International Journal of Computer Applications (0975 – 8887)
Volume 65– No.5, March 2013

A Proposed Solution to Travelling Salesman Problem using Branch and Bound

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ABSTRACT

Travelling salesperson problem is a well-known problem. In this problem the minimum cost tour of few cities is needed, which are connected. The cost of different paths is given. The tour should be started from a given node and after completing the tour the travelling salesman has to return to the starting node. The earlier method is given by Greedy programming. In this paper Branch & Bound method is used to solve this problem.

Keywords

Backtracking, Branch & Bound technique, Cost Matrix, Greedy Approach.

1. INTRODUCTION

If there is set of places which are connected to each other directly or indirectly. The path connecting them has some cost in terms of time or money. A sales person is standing at any source point. He needs to visit all the cities and return to the source city. The minimum path is to be found so that the length of the complete tour is minimum. There may be different paths to visit all the places. If the tour were represented on a paper in the form of a directed graph G, then it would be a set of vertices V (G) and a set of edges E (G) connecting them. We have to find the shortest path so that the distance would be minimum. If the graph consist exactly n vertices then the shortest path tour would be consisting of exactly n edges. The shortest path starts from a vertex V1 and after traversing all the places or vertices it ends at V1 itself. The cost of the path would be minimum possible value. This is parallel to the problem of finding the shortest path between two intersections on a road map: the graph's vertices correspond to intersections and the edges correspond to road segments, each weighted by the length of its road segment. The shortest paths are used for preparing driving directions, road maps, and web mapping sites. In computer networking it is used to send the data on communication channel so that the transmission time is minimum, means data needs to travel the minimum possible distance.

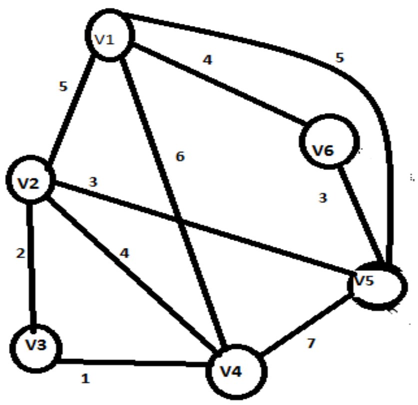


Fig 1: The Graph

As in the above figure given an undirected graph. This may be a paper presentation of a map of different cities. The edges are the roads connecting them with each other. And the distance in some units is given on edges. A person starts walking from V1 place and he needs to cover all the places means V2, V3, V4, V5, V6 and after that he needs to return to the node V1. The condition with this map is that the path covered by person must be minimum possible.

2. EXISTING SOLUTION USING GREEDY METHOD

Dynamic programming and Greedy Algorithms give existing solution to the problem. Especially Greedy algorithm is very efficient in solving this problem. Greedy algorithms solve the problem by making the choices that it feels best at particular moment. There are many optimization problems, which can be solved using Greedy Method i.e.: Activity selection Problem, Fractional Knapsack problem, Huffman Codes etc. The working culture followed by these algorithms is like human general tendency, which is Greed. As an example suppose a human is given a bundle of any currency, which is consisting of different denominations. So first he counts the greatest denomination, then second greatest denomination and so on. This method takes a graph in adjacency matrix form and after applying the algorithm it gives the result in the same form. Starting from V1 in the matrix it chooses the minimum but greater than zero value and the column from the row in

V1. Then the column with minimum value is chosen as the next row where we choose the minimum value again greater than zero. In the figure the chosen row, the minimum value in row and the corresponding chosen column is shown using bold fonts. Initially the matrix is as shown below:

$$\begin{array}{c|cccccc} & V1 & V2 & V3 & V4 & V5 & \mathbf{V6} \\ \hline \mathbf{V1} & 0 & 5 & 0 & 6 & 5 & \mathbf{4} \\ V2 & 5 & 0 & 2 & 4 & 3 & 0 \\ V3 & 0 & 2 & 0 & 1 & 0 & 0 \\ V4 & 6 & 4 & 1 & 0 & 7 & 0 \\ V5 & 5 & 3 & 0 & 7 & 0 & 3 \\ V6 & 4 & 0 & 0 & 0 & 3 & 0 \end{array}$$

Starting from V1 the minimum value except zero is 4. And corresponding column is V6, which is bold in the matrix. So next value is chosen from row V6. This is 2 and shown in next metrix. And so on.

$$\begin{array}{c|cccccc} & V1 & V2 & V3 & V4 & \mathbf{V5} & \mathbf{V6} \\ \hline \mathbf{V1} & 0 & 5 & 0 & 6 & 5 & 4 \\ V2 & 5 & 0 & 2 & 4 & 3 & 0 \\ V3 & 0 & 2 & 0 & 1 & 0 & 0 \\ V4 & 6 & 4 & 1 & 0 & 7 & 0 \\ V5 & 5 & 3 & 0 & 7 & 0 & 3 \\ \mathbf{V6} & 4 & 0 & 0 & 0 & \mathbf{3} & 0 \end{array}$$

Now the minimum value in row V5 is 3. So we need to choose V2 or V6 as next node. In which node V2 is still undiscovered. So V2 is the next node.

$$\begin{array}{c|cccccc} & V1 & \mathbf{V2} & V3 & V4 & \mathbf{V5} & \mathbf{V6} \\ \hline \mathbf{V1} & 0 & 5 & 0 & 6 & 5 & 4 \\ V2 & 5 & 0 & 2 & 4 & 3 & 0 \\ V3 & 0 & 2 & 0 & 1 & 0 & 0 \\ V4 & 6 & 4 & 1 & 0 & 7 & 0 \\ \mathbf{V5} & 5 & \mathbf{3} & 0 & 7 & 0 & 3 \\ \mathbf{V6} & 4 & 0 & 0 & 0 & 3 & 0 \end{array}$$

Now the minimum value in row V2 is 2. This corresponds to the column V3. So next node to discover is V3.

$$\begin{array}{c|cccccc} & V1 & \mathbf{V2} & \mathbf{V3} & V4 & \mathbf{V5} & \mathbf{V6} \\ \hline \mathbf{V1} & 0 & 5 & 0 & 6 & 5 & 4 \\ \mathbf{V2} & 5 & 0 & \mathbf{2} & 4 & 3 & 0 \\ V3 & 0 & 2 & 0 & 1 & 0 & 0 \\ V4 & 6 & 4 & 1 & 0 & 7 & 0 \\ \mathbf{V5} & 5 & 3 & 0 & 7 & 0 & 3 \\ \mathbf{V6} & 4 & 0 & 0 & 0 & 3 & 0 \end{array}$$

Now the minimum value in row V3 is 1. This corresponds to the column V4. So next node to discover is V4.

$$\begin{array}{c|cccccc} & V1 & \mathbf{V2} & \mathbf{V3} & \mathbf{V4} & \mathbf{V5} & \mathbf{V6} \\ \hline \mathbf{V1} & 0 & 5 & 0 & 6 & 5 & 4 \\ \mathbf{V2} & 5 & 0 & 2 & 4 & 3 & 0 \\ \mathbf{V3} & 0 & 2 & 0 & \mathbf{1} & 0 & 0 \\ V4 & 6 & 4 & 1 & 0 & 7 & 0 \\ \mathbf{V5} & 5 & 3 & 0 & 7 & 0 & 3 \\ \mathbf{V6} & 4 & 0 & 0 & 0 & 3 & 0 \end{array}$$

Now we see that all the nodes are discovered. So now we need to return back to V1. Which Gives the cost 6 to visit. So the path with minimum cost is:

$$V1 \xrightarrow{4} V6 \xrightarrow{3} V5 \xrightarrow{3} V2 \xrightarrow{2} V3 \xrightarrow{1} V4 \xrightarrow{6} V1$$

So the total cost of the complete tour is: 4+3+3+2+1+6= 19.

This is the minimum cost that can be found using any kind of traversing.

3.PROPOSED APPROACH USING BRANCH & BOUND

The method we are proposing to solve the problem is Branch and Bound Method. The term branch and bound refers to all state space search methods in which all the children of E-node are generated before any other live node can become the E-node. E-node is the node, which is being expended. State space tree can be expended in any method i.e. BFS or DFS. Both start with the root node and generate other nodes. A node which has been generated and all of whose children are not yet been expanded is called live-node. A node is called dead node, which has been generated, but it cannot be expanded further. The concept of dead node gives the birth to a new concept known as backtracking. Which says once we have traversed a node and it's a dead node and still we could not find the solution? So we need to come back to its parent and traverse its(parent's) another children for the solution. If it has no more children unexpended then we need to reach its parent (grand parent of dead node) and expend its children and so on. And we do so until we get the solution or complete tree is traversed. In this method at each node tree is de we need to expand the node, which is most promising, means the node which promises that expanding or choosing it will give us the optimal solution. So we prepare the tree starting form root then we expand it.

3.1Solution to the problem using Branch and Bound Method:

The input to the method is the cost matrix, which is prepared using the convention:

$$C_{ij}= \begin{cases} \infty \text{, If there is no direct path from } V_i \text{ to } V_j \\ W_{ij} \text{, If there is a direct path from } V_i \text{ to } V_j \end{cases}$$

While solving the problem, we first prepare the state space tree, which represents all possible solution.

Here in this problem |V|=6. Which is the number of total nodes on the graph or the cities in the map.

The input array for the method is given by:

$$\begin{array}{c|cccccc} & V1 & V2 & V3 & V4 & V5 & V6 \\ \hline V1 & \infty & 5 & \infty & 6 & 5 & 4 \\ V2 & 5 & \infty & 2 & 4 & 3 & \infty \\ V3 & \infty & 2 & \infty & 1 & \infty & \infty \\ V4 & 6 & 4 & 1 & \infty & 7 & \infty \\ V5 & 5 & 3 & \infty & 7 & \infty & 3 \\ V6 & 4 & \infty & \infty & \infty & 3 & \infty \end{array}$$

Step 1: Reduce each row and column in such a way that there must be at least one zero in each row and Column. For doing this, we need to reduce the minimum value from each element in each row and column.

a) After reducing the row:

$$\begin{array}{rrc} & & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{r} \text{Reduce 4} \\ \text{Reduce 2} \\ \text{Reduce 1} \\ \text{Reduce 1} \\ \text{Reduce 3} \\ \text{Reduce 3} \end{array} & \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & 1 & \infty & 2 & 1 & 0 \\ 3 & \infty & 0 & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 5 & 3 & 0 & \infty & 6 & \infty \\ 2 & 0 & \infty & 4 & \infty & 0 \\ 1 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} \end{array}$$

b) After reducing column

$$\begin{array}{rc} & \begin{array}{cccccc} \text{Reduce 1} & & & & & \\ \downarrow & & & & & \\ V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & 1 & \infty & 2 & 1 & 0 \\ 2 & \infty & 0 & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 4 & 3 & 0 & \infty & 6 & \infty \\ 1 & 0 & \infty & 4 & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M1} \end{array}$$

So the total expected cost at the root node is the sum of all reductions.

Total expected cost of expanding root node L (1)=4+2+1+1+3+3+1=15.

Because we have to plan the path starting from V1, so V1 will be the root of the tree and it would be the first node to be expanded.

Step 2: **We** have discovered the root node V1 so the next node to be expanded will be any node from V2, V3, V4, V5, V6. So we have to find out the expanding cost of each node. So which one would be the minimum we are going to expand it further. We will repeat a procedure for every node to find the expending cost for its expansion. The formula for finding the cost is:

L (node)=L (parent node)+P_{arent} (i, j) + total cost of reduction

a) Obtain cost of expanding using cost matrix for node 2 in the tree:

i) Change all the elements in 1st row and 2nd column and M_1 (2,1) to ∞.

$$\begin{array}{rc} & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & 1 & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ 4 & \infty & 0 & \infty & 6 & \infty \\ 1 & \infty & \infty & 4 & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M2'=M2} \end{array}$$

ii) Reduce M2’ in row and columns. As each row and column already has a zero so it cannot be reduced more. So reduction cost =0. So the total cost of expanding node 2 L (2)=L (1)+ M_1 (1,2)+r=15+1+0=16.

b) Obtain cost of expanding using cost matrix for node 3 in the tree:

i) Change all the elements in 1st row and 3rd column and M_1 (3,1) to ∞.

$$\begin{array}{rc} & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 4 & 3 & \infty & \infty & 6 & \infty \\ 1 & 0 & \infty & 4 & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M3'} \end{array}$$

ii) Reduce M3’ in row and columns.

$$\begin{array}{rrc} & & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{r} \\ \text{Reduce 1} \\ \\ \text{Reduce 3} \\ \\ \\ \end{array} & \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 1 & 0 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 1 & 0 & \infty & \infty & 3 & \infty \\ 1 & 0 & \infty & 4 & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M3'} \end{array}$$

So the total cost of expanding node 3, L (3)= L (1)+ M_1 (1,3)+r=15+∞+4=∞

c) Obtain cost of expanding using cost matrix for node 4 in the tree:

i) Change all the elements in 1st row and 4th column and M_1 (4,1) to ∞.

$$\begin{array}{rc} & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & 0 & \infty & 1 & \infty \\ \infty & 1 & \infty & \infty & \infty & \infty \\ \infty & 3 & 0 & \infty & 6 & \infty \\ 1 & 0 & \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M4'} \end{array}$$

ii) Reduce M3’ in row and columns.

$$\begin{array}{rrc} & & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{r} \\ \\ \text{Reduce 1} \\ \\ \\ \\ \end{array} & \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & 0 & \infty & 1 & \infty \\ \infty & 0 & \infty & \infty & \infty & \infty \\ \infty & 3 & 0 & \infty & 6 & \infty \\ 1 & 0 & \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & \infty & 0 & \infty \end{pmatrix} = \mathbf{M4} \end{array}$$

So the total cost of expanding node 4, L (4)= L (1)+ M_1 (1,4)+r=15+2+1=18.

d) Obtain cost of expanding using cost matrix for node 5 in the tree:

i) Change all the elements in 1st row and 5th column and M_1 (5,1) to ∞.

$$\begin{array}{rc} & \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{array} & \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & 0 & 2 & \infty & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 4 & 3 & 0 & \infty & \infty & \infty \\ 1 & 0 & \infty & 4 & \infty & 0 \\ 0 & \infty & \infty & \infty & \infty & \infty \end{pmatrix} = \mathbf{M5' = M5} \end{array}$$

ii) Reduce M2' in row and columns. As each row and column already has a zero so it cannot be reduced more. So reduction cost =0. So the total cost of expanding node 5 , L(5)= L(1)+ M_1 (1,5)+r=15+2+0=17.

e) Obtain cost of expanding using cost matrix for node 6 in the tree:

i) Change all the elements in 1st row and 6th column and M_1 (6,1) to ∞**.**

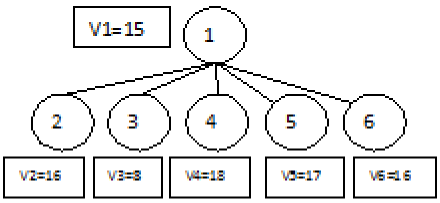
$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 2 & \infty & 0 & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 4 & 3 & 0 & \infty & 6 & \infty \\ 1 & 0 & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & \infty \end{pmatrix} \end{array} = \mathbf{M6'}$$

ii) Reduce M6' in row and columns.

Reduce 1

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 1 & \infty & 0 & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 3 & 3 & 0 & \infty & 6 & \infty \\ 0 & 0 & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & \infty \end{pmatrix} \end{array} = \mathbf{M6}$$

So the total cost of expanding node 6, L (6)= L (1)+ M_1 (1,6)+r=15+0+1=16.



Now we have two most promising V2 and V6. Suppose we choose V6 as the next node. So we will expand the tree on node 6, which belongs to V6. Till now two nodes have been traversed V1 and V6. So we have to find out the next node to be traversed.

Step3: As we are choosing V6 as the next node to be expand. So M6 will work as input matrix for this step. And we have 4 nodes still to be traversed. So we can expand V2, V3, V4, and V5 as the next node.

So using the same method we will find the expansion cost of each of these nodes.

a) Obtain cost of expanding using cost matrix for node 7 in the tree:

i) Change all the elements in 6th row and 2nd column and M_6 (2,6) to ∞**.**

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 1 & \infty & 0 & 2 & 1 & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ 3 & \infty & 0 & \infty & 6 & \infty \\ 0 & \infty & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = \mathbf{M7'}$$

ii) Reduce M7' in row and columns.

Reduce 1

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 0 & \infty \\ 1 & \infty & 0 & 2 & 0 & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ 3 & \infty & 0 & \infty & 5 & \infty \\ 0 & \infty & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = \mathbf{M7}$$

So the total cost of expanding node 7, L (7)= L (6)+ M_6 (6,2)+1=16+∞+1=∞.

b) Obtain cost of expanding using cost matrix for node 8 in the tree:

i) Change all the elements in 6th row and 3rd column and M_6 (3,6) to ∞**.**

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 1 & \infty & \infty & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 3 & 3 & \infty & \infty & 6 & \infty \\ 0 & 0 & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = \mathbf{M8'}$$

ii) Reduce M8' in row and columns.

Reduce 1

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 1 & \infty & \infty & 2 & 1 & \infty \\ \infty & 1 & \infty & 0 & \infty & \infty \\ 3 & 3 & \infty & \infty & 6 & \infty \\ 0 & 0 & \infty & 4 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = \mathbf{M8}$$

So the total cost of expanding node 8, L (8)= L (6)+ M_6 (6,3)+1=16+∞+1=∞.

c) Obtain cost of expanding using cost matrix for node 9 in the tree:

i) Change all the elements in 6th row and 4th column and M_6 (4,6) to ∞**.**

$$\begin{array}{cc} & \begin{matrix} V1 & V2 & V3 & V4 & V5 & V6 \end{matrix} \\ \begin{matrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{matrix} & \begin{pmatrix} \infty & \infty & \infty & \infty & 1 & \infty \\ 1 & \infty & 0 & \infty & 1 & \infty \\ \infty & 1 & \infty & \infty & \infty & \infty \\ 3 & 3 & 0 & \infty & 6 & \infty \\ 0 & 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & \infty \end{pmatrix} \end{array} = \mathbf{M9'}$$

ii) Reduce M9’ in row and columns.

$$\begin{array}{c|cccccc}
 & & & & & \text{Reduce 1} & \\
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & 0 & \infty \\
V2 & 1 & \infty & 0 & \infty & 0 & \infty \\
V3 & \infty & 1 & \infty & \infty & \infty & \infty \\
V4 & 3 & 3 & 0 & \infty & 5 & \infty \\
V5 & 0 & 0 & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M9$$

So the total cost of expanding node 9, L (9)= L (6)+ M_6 (6,4)+1=16+∞+1=∞.

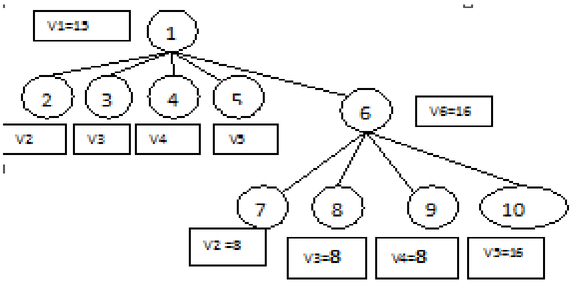
d) Obtain cost of expanding using cost matrix for node 9 in the tree:

i) Change all the elements in 6^{th} row and 5^{th} column and M_6 (5,6) to ∞**.**

$$\begin{array}{c|cccccc}
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 1 & \infty & 0 & 2 & \infty & \infty \\
V3 & \infty & 1 & \infty & 0 & \infty & \infty \\
V4 & 3 & 3 & 0 & \infty & \infty & \infty \\
V5 & 0 & 0 & \infty & 4 & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M10' = M10$$

ii) Reduce M10’ in row and columns. M10’is already reduced. So we don’t need to reduce it anymore. So it is final M_{10}.

So the total cost of expanding node 10, L (10)= L (6)+ M_6 (6,5)+0=16+0+1=16.



Step3: At this step node 10, V5 is the most promising node, because it gives the minimum cost of traversal. So we are going to expand it further. And we have 3 nodes still to be traversed. So we can expand V2, V3, and V4 as the next node.

So using the same method we will find the expansion cost of each of these nodes.

a) Obtain cost of expanding using cost matrix for node 11 in the tree:

i) Change all the elements in 5^{th} row and 2^{nd} column and M_{10} (2,5) to ∞**.**

$$\begin{array}{c|cccccc}
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 1 & \infty & 0 & 2 & \infty & \infty \\
V3 & \infty & \infty & \infty & 0 & \infty & \infty \\
V4 & 3 & \infty & 0 & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M11'$$

ii) Reduce M11’ in row and columns.

$$\begin{array}{c|cccccc}
 & \text{Reduce 1} & & & & & \\
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 0 & \infty & 0 & 2 & \infty & \infty \\
V3 & \infty & \infty & \infty & 0 & \infty & \infty \\
V4 & 2 & \infty & 0 & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M11$$

So the total cost of expanding node 11, L (11)= L (10)+ M_{10} (5,2)+r=16+0+1=17.

b) Obtain cost of expanding using cost matrix for node 12 in the tree:

i) Change all the elements in 5^{th} row and 3rd column and M_{10} (3,5) to ∞

$$\begin{array}{c|cccccc}
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 1 & \infty & \infty & 2 & \infty & \infty \\
V3 & \infty & 1 & \infty & \infty & \infty & \infty \\
V4 & 3 & 3 & \infty & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M12'$$

ii) Reduce M12’ in row and columns.

$$\begin{array}{r|cccccc}
 & & & & \text{Reduce 1} & & \\
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
\text{Reduce 1}\ V2 & 0 & \infty & \infty & 0 & \infty & \infty \\
\text{Reduce 1}\ V3 & \infty & 0 & \infty & \infty & \infty & \infty \\
V4 & 2 & 3 & \infty & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M12$$

So the total cost of expanding node 12, L (12)= L (10)+ M_{10} (5,3)+r=16+∞+3=∞.

c) Obtain cost of expanding using cost matrix for node 13 in the tree:

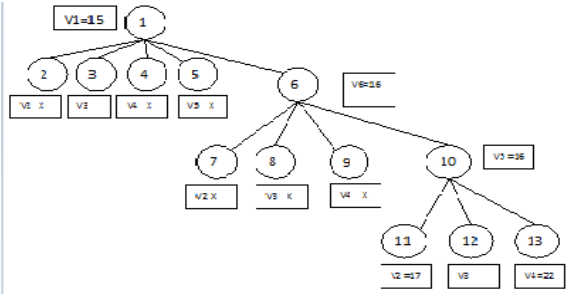
i) Change all the elements in 5^{th} row and 4^{th} column and M_{10} (4,5) to ∞

$$\begin{array}{c|cccccc}
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 1 & \infty & 0 & \infty & \infty & \infty \\
V3 & \infty & 1 & \infty & \infty & \infty & \infty \\
V4 & 3 & 3 & 0 & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M13'$$

ii) Reduce M13’ in row and columns.

$$\begin{array}{r|cccccc}
 & \text{Reduce 1} & & & & & \\
 & V1 & V2 & V3 & V4 & V5 & V6 \\
\hline
V1 & \infty & \infty & \infty & \infty & \infty & \infty \\
V2 & 0 & \infty & 0 & \infty & \infty & \infty \\
\text{Reduce 1}\ V3 & \infty & 0 & \infty & \infty & \infty & \infty \\
V4 & 2 & 3 & 0 & \infty & \infty & \infty \\
V5 & \infty & \infty & \infty & \infty & \infty & \infty \\
V6 & \infty & \infty & \infty & \infty & \infty & \infty
\end{array} = M13$$

So the total cost of expanding node 13, L (13)= L (10)+ M_{10} (5,4)+r=16+4+2=22.



Step 4: Here V2 is the most promising node which gives the minimum cost of expanding the tree which is 17. Now M11 will be the input matrix for this step. Now we are left with two nodes V3, V4 not traversed.
So using the same method we will find the expansion cost of each of these nodes.

a) Obtain cost of expanding using cost matrix for node 14 in the tree:

i) Change all the elements in 2nd row and 3rd column and M_{11} (3,2) to ∞**.**

$$\begin{array}{c} \\ V1\\V2\\V3\\V4\\V5\\V6 \end{array}\begin{array}{c} \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ 2 & \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = M14'$$

ii) Reduce M14' in row and columns.

Reduce 2

$$\begin{array}{c} \\ V1\\V2\\V3\\V4\\V5\\V6 \end{array}\begin{array}{c} \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = M14$$

So the total cost of expanding node 14, L (14)= L (11)+ M_{11} (2,3)+r=17+0+2=19.

b) Obtain cost of expanding using cost matrix for node 15 in the tree:

i) Change all the elements in 2nd row and

4th column and M_{11} (4,2) to ∞**.**

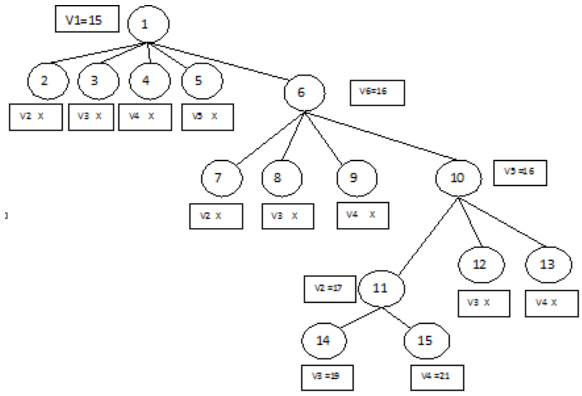
$$\begin{array}{c} \\ V1\\V2\\V3\\V4\\V5\\V6 \end{array}\begin{array}{c} \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = M15'$$

ii) Reduce M15' in row and columns.

Reduce 2

$$\begin{array}{c} \\ V1\\V2\\V3\\V4\\V5\\V6 \end{array}\begin{array}{c} \begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\ \begin{pmatrix} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix} \end{array} = M15$$

So the total cost of expanding node 15, L (15)= L (11)+ M_{11} (2,4)+r=17+2+2=21.



Step 5: Here V3 is the most promising node so next we are going to expand this node further. Now we are left with only one node not yet traversed which is V4. Then the tour is completed so we will return back to the node V1. So the sequence of traversal is:

$$V1 \xrightarrow{4} V6 \xrightarrow{3} V5 \xrightarrow{3} V2 \xrightarrow{2} V3 \xrightarrow{1} V4 \xrightarrow{6} V1$$

So the total cost of traversing the graph is: **4+3+3+2+1+6=19.**

4. CONCLUSION

The proposed method, which is using Branch & Bound, is better because it prepares the matrices in different steps. At each step the cost matrix is calculated. From the initial point we come to know that what can be the minimum cost of the tour. The cost in the initial stages is not exact cost but it gives some idea because it is the approximated cost. At each step it gives us the strong reason that which node we should travel the next and which one not. It gives this fact in terms of the cost of expanding a particular node.

5. REFERENCES

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