35] M. N. Do, "Contourlet toolbox." [Online]. Available: http://www.ifp.uiuc.edu/minhdo/software/contourlettoolbox.tar