On the Stability and Strong Convergence for Jungck-Agarwal et al. Iteration Procedure

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ABSTRACT

In this paper we introduce the Jungck-Agarwal et al. iteration procedure and obtain strong convergence as well as stability results for a pair of non-self mappings. The results obtained are generalization of some existing results in the literature. In addition, we show that the rate of convergence of this newly defined iteration procedure is better than Jungck- Mann, Jungck-Ishikawa and Jungck- Noor iteration procedures.

General Terms

Computational Mathematics.

Keywords

Jungck-Ishikawa iteration, Jungck-Agarwal et al. iteration , Jungck-Noor iteration, Jungck-Mann iteration, Stability.

1. INTRODUCTION

Let (E, d) be a complete metric space, T be *a* self map of E. Suppose that $F_T = \{p \in E \ s.t \ Tp = p\}$ is the set of fixed points of T in E. In metric space we have several iterative processes that have been defined by many researchers to approximate the fixed points of different operators. Some of them are the following:

Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving the operator T, that is,

$$x_{n+1} = f(T, x_n) \quad n = 0, 1, 2 \ldots \ldots \quad (1.1)$$

where $x_0 \in E$, be the initial approximation and f is some function. If in (1.1)

$$f(T, x_n) = Tx_n \quad (1.2)$$

Then we have **Picard iteration process** which has been used to approximate the fixed point of mappings satisfying the inequality:

$$d(Tx, Ty) \leq ad(x, y), \ \forall \ x, y \in E \ and \ a \in [0, 1) \quad (1.3)$$

Condition (1.3) is called Banach contraction condition and operator which satisfies the condition is called strict contraction. Also condition (1.3) is significant in Banach Fixed point theorem [2].

Let $x_0 \in E$, be the initial approximation, where E is any Banach space, then

In 1953, W.R. Mann defined the Mann iteration [10] as

$$x_{n+1} = (1 - \alpha_n)x_n + Tx_n \quad (1.4)$$

Where $\{\alpha_n\}_{n=0}^{\infty}$ is real sequence in [0, 1] satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

For $x_0 \in E$, Ishikawa Iteration [7] (S. Ishikawa 1974) $\{x_n\}_{n=0}^{\infty}$ is defined as

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Ty_n$$
$$y_n = (1 - \beta_n)x_n + \beta_n Tx_n \quad (1.5)$$

Where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0, 1] satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$ .Observe that if $\beta_n = 0$ for each *n*, then the Ishikawa iteration process (1.5) reduces to the Mann iteration scheme (1.4).

For $x_0 \in E$ such that $\{x_n\}_{n=0}^{\infty}$, the Noor three step iteration scheme [11] is defined as

$$x_{n+1} = (1 - \alpha_n)x_n + Ty_n$$
$$y_n = (1 - \beta_n)x_n + Tz_n \quad (1.6)$$
$$z_n = (1 - \gamma_n)x_n + Tx_n$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in [0, 1] satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

If γ_n=0 then Noor iteration process (1.6) reduces to Ishikawa Iteration scheme (1.5)

Agarwal Iteration Process

In 2007, Agarwal et al. defined the Agarwal et al. iterative scheme [1] as

$$x_{n+1} = (1-\alpha_n)Tx_n + \alpha_n Ty_n$$
$$y_n = (1-\beta_n)x_n + \beta_n Tx_n \quad (1.7)$$

Where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in [0, 1] satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

Zemfirescu [25] established a nice generalization of the Banach fixed point theorem as

Let T be a self map, there exists real numbers α, β, γ satisfying $0 \leq \alpha < 1, 0 \leq \beta < \frac{1}{2}, 0 \leq \gamma < \frac{1}{2}$ respectively such that for each $x, y \in E$, at least one of the following is true:

$(z_1) \, d(Tx, Ty) \le \alpha d(x, y)$

$(z_2) \, d(Tx, Ty) \le \beta[d(x, Tx) + d(y, Ty)]$ (1.8)

$(z_3) \, d(Tx, Ty) \le \gamma[d(x, Ty) + d(y, Tx)]$

Then the mapping T satisfying (1.8) is called Zamfirescu contraction.

Remark

Mapping which satisfy (z_2) is called a Kannan mapping. While the mapping satisfying (z_3) is called Chatterjea operator.

The contractive condition (1.8) implies

$$\|Tx - Ty\| \le 2\delta \|x - Tx\| + \delta \|x - y\|, \, \forall \, x, y \in E, \quad (1.9)$$

Where

$$\delta = \max \{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\} \, , \, 0 \le \delta < 1.$$

Rhoades [21, 22] used condition (1.8) in some convergence results for Mann and Ishikawa iteration process in a uniformly convex Banach space. Berinde [3] extended (1.8) to a new class of operators to an arbitrary Banach space and proved that this class is wider than class of Zamfirescu operators.

Several stability results are established by various authors in literature due to its increasing importance in computational mathematics. Some of the various authors whose contribution are much important in the study of stability of fixed point iteration process are Ostrowski [18], Harder and Hicks [6], Rhoades[20, 21], Osillike[16], Osilike and Udomene[15] ,Jachymski [8] ,Berinde [4, 5] and Singh et al. [24]. The summability theory of infinite matrices are used by Harder and Hicks [6], Rhoades [22], Osilike[23], and Singh et al.[24] to prove different stability results for certain contractive conditions. The first stability result on T- stable mappings was due to Ostrowski[18] for the stability of Picard iteration using Banach contraction condition. Osilike and Udomene[15] introduced a shorter method for stability results and has also been employed by Berinde[4] ,Imoru and Olatinwo[12],Olatinwo et al. [13] and some others.

2. Preliminaries

In 1976, Jungck introduced the Jungck iteration scheme [9] as

Let S, T be arbitrary operators on any set Y with values in E such that $T(Y) \subseteq S(Y)$, $S(Y)$ is complete subspace of E. for any $x_0 \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined as

$Sx_{n+1} = Tx_n \qquad n = 0, 1......$ (2.1)

For S=Id (2.1) becomes Picard iteration

Singh et al [24] in 2005 introduce the following iteration to establish some stability result as:

$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n \qquad$ n=0, 1, 2…..   (2.2)

Where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in [0, 1], called Jungck-Mann iteration process.

Jungck [9] established that the maps S and T satisfying

$d(Tx, Ty) \le kd(Sx, Sy), \qquad 0 \le k < 1 \, \forall \, x, y \in E$

has a unique common fixed point. For Y=E and S=Id above mapping becomes well known contraction map.

Olatinwo & Imoru in 2008 for $x_0 \in Y$ defined $\{Sx_n\}_{n=1}^{\infty}$ as

$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n$

$Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n$ (2.3)

Called Jungck-Ishikawa Iteration scheme [13]

Where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0, 1]

satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

Oltinwo in 2008 define Jungck-Noor iteration [14] process as

Let $x_o \in Y$ then $\{Sx_n\}_{n=1}^{\infty}$ is defined by

$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha Ty_n$

$Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n$ (2.4)

$Sz_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in [0, 1] satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

A more general contractive condition than (1.9) was used by Oltinwo[13] to prove stability and strong convergence in Jungck-Ishikawa iteration process.

(a) There exists a real number $a \in [0, 1)$ and a monotone increasing function $\varphi: R^+ \to R^+$,

such that φ (0) = 0 and $\forall x, y \in Y$, we have

$$\|Tx - Ty\| \le \varphi(\|Sx - Tx\|) + a\|Sx - Sy\| \qquad (2.5)$$

(b) There exist a real number $M \ge 0$, $a \in [0,1)$ and a monotone increasing function $\varphi: R^+ \to R^+$ such that $\varphi(0) = 0$ and $\forall$ x, y∈Y, we have

$$\|Tx - Ty\| \le \frac{\varphi(\|Sx - Tx\|) + a\|Sx - Sy\|}{1 + M\|Sx - Tx\|} \qquad (2.6)$$

In this paper, we define following new Jungck-Agarwal et al. iteration process and use it to approximate the common fixed point by using contractive condition (2.5, 2.6)

$Sx_{n+1} = (1-\alpha_n)Tx_n + \alpha_n Ty_n$

$Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n$ (2.7)

Where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0, 1]

satisfying $\sum_{k=0}^{\infty} \alpha_k = \infty$

We use following definition and lemma:

Definition 2.1. [12] Let S, T: Y→X such that $T(Y) \subseteq S(Y)$ and z a coincidence point of S and T, that is, $Sz = Tz = p$ (say). For any $x_0 \in Y$. Let the sequence $\{Sx_n\}_{n=1}^{\infty}$ generated by the iteration procedure (2.7) converges to p. Let $\{Sy_n\}_{n=0}^{\infty} \subseteq E$ be an arbitrary sequence, and set $\varepsilon_n = d(Sy_{n+1}, f(T, y_n)), \quad n = 0, 1, 2....$ Then, the iteration procedure (2.7) will be called (S, T)-stable if and only if $\lim_{n \to \infty} \varepsilon_n = 0$ implies that $\lim_{n \to \infty} S(y_n) = p$.

Lemma 2.1. (Berinde [3]):
If δ is a real number such that $0 \le \delta < 1$, and $\{\varepsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \varepsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying

$$u_{n+1} \le \delta u_n + \varepsilon_n, \quad n = 0,1,2.....,$$

We have $\lim_{n\to\infty} u_n = 0$

In this paper we shall prove convergence results for Jungck-Agarwal et al. iteration process for arbitrary Banach space and stability result for nonlinear mappings in normed linear space by using contractive condition (2.5, 2.6). Also by using C++ we shall prove that Jungck-Agarwal et al. iteration scheme converges faster than other jungck iteration scheme.

3. STABILITY RESULT IN NORMED LINEAR SPACE.

Theorem 3.1. Let $(E, \| \; \|)$ be a normed linear space and Y an arbitrary set. Suppose that S, T: Y→E are nonself operators such that $T(Y) \subseteq S(Y)$. S(Y) a complete subspace of E and S is an injective operator. Let z be a coincidence point of S and T (that is, Sz=Tz=p). Suppose that S and T satisfy condition (2.5, 2.6). Let φ: $R^+ \to R^+$ be monotone increasing function such that φ (0) =0. For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be Jungck- Agarwal et al. iteration process defined by (2.7) converging to p, where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in [0,1], such that $0 < \alpha \le \alpha_n$ and $0 < \beta \le \beta_n$. Then, the Jungck-Agarwal et al. iteration process is (S, T)-stable.

Proof: Suppose that $\{Sx_n\}_{n=0}^{\infty} \subset E$, $\varepsilon_n = \| Sx_{n+1} - (1-\alpha_n)Tx_n - \alpha_n Ty_n \|$, n==0, 1, 2….,

and let $\lim_{n\to\infty} \varepsilon_n = 0$ .Then, we shall establish that $\lim_{n\to\infty} Sx_n = p$, using the contractive condition (2.5, 2.6) and the triangle inequality:

$$\| Sx_{n+1} - p \| \le \| Sx_{n+1} - (1-\alpha_n)Tx_n - \alpha_n Ty_n \| +$$
$$\| (1-\alpha_n)Tx_n + \alpha_n Ty_n - (1 - \alpha_n + \alpha_n)p \|$$
$$\le \| Sx_{n+1} - (1-\alpha_n)Tx_n - \alpha_n Ty_n \|$$
$$+ (1-\alpha_n)\| Tx_n - p \| + \alpha_n \| Ty_n - p \|$$
$$\le \varepsilon_n + (1-\alpha_n)[\frac{\varphi(\| Sz - Tz \|) + a\| Sz - Sx_n \|}{1 + M\| Sz - Tz \|}] + \alpha_n[\frac{\varphi(\| Sz - Tz \|) + a\| Sz - Sy_n \|}{1 + M\| Sz - Tz \|}]$$
$$\le \varepsilon_n + (1-\alpha_n)a\| Sx_n - p \| + \alpha_n a \| Sy_n - p \| \qquad (3.1)$$

Therefore, we have

$$\| Sy_n - p \| = \| (1-\beta_n)Sx_n + \beta_n Tx_n + (1 - \beta_n + \beta_n)p \|$$
$$\le (1-\beta_n)\| Sx_n - p \| + \beta_n \| Tx_n - p \|$$
$$\le (1-\beta_n)\| Sx_n - p \| + a\beta_n \| Sx_n - p \| \qquad (3.2)$$
$$\le (1-\beta_n + a\beta_n)\| Sx_n - p \|$$

Using (3.2) in (3.1) yields

$$\| Sx_{n+1} - p \|$$
$$\le [a(1-\alpha_n) + \alpha_n a(1 - \beta_n + a\beta_n)]\| Sx_n - p \| + \varepsilon_n$$
$$\le a[1 - (1-a)\alpha_n \beta_n]\| Sx_n - p \| + \varepsilon_n \qquad (3.3)$$
$$\le [1 - (1-a)\alpha_n]\| Sx_n - p \| + \varepsilon_n$$
$$\le [1 - (1-a)\alpha]\| Sx_n - p \| + \varepsilon_n$$

Since $0 \le 1 - (1-a)\alpha < 1$, using Lemma 2.1 in (3.3) yields

$\lim_{n\to\infty} \| Sx_n - p \| = 0$, that is, $\lim_{n\to\infty} Sx_n = p$

Conversely, let $\lim_{n\to\infty} Sx_n = p$. Then, by using the triangle inequality and contractive definition, we have the following:

$$\varepsilon_n = \| Sx_{n+1} - (1-\alpha_n)Tx_n - \alpha_n Ty_n \|$$
$$\le \| Sx_{n+1} - p \| + \| (1 - \alpha_n + \alpha_n)p - (1-\alpha_n)Tx_n - \alpha_n Ty_n \|$$
$$\le \| Sx_{n+1} - p \| + (1-\alpha_n)\| Tx_n - p \| + \alpha_n \| Ty_n - p \| \qquad (3.4)$$
$$\le \| Sx_{n+1} - p \| + (1-\alpha_n)a\| Sx_n - p \| + \alpha_n a\| Sy_n - p \|$$

Again, by the contractive condition(2.6) , we get

$$\| Sy_n - p \| = \| (1 - \beta_n + \beta_n)p - (1-\beta_n)Sx_n - \beta_n Tx_n \|$$
$$= \| (1-\beta_n)(p - Sx_n) + \beta_n(p - Tx_n) \|$$
$$\le (1-\beta_n)\| p - Sx_n \| + \beta_n \| p - Tx_n \| \qquad (3.5)$$
$$= (1-\beta_n)\| p - Sx_n \| + \beta_n \| Tz - Tx_n \|$$
$$\le (1-\beta_n + a\beta_n)\| p - Sx_n \|$$

Using (3.5) in (3.4), we obtain

$$\varepsilon_n \le \| Sx_{n+1} - p \| + a[1 - (1-a)\alpha_n \beta_n]\| Sx_n - p \|$$
$$\le \| Sx_{n+1} - p \| + [1 - (1-a)\alpha_n]\| Sx_n - p \|$$
$$\le \| Sx_{n+1} - p \| + [1 - (1-a)\alpha]\| Sx_n - p \| \to 0 \; as \; n \to \infty$$

Hence, the iteration process defined in (2.7) is stable with respect to the pair (S, T).

4. STRONG CONVERGENCE IN AN ARBITRARY BANACH SPACE

Theorem 4.1: Let $(E, \| \; \|)$ be arbitrary Banach space and Y an arbitrary set. Suppose that S, T: Y→E are nonself operators such that $T(Y) \subseteq S(Y)$. S(Y) a complete subspace of E and S is an injective operator. Let z be a coincidence point of S and T (that is, Sz=Tz=p). Suppose that S and T satisfy condition (2.5, 2.6). Let φ: $R^+ \to R^+$ be monotone increasing function such that φ (0) =0. For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be Jungck- Agarwal et al. iteration process defined by (2.7) converging to p, where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in [0, 1], such that $\sum_{k=0}^{\infty} \alpha_k = \infty$. Then, $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p.

Proof: Let C(S, T) be the set of the coincidence points of S and T. We shall now use condition (2.5) to establish that S and T have a unique coincidence point z (i.e. Sz=Tz=p (say)): Suppose that there exists z_1, $z_2 \in C(S,T)$ such that $Sz_1=Tz_1=p_1$ and $Sz_2=Tz_2=p_2$. If $p_1=p_2$, then $Sz_1=Sz_2$ and since S is

injective, it follows that $z_1=z_2$. If $p_1 \neq p_2$, then we have by contractiveness condition (2.5) for S and T that

$$0 < \|p_1 - p_2\| = \|Tz_1 - Tz_2\| \leq \varphi(\|Sz_1 - Tz_2\|) + a\|Sz_1 - Sz_2\|$$
$$= a\|p_1 - p_2\|$$

Which leads to $(1-a)\|p_1 - p_2\| \leq 0$, from which it follows that 1-a>0 since a$\in$ [0, 1), but $\|p_1 - p_2\| \leq 0$, which is a contradiction since norm is nonnegative. Therefore, we have that $\|p_1 - p_2\| = 0$, that is, $p_1=p_2=p$. Since $p_1=p_2$, then we have that $p_1=Sz_1=Tz_1==Sz_2=Tz_2=p_2$, leading to $Sz_1=Sz_2 \Longrightarrow z_1=z_2=z$ (since S is injective). Hence, z$\in$ C(S, T), that is, z is unique coincidence point of S and T.

We now prove that $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p (where Sz=Tz=p) using again, condition (2.5, 2.6). Therefore, we have

$$\|Sx_{n+1} - p\| = \|(1-\alpha_n)Tx_n + \alpha_n Ty_n - (1-\alpha_n + \alpha_n)p\|$$
$$\leq (1-\alpha_n)\|p - Tx_n\| + \alpha_n\|p - Ty_n\|$$
$$= (1-\alpha_n)\|Tz - Tx_n\| + \alpha_n\|Tz - Ty_n\|$$
$$\leq (1-\alpha_n)[\frac{\varphi(\|Sz - Tz\|) + a\|Sz - Sx_n\|}{1 + M\|Sz - Tz\|}] \quad (4.1)$$
$$+ \alpha_n[\frac{\varphi(\|Sz - Tz\|) + a\|Sz - Sy_n\|}{1 + M\|Sz - Tz\|}]$$
$$\leq (1-\alpha_n)a\|Sx_n - p\| + \alpha_n a\|Sy_n - p\|$$

$$\|Sy_n - p\| \leq \|(1-\beta_n)Sx_n + \beta_n Tx_n +\|$$
$$\leq \|(1-\beta_n)Sx_n + \beta_n(Tz - Tx_n) - (1+\beta_n - \beta_n)p\|$$
$$\leq (1-\beta_n)\|Sx_n - p\| + \beta_n\|Tx_n - p\|$$
$$\leq (1-\beta_n)\|Sx_n - p\| \quad (4.2)$$
$$+ \beta_n[\frac{\varphi(\|Sz - Tz\|) + a\|Sz - Sx_n\|}{1 + M\|Sz - Tz\|}]$$
$$\leq (1-\beta_n + a\beta_n)\|Sx_n - p\|$$

By using (4.2) in (4.1) becomes

$$\|Sx_{n+1} - p\| \leq (1-\alpha_n)a\|Sx_n - p\|$$
$$+ \alpha_n a[\|(1-\beta_n)(p - Sx_n) + \beta_n(Tz - Tx_n)\|]$$
$$\leq (1-\alpha_n)a\|Sx_n - p\|$$
$$+ \alpha_n a[1 - \beta_n + a\beta_n]\|Sx_n - p\|$$
$$= a[1 - (1-a)\alpha_n\beta_n]\|Sx_n - p\|$$
$$\leq [1 - (1-a)\alpha_n]\|Sx_n - p\|$$
$$\leq \prod_{k=0}^{n}[1 - (1-a)\alpha_k]\|Sx_0 - p\|$$
$$\leq \prod_{k=0}^{n}e^{-(1-a)\alpha_k}\|Sx_0 - p\|$$
$$\leq e^{-[(1-a)\sum_{k=0}^{n}\alpha_k]}\|Sx_0 - p\| \to 0 \text{ as } n \to \infty \quad (4.3)$$

Since $\sum_{k=0}^{\infty}\alpha_k = \infty$ and a$\in$ [0, 1). Hence, we obtain from (4.3) that $\|Sx_n - p\| \to 0 \text{ as } n \to \infty$, that is, $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p.

5. Experiment

To highlight the practical implication, we apply the iteration to solve the following equation

$$x^3 + 4x^2 - 5x - 10 = 0 \quad (5.1)$$

and prove that Jungck-Agarwal et al. iteration converges faster than Jungck-Noor, Jungck-Mann and Jungck-Ishikawa used by Bhagwati and Ritu [19] . We solve the equation by rewriting as by Bhagwati and Ritu [19]

$$Sx = x \text{ and } Tx = x^3 + 4x^2 - 10 \quad (5.2)$$

Then the coincidence point of S and T in (5.2) leads to the solution of cubic equation (5.1). We compile and execute separate program in C++ and show our output in table1 by taking initial approximation

$$x_0 = 1 \text{ and } \alpha_n = \beta_n = \gamma_n = 0.9$$

Table-1

	Jungck-Agarwal et al. Iteration Scheme			Jungck-Noor Iteration Scheme			Jungck-Mann Iteration Scheme			Jungck-Ishikawa Iteration Scheme		
0	-7.6568	-5	-1.53136	-4.24281	-5	-0.848561	-4	-5	-0.8	-6.6568	-5	-1.33136
1	-7.11342	-4.21088	-1.42268	-7.18707	-7.73079	-1.43741	-7.5568	-7.952	-1.51136	-6.59253	-5.26978	-1.31851
2	-6.76354	-4.78344	-1.35271	-5.0744	-4.70529	-1.01488	-4.63956	-4.31543	-0.927913	-6.53485	-5.33834	-1.30697
3	-6.52719	-5.15593	-1.30544	-6.65334	-6.92539	-1.33067	-7.08334	-7.35487	-1.41667	-6.48296	-5.39985	-1.29659
4	-6.36414	-5.40802	-1.27283	-5.47281	-5.27347	-1.09456	-5.04219	-4.81539	-1.00844	-6.43619	-5.45517	-1.28724
5	-6.25059	-5.58173	--1.25012	-6.36812	-6.51909	-1.27362	-6.76619	-6.95774	-1.35324	-6.39397	-5.505	-1.27879
41	**-5.98443**	**-5.98443**	**-1.19689**	-5.98442	-5.98441	-1.19688	-5.98665	-5.98719	-1.19733	-5.99572	-5.9713	-1.19914
42	**-5.98443**	**-5.98443**	**-1.19689**	-5.98445	-5.98445	-1.19689	-5.98255	-5.98209	-1.19651	-5.99468	-5.97252	-1.19894
43	**-5.98443**	**-5.98443**	**-1.19689**	-5.98442	-5.98442	-1.19688	-5.98604	-5.98642	-1.19721	-5.99373	-5.97362	-1.19875
51				**-5.98443**	**-5.98443**	**-1.19689**	-5.98487	-5.98498	-1.19697	-5.98871	-5.97946	-1.19774
52				**-5.98443**	**-5.98443**	**-1.19689**	-5.98406	-5.98397	-1.19681	-5.98831	-5.97992	-1.19766
53				**-5.98443**	**-5.98443**	**-1.19689**	-5.98475	-5.98483	-1.19695	-5.98795	-5.98034	-1.19759
88							**-5.98443**	**-5.98443**	**-1.19689**	-5.98455	-5.9843	-1.19691
89							**-5.98443**	**-5.98443**	**-1.19689**	-5.98454	-5.98431	-1.19691
90							**-5.98443**	**-5.98443**	**-1.19689**	-5.98453	-5.98432	-1.19691
138										**-5.98443**	**-5.98443**	**-1.19689**
139										**-5.98443**	**-5.98443**	**-1.19689**
140										**-5.98443**	**-5.98443**	**-1.19689**

6. Observation

We observe that Jungck-Agarwal et al. iteration scheme converges faster than other Jungck Iteration schemes and decreasing order of convergence of Jungck iteration schemes is: Jungck-Agarwal et al., Jungck-Noor, Jungck-Mann, Jungck-Ishikawa.

7. References

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