Two Server (s, S) Inventory System with Positive Service Time, Positive Lead Time, Retrial Customers and Negative Arrivals

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ABSTRACT
This paper deals with a two server (s,S) inventory system with positive service time, positive lead time, retrial of customers and negative arrivals. In this system, arrival of customers form a Poisson process, lead time and service time are exponentially distributed. The system starts with S units of inventory on hand. Each arriving customer is served a single unit of the item by any one of the servers. When the inventory level reaches s, an order is placed for (S-s) units. If the inventory level is zero or both servers are busy, then the arriving customer goes to orbit and becomes a source of repeated calls. Assume that the capacity of the orbit is infinite. The negative arrival plays an important role in this paper and it controls the congestion in the orbit by removing one customer from the orbit and further it is assumed that it removes the customer from the orbit only if inventory level is zero or both servers are busy. It is also assumed that the access from orbit to the service facility is governed by the classical retrial policy. This model is solved by using Direct Truncation Method. Numerical and graphical studies have been done for analysis of mean number of customers in the orbit, average inventory level, Truncation level, mean number of busy servers and system performance measures. A suitable cost function is defined.

General Terms
Stochastic Process, Queueing theory, Retrial queues, Inventory system, Negative arrivals.

Keywords
(s,S) inventory system, positive service time, Positive lead time, retrial customers, LDQBD Process, negative arrivals

1. INTRODUCTION
Many researchers have studied inventory systems for decades but only recently their attention has turned towards retrial customers in inventory system. These models consider situations in which the arriving customers who find all servers busy or the inventory level zero may retry for service after sometime. A .Krishnamoorthy and Islam [9] have studied inventory system with postponed demands. J. R. Artalejo and A. Krishnamoorthy [8] have studied (s,S) inventory systems with repeated attempts and P.V. Ushakumari [13] considering random lead time. A .Krishnamoorthy and K.P.Jose [10] have studied the system with positive lead time, loss and retrial of customers. Gelenbe [5], Harrison and Pitel [6], Artalejo and Gomez-Corral [2] who have studied the negative arrivals which removes a customer or a batch of customers from the system according to some strategy.

This paper is organized as follows. Section 2 gives the model description and analysis while section 3 highlights the direct truncation method used. Section 4 discusses the stability condition and the performance measures and section 5 provides the cost analysis and numerical results followed by conclusion as section 6.

2. MODEL DESCRIPTION
In this paper, a (s,S) inventory retrial queueing system in which arrival follows a Poisson process with parameter λ is considered. Lead time and service time are exponentially distributed with parameter ν and μ respectively. The negative arrival rate follows a Poisson distribution with parameter λ_{-1}. The system starts with S units of inventory on hand. Each arriving customer is served a single unit of the item by any one of the servers. When the inventory level reaches s, an order is placed for (S-s) units. If the inventory level is zero or both servers are busy, then the arriving customer goes to orbit and becomes a source of repeated calls. Assume that the capacity of the orbit is infinite. The negative arrival plays an important role in this paper and it controls the congestion in the orbit by removing one customer from the orbit and further it is assumed that it removes the customer from the orbit only if inventory level is zero or both servers are busy.

Most of the queueing system with repeated attempts assume that each customer in the retrial group seeks service independently of each other after a random time exponentially distributed with rate σ so that the probability of repeated attempt during the interval $(t, t+\Delta t)$ given that there were n customers in orbit at time t is $n\sigma\,\Delta t + O(\Delta t)$. This discipline of access to the server from the retrial group is called classical retrial policy. The input flow of primary calls, interval between repeated calls, negative arrival and service time are mutually independent.

Let N(t) be the random variable which represents the number of customers in orbit at time t , B(t) be the number of busy servers at time t and I(t) be the random variable which represents the inventory level at time t.

The random process is described as

$\{< N(t), B(t), I(t) > / N(t) = 0, 1, 2, 3, \ldots ; B(t) = 0, 1, 2 ;$

$I(t) = 0, 1, 2, 3, \ldots s, s+1, \ldots S\}$

The possible state space is

$\{(i, 0, j), i \geq 0, 0 \leq j \leq S\} \cup \{(i, 1, j), i \geq 0, 1 \leq j \leq S\} \cup$

$\{(i, 2, j), i \geq 0, 2 \leq j \leq S\}$

The infinitesimal generator matrix **Q** is a block tri diagonal matrix given below

$$
\begin{pmatrix}
A_{10} & A_0 & O & O & O & \dots \\
A_{10} & A_{11} & A_0 & O & O & \dots \\
O & A_{21} & A_{22} & A_0 & O & \dots \\
O & O & A_{32} & A_{33} & A_0 & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots
\end{pmatrix}
$$

Where the blocks A_0, $A_{1,i}$ $(i \geq 0)$, and $A_{2,i}$ $(i \geq 1)$, are square matrices of order 3S

they are given by

$$A_0 = \begin{pmatrix} B_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_1 \end{pmatrix}$$

$$A_{1,i} = \begin{pmatrix} E_0 & E_1 & 0 \\ E_2 & E_3 & E_4 \\ 0 & E_5 & E_6 \end{pmatrix} \qquad A_{2,i} = \begin{pmatrix} C_0 & C_1 & 0 \\ 0 & 0 & C_2 \\ 0 & 0 & C_3 \end{pmatrix}$$

Where

$$B_0 = \begin{pmatrix} \lambda & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{(S+1)\times(S+1)}$$

$$B_1 = \begin{pmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda \end{pmatrix}_{(S-1)\times(S-1)}$$

$$E_0 = \begin{matrix} 0 \\ 1 \\ \vdots \\ s \\ s+1 \\ \vdots \\ S-s \\ S-s+1 \\ \vdots \\ S-1 \\ S \end{matrix}
\begin{pmatrix}
H_1 & 0 & \cdots & 0 & 0 & \cdots & \nu & 0 & \cdots & 0 & 0 \\
0 & H_6 & \cdots & 0 & 0 & \cdots & 0 & \nu & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & H_6 & 0 & \cdots & 0 & 0 & \cdots & 0 & \nu \\
0 & 0 & \cdots & 0 & H_2 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & H_2 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_2 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & H_2 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_2
\end{pmatrix}_{(S+1)\times(S+1)}$$

$$E_1 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda \end{pmatrix}_{(S+1)\times S}$$

$$E_2 = \begin{pmatrix} \mu & 0 & \cdots & 0 \\ 0 & \mu & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \mu \end{pmatrix}_{S\times(S+1)}$$

$$E_3 = \begin{matrix} 1 \\ 2 \\ \vdots \\ s \\ s+1 \\ \vdots \\ S-s \\ S-s+1 \\ \vdots \\ S-1 \\ S \end{matrix}
\begin{pmatrix}
H_9 & 0 & \cdots & 0 & 0 & \cdots & \nu & 0 & \cdots & 0 & 0 \\
0 & H_9 & \cdots & 0 & 0 & \cdots & 0 & \nu & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & H_9 & 0 & \cdots & 0 & 0 & \cdots & 0 & \nu \\
0 & 0 & \cdots & 0 & H_7 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & H_7 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_7 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & H_7 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_7
\end{pmatrix}_{S\times S}$$

$$E_4 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda \end{pmatrix}_{S\times(S-1)}$$

$$E_6 = \begin{matrix} 2 \\ 3 \\ \vdots \\ s \\ s+1 \\ \vdots \\ S-s \\ S-s+1 \\ \vdots \\ S-1 \\ S \end{matrix}
\begin{pmatrix}
H_{10} & 0 & \cdots & 0 & 0 & \cdots & 0 & \nu & \cdots & 0 & 0 \\
0 & H_{10} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & H_{10} & 0 & \cdots & 0 & 0 & \cdots & 0 & \nu \\
0 & 0 & \cdots & 0 & H_8 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & H_8 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_8 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & H_8 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & H_8
\end{pmatrix}_{(S-1)\times(S-1)}$$

$$C_0 = \begin{pmatrix} \lambda_{-1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{(S+1)\times(S+1)}$$

$$C_1 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ i\sigma & 0 & \cdots & 0 \\ 0 & i\sigma & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & i\sigma \end{pmatrix}_{(S+1)\times S}$$

$$C_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ i\sigma & 0 & \cdots & 0 \\ 0 & i\sigma & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & i\sigma \end{pmatrix}_{S\times(S-1)}$$

$$C_3 = \begin{pmatrix} \lambda_{-1} & 0 & \cdots & 0 \\ 0 & \lambda_{-1} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda_{-1} \end{pmatrix}_{(S-1)\times(S-1)}$$

3. DESCRIPTION OF COMPUTATIONAL METHOD

Retrial queueing models can be solved computationally by the various techniques. One of the feasible techniques is Direct Truncation Method. This method is applied in this paper for computational purpose for finding the Steady state probability vector and described as below

DIRECT TRUNCATION METHOD

Let **X** be the steady-state probability vector of **Q,** partitioned as **X** = (**x(0), x(1), x(2), . . .**) where **X** satisfies

$$\mathbf{XQ} = \mathbf{0} \quad \text{and} \quad \mathbf{Xe} = \mathbf{1}$$

where **x(i)** = $(y_{i,0,0}, y_{i,0,1}, \ldots, y_{i,0,S}, y_{i,1,1}, y_{i,1,2}, \ldots, y_{i,1,S}, y_{i,2,2}, y_{i,2,3}, \ldots, y_{i,2,S})$; i = 0, 1, 2, **. . .**

The above system of equations can be solved by means of truncating the system of equations for sufficiently large value of the number of customers in the orbit, say M. That is, the orbit size is restricted to M such that any arriving customer finding the orbit full is considered lost. The value of M can be chosen so that the loss probability is small. Due to the intrinsic nature of the system, the only choice available for studying M is through algorithmic methods. While a number of approaches are available for determining the cut-off point, M, the one that seems to perform well is to increase M until the largest individual change in the elements of **X** for successive values is less than ε a predetermined infinitesimal value.

If M denotes the cut-off point or Truncation level, then the steady state probability vector $\mathbf{X}^{(M)}$ is partitioned as $\mathbf{X}^{(M)}$ = **(x(0) , x(1), x(2) , . . . , x(M)),** where $\mathbf{X}^{(M)}$ satisfies

$$\mathbf{X}^{(M)}\mathbf{Q}^{(M)} = \mathbf{0} \quad \text{and} \quad \mathbf{X}^{(M)}\mathbf{e} = 1,$$

where x(i) = $(y_{i,0,0}, y_{i,0,1}, \ldots, y_{i,0,S}, y_{i,1,1}, y_{i,1,2}, \ldots, y_{i,1,S}, y_{i,2,2}, y_{i,2,3}, \ldots, y_{i,2,S})$; i = 0, 1, 2, **. . .** , M.

$$Q = \begin{pmatrix} A_{1,0} & A_0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ A_{2,1} & A_{1,1} & A_0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & A_{2,2} & A_{1,2} & A_0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & A_{2,M} & A_{1,M} \end{pmatrix}$$

The above system of equations is solved exploiting the special structure of the co-efficient matrix. It is solved using Numerical methods. Since there is no clear cut choice for M, the iterative process may be started by taking, say M = 1 and increase it until the individual elements of **X** do not change significantly. That is, if M^* denotes the truncation point then

$$\| \mathbf{x}^{M^*}(\mathbf{i}) - \mathbf{x}^{M^*-1}(\mathbf{i}) \|_\infty < \boldsymbol{\varepsilon}$$

where **ε** is an infinitesimal quantity.

4. STABILITY ANALYSIS

Let π denote the steady state probability vector of the generator $A=A_0+A_1+A_2$. The vector π partitioned as $\pi = (\pi_0, \pi_1, \ldots \pi_s, \ldots. \pi_{3S-1})$ can be got by solving $\pi A = 0$ and $\pi e = 1$. The steady state probability vector X exists if and only if the condition

$\pi A_0 e < \pi A_2 e$ --------------------------------(4.1)

holds good. This reduces to

$$\lambda[\pi_0 + \sum_{j=2S+1}^{3S-1} \pi_j] < M\sigma \left[\sum_{j=1}^{S} \pi_j + \sum_{j=S+2}^{2S} \pi_j \right] + \lambda_{-1} [\pi_0 + \sum_{j=2S+1}^{3S-1} \pi_j]$$

which gives the required stability condition for this model.

SYSTEM PERFORMANCE MEASURES

The expected inventory level is given by

$$EIL = \sum_{i=0}^{\infty}\sum_{j=0}^{S} j\, y_{i,0,j} + \sum_{i=0}^{\infty}\sum_{j=1}^{S} j\, y_{i,1,j} + \sum_{i=0}^{\infty}\sum_{j=2}^{S} j\, y_{i,2,j}$$

The expected number of customers in the orbit is given by

$$ENCO = \sum_{i=0}^{\infty} i \left(\sum_{j=0}^{S} y_{i,0,j} + \sum_{j=1}^{S} y_{i,1,j} + \sum_{j=2}^{S} y_{i,2,j} \right)$$

The expected reorder rate is given by

$$EROR = \mu \sum_{i=0}^{\infty} y_{i,1,s+1} + 2\mu \sum_{j=2}^{\infty} y_{i,2,j}$$

The expected number of departures after completing service (ENDS) is given by

$$ENDS = \mu \sum_{I=0}^{\infty}\sum_{j=1}^{S} y_{i,1,j} + 2\mu \sum_{I=0}^{\infty}\sum_{j=1}^{S} y_{i,2,j}$$

5. COST ANALYSIS

Different costs are defined as

K = fixed cost,
C_1 = procurement cost/unit,
C_2 = holding cost of inventory/unit /unit time,
C_3 = holding cost of customers/unit /unit time,
C_4 = cost due to service/unit /unit time,
C_5 = revenue from service/unit/unit time.

A cost function defined as the expected total cost (ETC) of the system is introduced, which is given by

$$ETC = [K + (S-s)C_1]EROR + C_2 EIL + C_3 ENCO + (C_5 - C_4) ENDS$$

Numerical Examples

This section gives the numerical results of this model for various values of the parameters λ, λ_{-1}, μ, ν, σ, s and S chosen in such a way that they satisfy the stability conditions given in section 4.

System performance measures like Expected Number of Customers in the Orbit (ENCO) and Expected Inventory Level (EIL) are found using the steady state probability vector X for different values of ν and σ fixing λ, μ, s and S and some are tabulated below.

Table 5.1 Variations in Retrial rate for
λ=10 μ=20 ν=10 s=3 S=6 λ_{-1}=6

σ	M	ENCO	EIL
10	17	0.2013	3.9616
20	17	0.1457	3.9544
30	17	0.1264	3.951
40	17	0.1164	3.949
50	17	0.1104	3.9477
60	17	0.1063	3.9467
70	17	0.1033	3.946
80	17	0.1011	3.9455
90	17	0.0993	3.9451
100	17	0.0979	3.9447
200	17	0.0915	3.9431
300	17	0.0893	3.9425
400	17	0.0882	3.9423
500	17	0.0875	3.9421
600	17	0.0871	3.942
700	17	0.0868	3.9419
800	17	0.0865	3.9418
900	17	0.0863	3.9418
1000	17	0.0862	3.9417
2000	17	0.0855	3.9416
3000	17	0.0853	3.9415
4000	17	0.0852	3.9415
5000	17	0.0851	3.9415
6000	17	0.0851	3.9414
7000	17	0.085	3.9414
8000	17	0.085	3.9414
9000	17	0.085	3.9414

Table 5.2 Variations in Retrial rate for
λ=10 μ=20 ν=20 s=3 S=6 λ_{-1}=6

σ	M	ENCO	EIL
10	11	0.1189	4.5003
20	11	0.0778	4.4985
30	11	0.0635	4.4976
40	11	0.0562	4.497
50	11	0.0517	4.4967
60	11	0.0487	4.4964
70	11	0.0466	4.4962
80	11	0.0449	4.4961
90	11	0.0436	4.4959
100	11	0.0426	4.4958
200	11	0.0379	4.4954
300	11	0.0362	4.4952
400	11	0.0354	4.4951
500	11	0.0349	4.4951
600	11	0.0346	4.495
700	11	0.0344	4.495
800	11	0.0342	4.495
900	11	0.0341	4.495
1000	11	0.0339	4.495
2000	11	0.0334	4.4949
3000	11	0.0333	4.4949
4000	11	0.0332	4.4949
5000	11	0.0331	4.4949
6000	11	0.0331	4.4949
7000	11	0.0331	4.4949
8000	11	0.0331	4.4949
9000	11	0.033	4.4949

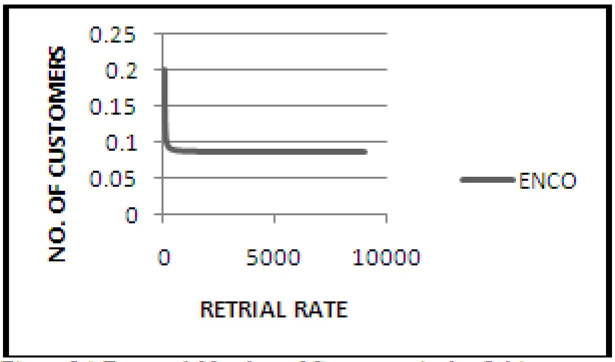

Figure 5.1 Expected Number of Customers in the Orbit
When λ=10 μ=20 ν=10 s=3 S=6 λ_{-1}=6

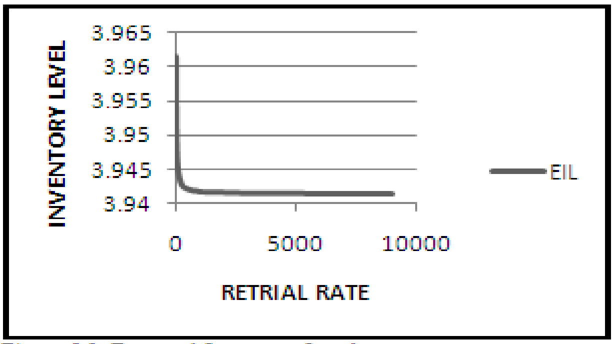

Figure 5.2 Expected Inventory Level
When λ=10 μ=20 ν=10 s=3 S=6 λ_{-1}=6

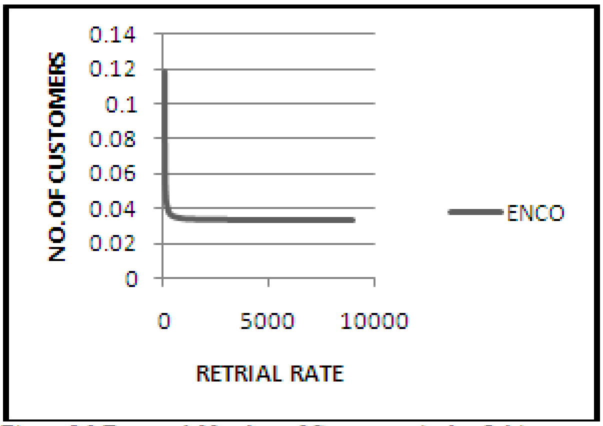

Figure 5.3 Expected Number of Customers in the Orbit
When λ=10 μ=20 ν=20 s=3 S=6 λ_{-1}=6

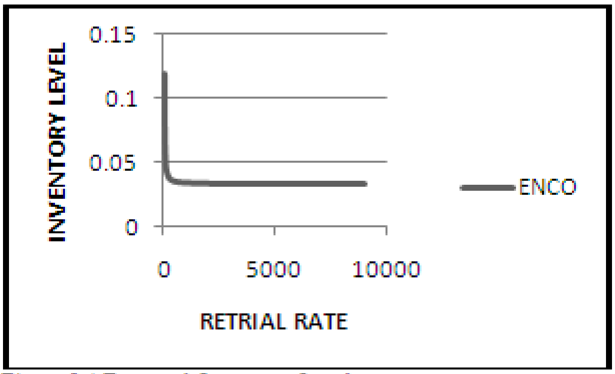

Figure 5.4 Expected Inventory Level
When λ=10 μ=20 ν=20 s=3 S=6 λ_{-1}=6

Table 5.3 Variations in the Negative arrival Rate λ=10 μ=20 σ=10 s=3 S=6 γ = 10

λ_{-1}	M	ENCO	EIL
2	21	0.2669	3.9345
4	19	0.2299	3.9494
6	11	0.1189	4.5003

Table 5.4 Variations in the Negative arrival Rate λ=10 μ=20 σ=9000 s=3 S=6 γ = 10

λ_{-1}	M	ENCO	EIL
2	21	0.1183	3.9129
4	19	0.0992	3.9286
6	17	0.0850	3.9414

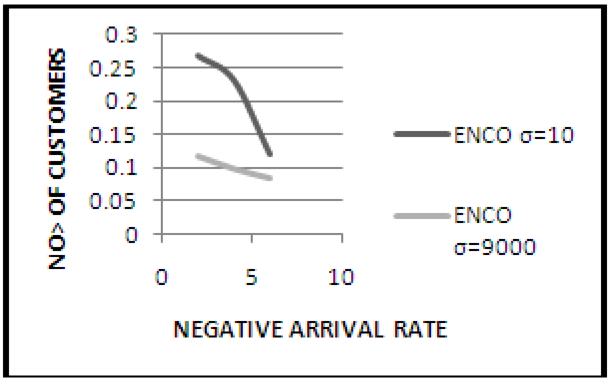

Figure 5.5 Expected Number of Customers in the Orbit When λ=10 μ=20 s=3 S=6 γ = 10

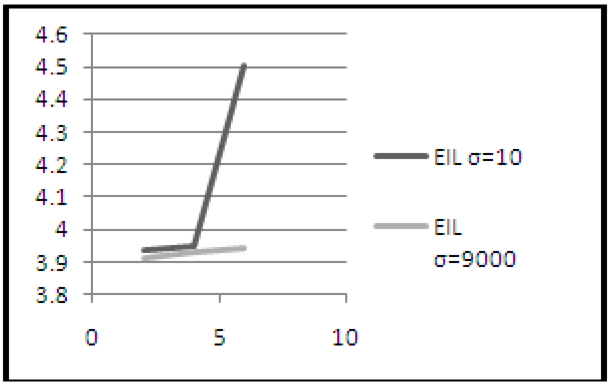

Figure 5.6 Expected Inventory Level When λ=10 μ=20 s=3 S=6 γ = 10

6. CONCLUSIÓN

In this paper a (s,S) inventory system with positive service time, lead time , retrial of customers and negative arrivals using matrix analytic method is studied and the following observations have been made.

From Tables 5.1, 5.2 and the given figures 5.1, 5.2, 5.3 and 5.4 it is seen that

- the ENCO decreases as the retrial rate increases
- the EIL decreases as the retrial rate increases

From Tables 5.3, 5.4 and the given figures 5.5, and 5.6 it is seen that

- the ENCO decreases as the negative arrival rate increases
- the EIL increases as the negative arrival rate increases

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