On Intuitionistic Fuzzy Generalized b Closed Sets

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ABSTRACT
The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy generalized b closed sets and intuitionistic fuzzy generalized b open sets in intuitionistic fuzzy topological space. We investigate some of their properties. Further the notion of intuitionistic fuzzy ${}_{gb}T_{1/2}$ spaces and intuitionistic fuzzy ${}_{gb}T_b$ spaces are introduced and studied.

KEYWORDS
Intuitionistic fuzzy topology, intuitionistic fuzzy generalized b closed sets, intuitionistic fuzzy generalized b open sets, intuitionistic fuzzy ${}_{gb}T_{1/2}$ spaces and intuitionistic fuzzy ${}_{gb}T_b$ spaces.

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1. INTRODUCTION
Fuzzy set (FS) as proposed by Zadeh [11] in 1965 is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [3] in 1968, there have been several generalizations of notions of fuzzy set and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In the present paper, we extend the concept of generalized b closed sets due to Benchalli and Jenifer [2] in intuitionistic fuzzy topology and study some of the basic properties regarding it. We also introduce the applications of intuitionistic fuzzy generalized b closed sets namely intuitionistic fuzzy ${}_{gb}T_{1/2}$ spaces, intuitionistic fuzzy ${}_{gb}T_b$ spaces and obtained some characterizations and several preservation theorems of such spaces.

2. PRELIMINARIES
Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A \rangle$ shall be used instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3: [4] An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Result 2.4: [4] Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space (X, τ). Then

(a) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ $cl(A) = A$,

(b) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ $int(A) = A$,

(c) $cl(A^c) = (int(A))^c$,

(d) $int(A^c) = (cl(A))^c$,

(e) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$,

(f) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,

(g) $cl(A \cup B) = cl(A) \cup cl(B)$,

(h) $int(A \cap B) = int(A) \cap int(B)$.

Definition 2.5: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Definition 2.6: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ in an IFTS (X, τ) is said to be

(a) *intuitionistic fuzzy b closed set* [6] (IFbCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$,

(b) *intuitionistic fuzzy α-closed set* [5] (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,

(c) *intuitionistic fuzzy pre-closed set* [5] (IFPCS in short) if $cl(int(A)) \subseteq A$,

(d) *intuitionistic fuzzy regular closed set* [5] (IFRCS in short) if $cl(int(A)) = A$,

(e) *intuitionistic fuzzy semi closed set* [5] (IFSCS in short) if $int(cl(A)) \subseteq A$,

(f) *intuitionistic fuzzy generalized closed set* [10] (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(g) *intuitionistic fuzzy generalized pre closed set* [8] (IFGPCS in short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(h) *intuitionistic fuzzy α generalized closed set* [9] (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(i) *intuitionistic fuzzy weakly generalized closed set* [7] (IFWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called *intuitionistic fuzzy b open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy semi open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized pre open set, intuitionistic fuzzy α generalized open set* and *intuitionistic fuzzy weakly generalized open set* (IFbOS, IFαOS, IFPOS, IFROS, IFSOS, IFGOS, IFGPOS, IFαGOS and IFWGOS) if the complement of A^c is an IFbCS, IFαCS, IFPCS, IFRCS, IFSCS, IFGCS, IFGPCS, IFαGCS and IFWGCS respectively.

3. INTUITIONISTIC FUZZY GENERALI - ZED b CLOSED SETS

In this section, we introduce intuitionistic fuzzy generalized b closed sets in intuitionistic fuzzy topological space and study some of their properties.

Definition 3.1: Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy b closure* of A (bcl(A) in short) and *intuitionistic fuzzy b interior* of A (bint(A) in short) are defined as

$bint(A) = \cup \{ G / G$ is an IFbOS in X and $G \subseteq A\}$,

$bcl (A) = \cap \{ K / K$ is an IFbCS in X and $A \subseteq K \}$.

Theorem 3.2: If A is an IFS in X then $A \subseteq bcl(A) \subseteq cl(A)$.
Proof: The result follows from the definition 3.1.

Theorem 3.3: If A is an IFbCS then $bcl(A) = A$.
Proof: Since A is an IFbCS, bcl(A) is the smallest IFbCS which contains A, which is nothing but A. Hence $bcl(A) = A$.

Theorem 3.4: If A is an IFbOS then $bint(A) = A$.
Proof: Since A is an IFbOS, bint(A) is the largest IFbOS which contains A, which is nothing but A. Hence $bint(A) = A$.

Proposition 3.5: Let (X, τ) be any IFTS. Let A and B be any two intuitionistic fuzzy sets in (X, τ). Then the intuitionistic fuzzy generalized b closure operator satisfies the following properties.

(a) $bcl(0_\sim) = 0_\sim$ and $bcl(1_\sim) = 1_\sim$,

(b) $A \subseteq bcl(A)$,

(c) $bint(A) \subseteq A$,

(d) If A is an IFbCS then $A = bcl(bcl(A))$,

(e) $A \subseteq B \Rightarrow bcl(A) \subseteq bcl(B)$,

(f) $A \subseteq B \Rightarrow bint(A) \subseteq bint(B)$.

Definition 3.6: An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized b closed set* (IFGbCS in short) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

The family of all IFGbCSs of an IFTS (X, τ) is denoted by IFGbC(X).

Example 3.7: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.3), (0.8, 0.7) \rangle$. Then the IFS $A = \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$ is an IFGbCS in X.

Example 3.8: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.6, 0.8), (0.3, 0.2) \rangle$. Then the IFS $A = \langle x, (0.6, 0.3), (0.4, 0.7) \rangle$ is not an IFGbCS in X.

Theorem 3.9: Every IFCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFCS and $bcl(A) \subseteq cl(A)$, $bcl(A) \subseteq cl(A) = A \subseteq U$. Therefore A is an IFGbCS in X.

Example 3.10: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle$. Then the IFS $A = \langle x, (0.5, 0.6), (0.4, 0.3) \rangle$ is an IFGbCS but not an IFCS in X, since $cl(A) = T \neq A$.

Theorem 3.11: Every IFαCS is an IFGbCS but not conversely.
Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFαCS, $\alpha cl(A) = A$. Therefore $bcl(A) \subseteq \alpha cl(A) = A \subseteq U$. Hence A is an IFGbCS in X.

Example 3.12: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.3, 0.2), (0.6, 0.8) \rangle$. Then the IFS $A = \langle x, (0.5, 0.6), (0.3, 0.3) \rangle$ is an IFGbCS but not an IFαCS in X, since $cl(int(cl(A))) = T^c \not\subseteq A$.

Theorem 3.13: Every IFPCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFPCS, $cl(int(A)) \subseteq A$. Therefore $cl(int(A)) \cap int(cl(A)) \subseteq cl(A) \cap cl(int(A)) \subseteq A$. This implies $bcl(A) \subseteq U$. Hence A is an IFGbCS in X.

Example 3.14: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.3), (0.8, 0.7) \rangle$. Then the IFS $A = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$ is an IFGbCS but not an IFPCS in X, since $cl(int(A)) = T^c \not\subseteq A$.

Theorem 3.15: Every IFbCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFbCS, $bcl(A) = A$. Therefore $bcl(A) = A \subseteq U$. Hence A is an IFGbCS in X.

Example 3.16: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.6, 0.8), (0.4, 0.2) \rangle$. Then the IFS $A = \langle x, (0.8, 0.9), (0.2, 0.1) \rangle$ is an IFGbCS but not an IFbCS in X, since $cl(int(A)) \cap int(cl(A)) = 1_\sim \not\subseteq A$.

Theorem 3.17: Every IFRCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFRCS, $cl(int(A)) = A$. This implies $cl(A) = cl(int(A))$. Therefore $cl(A) = A$. Hence A is an IFCS in X. By theorem 3.9, A is an IFGbCS in X.

Example 3.18: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.4), (0.8, 0.6) \rangle$. Then the IFS $A = \langle x, (0.7, 0.5), (0.3, 0.5) \rangle$ is an IFGbCS but not an IFRCS in X, since $cl(int(A)) = T^c \neq A$.

Theorem 3.19: Every IFGCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFGCS, $cl(A) \subseteq U$. Therefore $bcl(A) \subseteq cl(A)$, $bcl(A) \subseteq U$. Hence A is an IFGbCS in X.

Example 3.20: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.4), (0.8, 0.6) \rangle$. Then the IFS $A = \langle x, (0.1, 0.3), (0.8, 0.7) \rangle$ is an IFGbCS but not an IFGCS in X, since $cl(A) = T^c \not\subseteq T$.

Theorem 3.21: Every IFαGCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFαGCS, $\alpha cl(A) \subseteq U$. Therefore $bcl(A) \subseteq \alpha cl(A)$, $bcl(A) \subseteq U$. Hence A is an IFGbCS in X.

Example 3.22: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.5, 0.5), (0.4, 0.5) \rangle$. Then the IFS $A = \langle x, (0.5, 0.3), (0.5, 0.7) \rangle$ is an IFGbCS but not an IFαGCS in X, since $\alpha cl(A) = 1_\sim \not\subseteq T$.

Theorem 3.23: Every IFGPCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFGPCS, $pcl(A) \subseteq U$. Therefore $bcl(A) \subseteq pcl(A)$, $bcl(A) \subseteq U$. Hence A is an IFGbCS in X.

Example 3.24: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ be an IFT on X where $T_1 = \langle x, (0.2, 0.3), (0.8, 0.7) \rangle$ and $T_2 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$. Then the IFS $A = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$ is an IFGbCS but not an IFGPCS in X, since $pcl(A) = T_2^{\,c} \not\subseteq T_2$.

Theorem 3.25: Every IFWGCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFWGCS, $cl(int(A)) \subseteq U$. This implies A is an IFPCS in X. By theorem 3.13, A is an IFGbCS in X.

Example 3.26 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle$. Then the IFS $A = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle$ is an IFGbCS but not an IFWGCS in X, since $cl(int(A)) = T^c \not\subseteq T$.

Theorem 3.27: Every IFSCS is an IFGbCS but not conversely.

Proof: Let $A \subseteq U$ and U is an IFOS in (X, τ). Since A is an IFSCS, $bcl(A) \subseteq scl(A) \subseteq U$. Therefore A is an IFGbCS in X.

Example 3.28: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where $T = \langle x, (0.9, 0.7), (0.1, 0.2) \rangle$. Then the IFS $A = \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$ is an IFGbCS but not an IFSCS in X, since $int(cl(A)) = 1_\sim \not\subseteq A$.

The following implications are true:

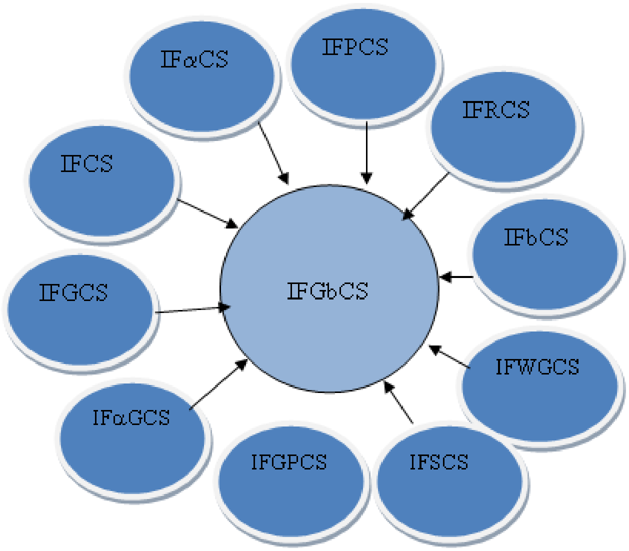

Fig.1 Relation between intuitionistic fuzzy generalized b closed set and other existing intuitionistic fuzzy closed sets.

In this diagram by "$A \longrightarrow B$" we mean A implies B but not conversely.

None of them is reversible.

Remark 3.29: The union of any two IFGbCSs need not be an IFGbCS in general as seen from the following example.

Example 3.30: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, T, 1_\sim\}$be an IFTS on X where $T = \langle x, (0.6, 0.8), (0.4, 0.2) \rangle$. Then the IFSs $A = \langle x, (0.1, 0.8), (0.9, 0.2) \rangle$ and $B = \langle x, (0.6, 0.7), (0.4, 0.3) \rangle$ are IFGbCSs but $A \cup B$ is not an IFGbCS in X, since $bcl(A \cup B) = 1_\sim \not\subseteq T$.

Theorem 3.31: If A is an IFGbCS in (X, τ) such that $A \subseteq B \subseteq bcl(A)$ then B is an IFGbCS in (X, τ).

Proof: Let B be an IFS in an IFTS (X, τ) such that $B \subseteq U$ and U is an IFOS in X. This implies $A \subseteq U$. Since A is an IFGbCS, $bcl(A) \subseteq U$. By hypothesis, we have $bcl(B) \subseteq bcl(bcl(A)) = bcl(A) \subseteq U$. Hence B is an IFGbCS in X.

Theorem 3.32: If A is intuitionistic fuzzy b open and intuitionistic fuzzy generalized b closed in an IFTS (X, τ) then A is intuitionistic fuzzy b closed in (X, τ).

Proof: Since A is intuitionistic fuzzy b open and intuitionistic fuzzy generalized b closed in (X, τ), $bcl(A) \subseteq A$. But $A \subseteq bcl(A)$. Thus $bcl(A) = A$ and hence A is intuitionistic fuzzy b closed in (X, τ).

4. INTUITIONISTIC FUZZY GENERALI - ZED b OPEN SETS

In this section, we introduce intuitionistic fuzzy generalized b open sets in intuitionistic fuzzy topological space and study some of their properties.

Definition 4.1: An IFS A is said to be an *intuitionistic fuzzy generalized b open set* (IFGbOS in short) in (X, τ) if the complement A^c is an IFGbCS in X.

The family of all IFGbOSs of an IFTS (X, τ) is denoted by IFGbO(X).

Example 4.2: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where T = ⟨ x, (0.3, 0.4), (0.7, 0.6) ⟩. Then the IFS A = ⟨ x, (0.4, 0.5), (0.6, 0.5) ⟩ is an IFGbOS in X.

Theorem 4.3: For any IFTS (X, τ), we have the following:

(a) Every IFOS is an IFGbOS.

(b) Every IFbOS is an IFGbOS.

(c) Every IFαOS is an IFGbOS.

(d) Every IFGOS is an IFGbOS.

(e) Every IFGPOS is an IFGbOS.

Proof: Straight forward.

The converse of the above statements need not be true in general as seen from the following examples.

Example 4.4: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFTS on X where T = ⟨ x, (0.4, 0.3), (0.5, 0.4) ⟩. Then IFS A = ⟨ x, (0.4, 0.4), (0.6, 0.5) ⟩ is an IFGbOS but not an IFOS in X.

Example 4.5: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFTS on X where T = ⟨ x, (0.6, 0.8), (0.4, 0.2) ⟩. Then IFS A = ⟨ x, (0.2, 0.1), (0.8, 0.9) ⟩ is an IFGbOS but not an IFbOS in X.

Example 4.6: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where T = ⟨ x, (0.3, 0.2), (0.6, 0.8) ⟩. Then the IFS A = ⟨ x, (0.3, 0.3), (0.5, 0.6) ⟩ is an IFGbOS but not an IFαOS in X.

Example 4.7: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where T = ⟨ x, (0.2, 0.4), (0.7, 0.6) ⟩. Then the IFS A = ⟨ x, (0.8, 0.7), (0, 0.3) ⟩ is an IFGbOS but not an IFGOS in X.

Example 4.8: Let X = {a, b} and let $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ be an IFT on X where T_1 = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩ and T_2 =⟨ x, (0.4, 0.5), (0.6, 0.5)⟩. Then the IFS A = ⟨ x, (0.6, 0.5), (0.4, 0.5) ⟩ is an IFGbOS but not an IFGPOS in X.

Remark 4.9: The intersection of any two IFGbOSs need not be an IFGbOS in general.

Example 4.10: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFT on X where T = ⟨ x, (0.6, 0.8), (0.3, 0.2) ⟩. The IFSs A= ⟨ x, (0.6, 0.9), (0.3, 0.1) ⟩ and B = ⟨ x, (0.7, 0.8), (0.2, 0.2) ⟩ are IFGbOSs but $A \cap B$ is not an IFGbOS in X.

Theorem 4.11: An IFS A of an IFTS (X, τ) is an IFGbOS if and only if $F \subseteq$ bint(A) whenever F is an IFCS and $F \subseteq A$.

Proof: Necessity: Suppose A is an IFGbOS in X. Let F be an IFCS and $F \subseteq A$. Then F^c is an IFOS in X such that $A^c \subseteq F^c$. Since A^c is an IFGbCS, $bcl(A^c) \subseteq F^c$. Hence $(bint(A))^c \subseteq F^c$. This implies $F \subseteq bint(A)$.

Sufficiency: Let A be any IFS of X and let $F \subseteq bint(A)$ whenever F is an IFCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFOS. By hypothesis, $(bint(A))^c \subseteq F^c$. Hence $bcl(A^c) \subseteq F^c$. Hence A is an IFGbOS in X.

Theorem 4.12: If A is an IFGbOS in (X, τ) such that $bint(A) \subseteq B \subseteq A$ then B is an IFGbOS in (X, τ).

Proof: By hypothesis, we have $bint(A) \subseteq B \subseteq A$. This implies $A^c \subseteq B^c \subseteq (bint(A))^c$. That is, $A^c \subseteq B^c \subseteq bcl(A^c)$. Since A^c is an IFGbCS, by theorem 3.31, B^c is an IFGbCS. Hence B is an IFGbOS in X.

5. APPLICATIONS OF INTUITIONISTIC FUZZY GENERALIZED b CLOSED SETS

In this section, we introduce intuitionistic fuzzy ${}_bT_{1/2}$ spaces, intuitionistic fuzzy ${}_{gb}T_{1/2}$ spaces and intuitionistic fuzzy ${}_{gb}T_b$ spaces in intuitionistic fuzzy topological space and study some of their properties.

Definition 5.1: An IFTS (X, τ) is called an *intuitionistic fuzzy ${}_bT_{1/2}$ space* ($IF_bT_{1/2}$ space in short) if every IFbCS in X is an IFCS in X.

Definition 5.2: An IFTS (X, τ) is called an *intuitionistic fuzzy ${}_{gb}T_{1/2}$ space* ($IF_{gb}T_{1/2}$ space in short) if every IFGbCS in X is an IFCS in X.

Definition 5.3: An IFTS (X, τ) is called an *intuitionistic fuzzy ${}_{gb}T_b$ space* ($IF_{gb}T_b$ space in short) if every IFGbCS in X is an IFbCS in X.

Theorem 5.4: Every $IF_{gb}T_{1/2}$ space is an $IF_{gb}T_b$ space.

Proof: Let (X, τ) be an $IF_{gb}T_{1/2}$ space and let A be an IFGbCS in X. By hypothesis, A is an IFCS in X. Since every IFCS is an IFbCS, A is an IFbCS in X. Hence (X, τ) is an $IF_{gb}T_b$ space.

The converse of the above theorem need not be true in general as seen from the following example.

Example 5.5: Let X = {a, b} and let $\tau = \{0_\sim, T, 1_\sim\}$ be an IFTS on X where T = ⟨ x, (0.9, 0.9), (0.1, 0.1) ⟩. Then (X, τ) is an $IF_{gb}T_b$ space but not an $IF_{gb}T_{1/2}$ space, since the IFS A = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩ is an IFGbCS but not IFCS in X.

Theorem 5.6: Let (X, τ) be an IFTS and (X, τ) an $IF_{gb}T_{1/2}$ space. Then the following statements hold.

(i) Any union of IFGbCS is an IFGbCS.

(ii) Any intersection of IFGbOS is an IFGbOS.

Proof:

(i): Let $\{A_i\}_{i \in J}$ be a collection of IFGbCS in an $IF_{gb}T_{1/2}$ space (X, τ). Therefore every IFGbCS is an IFCS. But the union of IFCS is an IFCS. Hence the union of IFGbCS is an IFGbCS in X.

(ii): It can be proved by taking complement in (i).

Theorem 5.7: An IFTS (X, τ) is an $IF_{gb}T_b$ space if and only if IFGbO(X) = IFbO(X).

Proof:

Necessity: Let A be an IFGbOS in X. Then A^c is an IFGbCS in X. By hypothesis, A^c is an IFbCS in X. Therefore A is an IFbOS in X. Hence IFGbO(X) = IFbO(X).

Sufficiency: Let A be an IFGbCS in X. Then A^c is an IFGbOS in X. By hypothesis, A^c is an IFbOS in X. Therefore A is an IFbCS in X. Hence (X, τ) is an $IF_{gb}T_b$ space.

Theorem 5.8: An IFTS (X, τ) is an $IF_{gb}T_{1/2}$ space if and only if IFGbO(X) = IFO(X).

Proof:

Necessity: Let A be an IFGbOS in X. Then A^c is an IFGbCS in X. By hypothesis, A^c is an IFCS in X. Therefore A is an IFOS in X. Hence IFGbO(X) = IFO(X).

Sufficiency: Let A be an IFGbCS in X. Then A^c is an IFGbOS in X. By hypothesis, A^c is an IFOS in X. Therefore A is an IFCS in X. Hence (X, τ) is an $IF_{gb}T_{1/2}$ space.

6. CONCLUSION

The theory of g-closed sets plays an important role in the general topology. Since its inception many weak forms of g-closed sets have been introduced in general topology as well as in fuzzy topology and intuitionistic fuzzy topology. The present paper investigated in new weak form of intuitionistic fuzzy g closed sets namely intuitionistic fuzzy generalized b closed sets which contain the classes of intuitionistic fuzzy g closed sets, intuitionistic fuzzy α generalized closed sets and intuitionistic fuzzy weakly generalized closed sets. Several properties and applications of intuitionistic fuzzy generalized b closed sets are studied. Many examples are given to justify the results.

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