MIMO Radar Ambiguity Optimization using Phase Coded Pulse Waveforms

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ABSTRACT
Multiple input multiple output radars transmit partially coherent or noncoherent waveforms for improving spatial resolution or spatial diversity. While waveform design for SIMO radars concentrate on improving the delay and Doppler resolution characteristics, waveform design for MIMO radars involve optimization of delay, Doppler and spatial resolution characteristics. The resolution properties of MIMO radar transmit waveforms is studied using the MIMO radar ambiguity function. MIMO radar ambiguity function of frequency hopping waveforms is recently derived. Considering the many advantages of phase coded pulse waveforms in radar applications, this paper derives the MIMO radar ambiguity function of phased coded pulse waveforms. Further, a numerical optimization algorithm based on simulated annealing is proposed for designing parameters of the phase coded pulse waveforms that minimize the peak of the ambiguity function at all mismatched values of delay, Doppler and angular dimensions.

General Terms
Signal Processing, Radar Signal Processing, Simulated Annealing, Evolutionary Algorithms.

Keywords
MIMO radar; ambiguity function; pulse coded waveforms; spatial resolution

1. INTRODUCTION
Radar systems transmit electromagnetic energy into free space and use the reflected energy from the objects to detect the range, velocity of the desired objects. Modern radar systems employ multiple transmit antenna elements on transmit and multiple receive antenna elements on receive. Multiple transmit antenna elements allow directive radiation of transmit power and multiple receive antenna elements allow directive reception of reflected echoes. These systems further allow angular parameters of the target to be determined. Conventional phased array radars transmit fully coherent waveforms (possibly scaled by a complex constant) from their M different transmit antenna elements forming a strong transmit beam in the desired direction. Beamforming is performed only by the receive array (containing N antenna elements) to estimate the angular parameters of the target. Thus the transmit degrees of freedom are limited to one and receive degrees of freedom are N. However multiple input multiple output (MIMO) radars transmit diverse waveforms from their different transmit antenna elements and use joint processing of the received signals from the different receive array elements. While phased array radars employ only spatial diversity, MIMO radars employ both spatial and waveform diversity to improve many aspects of system performance. MIMO radars can employ widely spaced antennas [1] or collocated antennas [2]. While the former configuration offers improved spatial diversity to improve target detection capabilities the latter configuration improves the spatial resolution, parameter identifiability and interference rejection capability. This paper deals with collocated MIMO radar configuration only.

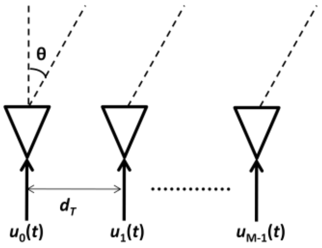
Several waveforms [6] have been designed for single input single output (SISO) radars with wide range of delay and Doppler resolution characteristics. Design of radar waveforms with an information theoretic perspective is proposed in [3]. The choice of waveforms for MIMO radars affect the range, Doppler and spatial resolution characteristics. While waveforms for SISO radar are designed for desired delay and Doppler resolution characteristics, waveforms chosen for MIMO radars should have desirable ambiguity properties in the delay, Doppler and spatial dimensions. The resolution properties of the transmit waveforms is studied using the concept of ambiguity function [6]. The ambiguity function of a transmit waveform represents the output of the matched filter in the presence of delay and Doppler mismatch. For high delay and Doppler resolution, the ambiguity function should resemble a "thumbtack" function around zero delay and Doppler mismatch. The concept of ambiguity function is extended to the MIMO system case by Fuhrmann [4]. Chen and Vaidyanathan [5] have derived the properties of MIMO ambiguity function and further extended the MIMO ambiguity function for frequency hopping waveforms.

Phase coded pulse waveforms are good candidates for radar waveforms since they offer the high time-bandwidth product, have constant modulus and can be easily generated and processed. Several phase coded waveforms have been proposed previously [7]-[10] with good autocorrelation and crosscorrelation properties. These waveforms are designed for good delay and Doppler resolution characteristics but spatial resolution characteristics are not considered. However designing the phase coded waveforms by optimizing the MIMO ambiguity function improves the delay, Doppler and also spatial resolution characteristics.

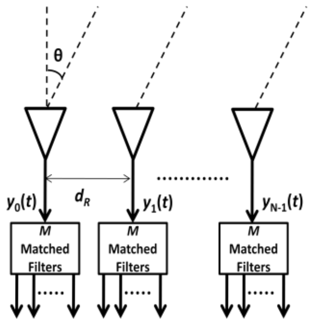
In this paper, ambiguity function of MIMO radar employing phase coded pulse waveforms is derived. The MIMO radar ambiguity function is expressed in terms of the parameters of the phase coded pulse waveforms. These parameters are further optimized using a simulated annealing algorithm that provides good resolution in the delay, Doppler and spatial dimensions. Section 2 describes MIMO radar signal model and structure of phase coded waveforms. Section 3 derives the MIMO radar ambiguity function employing phase coded pulse waveforms. Section 4 presents the design of parameters of phase coded pulse waveforms using the simulated annealing algorithm. Section 5 presents the numerical results and Section 6 concludes the paper.

2. MIMO RADAR SYSTEM MODEL
Consider a monostatic MIMO radar that contains M transmitters and N receivers with their antennas configured

as uniform linear arrays (ULAs). A point target is assumed and also that the target, transmitters and receivers lie in the same 2-D plane (see Fig. 1). Let d_T and d_R represent the spacing between consecutive transmitters and receivers respectively, and let $\gamma = d_T / d_R$. Define the spatial frequency of the target as $f = d_R \sin(\theta)/\lambda$, where θ is the target angle with respect to the broadside direction and λ is the carrier wavelength of the transmitted waveforms.



a) Transmitter Model



b) Receiver Model

Figure 1: Transmitter and Receiver Models

Let τ and v be the target delay and Doppler frequency respectively. Let $\{u_m(t)\},\ m \in \{0, 1, \dots, M-1\}$ represent the M transmitter waveforms. Then the waveform received at the n^{th} receiver antenna can be expressed as [5]

$$y_n^{\tau,\ v,\ f}(t) \approx \sum_{m=0}^{M-1} u_m(t-\tau) e^{j2\pi v t} e^{j2\pi f(\gamma m + n)} \qquad (1)$$

for $n = 0, 1, \dots, N-1$.

2.1 Phase Coded Pulse Waveforms

The phase coded pulse waveform emitted by the m^{th}transmitter can be represented as

$$u_m(t) = \sum_{l=0}^{L-1} \phi_m(t - T_l), \qquad (2)$$

where

$$\phi_m(t) = \frac{1}{\sqrt{T_p}} \sum_{q=0}^{Q-1} c_{m,q}\, s\left(\frac{t - q\Delta t}{\Delta t}\right) \qquad (3)$$

$$s(t) = \begin{cases} 1 & \text{if } 0 < t < 1 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Here L represents the number of pulses emitted by each transmitter. Here, $c_{m,q}$ is the $(m,q)^{\text{th}}$ element of the code matrix $[\mathbf{C}]_{M \times Q}$ and it can assume a value from the set $\left\{e^{j\frac{2\pi}{K}0}, e^{j\frac{2\pi}{K}1}, \dots, e^{j\frac{2\pi}{K}(K-1)}\right\}$. $T_p = Q\Delta t$ is the duration of each pulse and Δt is the duration of each subpulse. K is the phase number and represents the number of phases allowed by each polyphase waveform. Each row of the code matrix $\mathbf{C}$ represents the phase code associated with each transmitted waveform. Each column of the code matrix corresponds to the phase code transmitted by each of the M transmitters during the q^{th} subpulse. As shown in Fig. 2, each transmitter waveform $u_m(t)$ consists of a stream of L identical pulses $\phi_m(t)$. Each pulse in turn contains Q phase coded subpulses each having width Δt. For each of the transmitter waveforms $u_m(t)$to be orthogonal (at zero Doppler and zero delay mismatch) i.e.,

$$\int_{-\infty}^{\infty} u_m(t) u_{m'}^*(t)\, dt = 0, \qquad \forall m \neq m' \qquad (5)$$

we require

$$\mathbf{C}\mathbf{C}^T = \mathrm{I}_{M \times M} \qquad (6)$$

Orthogonal waveforms result in uniform illumination in all directions. For fixed Δt, these waveforms can be completely described by the code matrix $\mathbf{C} = [c_{m,q}]_{M \times Q}$and the pulse spacings ($T_0$, T_1, ... T_{L-1}). The pulse spacings affect the Doppler resolution of the waveforms.

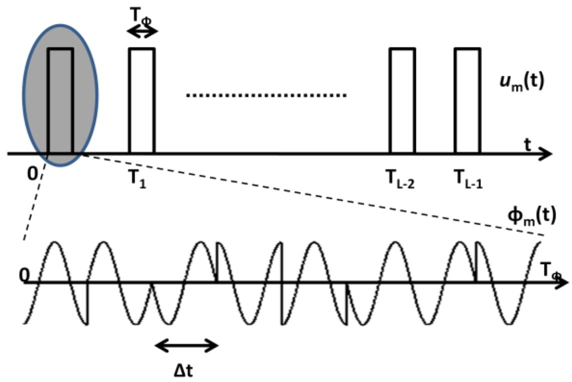


Figure 2: Structure of Phase-Coded Pulse waveforms

3. MIMO RADAR AMBIGUITY FUNCTION

The resolution of a radar system is determined by the response to a point target in the matched filter output. This response can be characterized by a function called the ambiguity function. The ambiguity function for SIMO radar is given as

$$\chi(\tau, v) = \int_{-\infty}^{\infty} u(t) u^*(t-\tau) e^{-j2\pi v t}\, dt \qquad (7)$$

Here τ and v represent the delay and Doppler mismatch at the receiver respectively. The ideal ambiguity function should have a “thumbtack” shape around the zero-mismatch region i.e $(\tau, v) = (0, 0)$so that good delay and Doppler resolution are achieved. The ambiguity function for MIMO case is derived in [4]. Consider the expression for the received signal in MIMO radar given in (2). Let (τ_1, v_1, f_1) represent the true parameters of the target and (τ_2, v_2, f_2) be the assumed parameters at the receiver. The summed matched filter output is given as

$$\sum_{n=0}^{N-1} \int_{-\infty}^{\infty} y_n^{\tau_1, v_1, f_1}(t) \left(y_n^{\tau_2, v_2, f_2}\right)^*(t)\, dt$$
$$= \left(\sum_{n=0}^{N-1} e^{j2\pi(f_1 - f_2)n}\right) \qquad (8)$$

$$\times \left(\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \int_{-\infty}^{\infty} u_m(t-\tau_1) u_{m'}^*(t-\tau_2) \times e^{j2\pi(v_1-v_2)t} e^{j2\pi(f_1 m - f_2 m')\gamma} \right)$$

In the above expression, the second term corresponds to the ambiguity function when only one receiver is present, while the first term brings out the effect of having multiple receivers. To simplify the ambiguity function and the waveform design problem, the first term can be decoupled from the above expression. The resulting expression is termed the "MIMO ambiguity function". Consider $\tau = (\tau_1 - \tau_2)$ to be the delay mismatch and $v = (v_1 - v_2)$ to be the Doppler mismatch, and rewrite the expression. The MIMO radar ambiguity function is thus given as [5]

$$\chi_{m,m'}(\tau, v, f_1, f_2) = \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \chi_{m,m'}(\tau, v) e^{j2\pi(f_1 m - f_2 m')\gamma} \quad (9)$$

where τ and v represent the delay and Doppler mismatch at the receiver, f_1 represents the target's true spatial frequency, and f_2 represents the assumed spatial frequency at the receiver. $\chi_{m,m'}(\tau, v)$ represents the cross-ambiguity function between the waveforms $u_m(t)$ and $u_{m'}(t)$, given by

$$\chi_{m,m'}(\tau, v) = \int_{-\infty}^{\infty} u_m(t) u_{m'}^*(t+\tau) e^{j2\pi vt} dt \quad (10)$$

For phase coded waveforms, (10) becomes

$$\chi_{m,m'}(\tau, v) = \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} \chi_{m,m'}^{\phi}(\tau - T_l - T_{l'}, v) e^{j2\pi v T_l} \quad (11)$$

where

$$\chi_{m,m'}^{\phi}(\tau, v) = \int_{0}^{Q\Delta t} \phi_m(t) \phi_{m'}^*(t+\tau) e^{j2\pi vt} dt \quad (12)$$

All the target returns due to the transmission of a phase coded pulse are assumed to arrive before the transmission of the new pulse. This assumption implies that $|\tau| < \min_{l,l'} (|T_l - T_{l'}| - Q\Delta t)$ due to which

$$\chi_{m,m'}^{\phi}(\tau - T_l - T_{l'}, v) = 0 \text{ for } l \neq l' \quad (13)$$

and hence

$$\chi_{m,m'}(\tau, v) = \chi_{m,m'}^{\phi}(\tau, v) \sum_{l=0}^{L-1} e^{j2\pi v T_l} \quad (14)$$

$\chi_{m,m'}^{\phi}(\tau, v)$ is the cross-ambiguity between two individual pulses of different waveforms $\phi_m(t)$ and $\phi_{m'}(t)$.

$$\chi_{m,m'}^{\phi}(\tau, v) = \frac{1}{T_p} \sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} c_{m,q} c_{m'q'}^* \times \int_{-\infty}^{\infty} s\left(\frac{t - q\Delta t}{\Delta t}\right) s\left(\frac{t + \tau - q'\Delta t}{\Delta t}\right) e^{j2\pi vt} dt \quad (15)$$

$$= \frac{1}{T_p} \sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} c_{m,q} c_{m'q'}^* \Gamma_1 \quad (16)$$

where

$$\Gamma_1 = \int_{-\infty}^{\infty} s\left(\frac{t - q\Delta t}{\Delta t}\right) s\left(\frac{t + \tau - q'\Delta t}{\Delta t}\right) e^{j2\pi vt} dt \quad (17)$$

In order to solve (17), the change of variables is made as $t_1 = t - q\Delta t$, and then integrate over the range $(-\infty, \infty)$.

$$\Gamma_1 = e^{j2\pi vq\Delta t} \chi_{rect}(\tau + (q - q')\Delta t, v) \quad (18)$$

where $\chi_{rect}(\tau, v)$ given by

$$\chi_{rect}(\tau, v) = \int_0^{\Delta t} s(t) s(t+\tau) e^{j2\pi vt} dt \quad (19)$$

$$= \begin{cases} \left(\frac{\Delta t - |\tau|}{\Delta t}\right) \text{sinc}(v(\Delta t - |\tau|)) e^{j\pi v(\tau + \Delta t)} & \text{for} |\tau| < \Delta t \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

denotes the triangular ambiguity function of a rectangular pulse $s(t)$ and represents the output of a matched filter for a single pulse. By substituting (18) into (16), the cross ambiguity function can be written as

$$\chi_{m,m'}^{\phi}(\tau, v) = \frac{1}{T_p} \sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} c_{m,q} c_{m'q'}^* \times e^{j2\pi vq\Delta t} \chi_{rect}(\tau + (q - q')\Delta t, v) \quad (21)$$

Utilizing the relation $p = q - q'$ and collecting the terms centered at the same shift $\tau = p\Delta t$ the double sum in (21) can be written as

$$\sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} = \sum_{p=0}^{Q-1} \sum_{q'=0}^{Q-1} \Bigg|_{q=q'+p} + \sum_{p=-(Q-1)}^{-1} \sum_{q'=0}^{Q-1-|p|} \Bigg|_{q'=q-p} \quad (22)$$

The cross-ambiguity function $\chi_{m,m'}^{\phi}(\tau, v)$ between the phase coded pulses $\phi_m(t)$ and $\phi_{m'}(t)$can then be written with (22) as a series of shifted ambiguity functions $\chi_{rect}(\tau, v)$ of the rectangular pulse as

$$\chi_{m,m'}^{\phi}(\tau, v) = \frac{1}{T_p} \sum_{p=0}^{Q-1} \chi_{rect}(\tau + p\Delta t, v) e^{j2\pi vp\Delta t}. S_1 + \frac{1}{T_p} \sum_{p=0}^{Q-1} \chi_{rect}(\tau + p\Delta t, v). S_2 \quad (23)$$

$$S_1 = \sum_{q'=0}^{Q-1-p} c_{m,(q'+p)} c_{m',q'}^* e^{j2\pi vq'\Delta t}$$
$$S_2 = \sum_{q=0}^{Q-1-|p|} c_{m,q} c_{m',(q-p)}^* e^{j2\pi vq\Delta t} \quad (24)$$

Equations (9), (14), (23) and (24) constitute the complete MIMO ambiguity function for phase coded pulse waveforms. The objective of phase coded pulse waveform design for MIMO radars is to design the code matrix **C**such that the cost function

$$g_p(\mathbf{C}) \triangleq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{1} \int_{0}^{1} |\chi_{m,m'}(\tau, v, f_1, f_2)|^p d\tau dv df_1 df_2 \quad (25)$$

is minimized. To obtain good resolutions, the peaks in $\chi_{m,m'}(\tau, v, f_1, f_2)$ need to be minimized which are not along the line $(0, 0, f_1, f_1)$.Typically p is chosen to be 3 so that sidelobe peak values of the MIMO radar ambiguity function at $\tau \neq 0, v \neq 0, f_1 \neq f_2$ are minimized. This forces the energy of the function $\chi_{m,m'}(\tau, v, f_1, f_2)$ to the evenly spread in the delay, Doppler and angular dimensions.

4. ALGORITHM FOR OPTIMIZATION OF PHASE CODED WAVEFORM SETS

The computational complexity involved in searching the best polyphase code set $\mathbf{C} = [c_{m,q}]_{M \times Q}$ with set size of M, code length of Q, and distinct phasenumber K, through exhaustive search, i.e., minimizing (25), is on the order of M^{KQ}, which grows exponentially with the code length and the set size. Simulated annealing algorithm has been widely used and found to be very efficient in the design of sequences for radar applications [7]-[8].

4.1 Simulated Annealing

Simulated annealing (also called as statistical cooling) optimizes a cost function that is analogous to a metallurgical process. In a typical process, a material is heated up to a temperature that allows its molecules to move freely around and then cooled down slowly (annealing). The freedom of movement for the molecules decreases gradually until all the molecules take a fixed position. At the end of the process, the total energy of the material is minimal provided that the cooling is very slow. The idea of applying this method to combinatorial optimization was first published by Kirkpatrick [11]. The analogy with physical model has the correspondence with optimization a) energy corresponds to the cost function b) the movement of molecules corresponds to a sequences of moves in the set of feasible solutions c) temperature corresponds to a control parameter T which controls the acceptance probability for a move from $f \in F$ to $g \in N(f)$. A good move, i.e., a move for which $c(g) \leq c(f)$, is always accepted irrespective of the value T. A bad move for which $c(g) > c(f)$, is accepted with a probability $e^{-\Delta c/T}$, where $\Delta c = c(g) - c(f)$. So, for high values of T nearly all bad moves are accepted, while hardly any bad move is accepted when T is low. The following section describes the design of the parameters of the phase coded pulse waveforms using simulated annealing algorithm.

4.2 Optimization Algorithm

The design starts with a initial code matrix with randomly selected entries from $\left\{e^{j\frac{2\pi}{K}0}, e^{j\frac{2\pi}{K}1}, \ldots, e^{j\frac{2\pi}{K}(K-1)}\right\}$. The initial temperature is chosen [7] to be $T = 10\sigma$ where σ is the standard deviation of the cost function values obtained by evaluating the cost function of $R = MQ$ different code matrices obtained by perturbation of an initial randomly chosen code matrix. In the optimization process of the code matrix, random search is carried out using code phase value "perturbation" i.e randomly selecting an entry in the code matrix and replacing it with another permissible value. With each phase perturbation, the cost function (25) is evaluated before and after perturbation and the cost function change Δc is calculated. The phase change is accepted with probability 1 if $\Delta c < 0$, otherwise it is accepted with probability $e^{-\Delta c/T_i}$. This process is repeated until the cost function distribution reaches equilibrium state. Then, the temperature is reduced to $T_{i+1} = \beta T_i (0 < \beta < 1)$ where β is smaller and very close to one. In this design it is chosen as 0.98. The perturbation is repeated with the updated temperature until equilibrium and the temperature is further reduced. The annealing process is terminated until no "perturbed" phase is accepted during four consecutive temperature updations.

5. DESIGN RESULTS

With the above proposed algorithm, a range of polyphase code sets with various values of M and Q are designed. Table-1 shows the sequence matrix for a designed polyphase sequence set with $M = 5$, $Q = 63$ and $K = 4$. Phase sequences within the code matrix are obtained by mapping the elements {0, 1, 2, 3} to $\{e^{j0}, e^{j\pi/2}. e^{j\pi}, e^{j3\pi/2}\}$. The MIMO radar ambiguity function of the code matrix in Table 1 and its corresponding delay and Doppler cut are plotted in Fig. 3, Fig. 4 and Fig. 5 respectively. Table 2 shows the sequence matrix for a designed polyphase sequence set with $M = 32$, $Q = 127$ and $K = 4$. The MIMO radar ambiguity function of the code matrix in Table-2 and its corresponding delay and Doppler cut are plotted in Fig. 6, Fig. 7 and Fig. 8 respectively. Fig. 9 shows the spatial ambiguity function of the phase coded waveforms generated using sequences in Table 2. Fig. 9 displays cuts of $|\boldsymbol{\Omega}(\boldsymbol{\tau}, \boldsymbol{v}, \boldsymbol{f_1}, \boldsymbol{f_2})|$ at various values of $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$ of the phase coded waveforms generated using sequences in Table 1. It is seen that the ambiguity function has a very low magnitude when $(\boldsymbol{\tau}, \boldsymbol{v}) \neq (\mathbf{0}, \mathbf{0})$ for all mismatched values of of $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$.

Table 1: Sequences corresponding to the designed polyphase code set with $M = 5, Q = 63$ and $K = 4$

1 1 1 3 1 3 0 1 2 3 0 1 3 2 2 1 1 0 3 3 1 2 3 1 2 1 2 0 3 2 0 1 3 3 2 2 0 3 0 1 1 3 1 0 2 3 2 3 0 0 0 1 3 0 2 0 0 3 0 0 0 0 0
1 1 3 3 1 3 0 1 0 1 2 3 1 0 2 3 1 2 3 1 3 2 3 3 0 1 0 2 1 2 2 3 3 1 2 2 2 3 0 3 3 1 1 0 2 1 2 1 2 2 2 1 3 2 2 2 0 3 0 2 2 0 0
1 2 2 2 1 1 0 0 0 2 1 0 0 2 2 0 3 3 3 3 3 1 0 3 2 3 2 1 3 0 0 3 1 0 3 1 1 0 1 3 0 1 2 2 3 0 0 1 3 1 0 2 0 3 0 3 1 3 1 0 2 1 0
1 2 2 3 3 3 0 0 0 1 2 3 1 1 3 0 0 3 2 1 0 0 2 3 1 0 2 0 2 3 2 2 2 2 0 1 3 1 1 1 3 0 3 0 3 0 1 3 0 2 1 1 0 3 1 3 0 1 3 0 1 0 0
1 3 2 3 1 2 3 2 0 3 2 0 3 1 1 0 0 2 3 2 1 3 3 3 2 2 3 0 1 0 2 2 3 3 2 0 0 2 3 0 2 0 0 2 3 3 1 3 1 1 0 3 0 1 2 1 3 0 2 3 1 0 3

Table 2: Sequences corresponding to the designed polyphase code set with $M = 16, Q = 127$ and $K = 4$

1 1 1 2 1 1 3 0 3 2 2 3 2 1 1 2 0 0 1 3 0 1 1 3 3 2 3 1 0 1 2 3 1 2 3 1 0 2 3 2 0 3 2 2 0 1 3 3 2 2 2 0 1 0 3 1 1 3 1 0 2 3 2 3 2 1 1 1 0 0 1 1 2 3 2 0 0 3 0 2 1 1 1 1 2 0 0 3 0 1 2 2 2 0 3 3 0 2 2 2 2 1 0 0 2 2 0 2 3 3 3 1 1 3 3 0 1 2 1 2 3 2 0 1 1 2 0
1 1 1 2 1 2 2 3 2 3 1 0 2 2 3 1 0 1 3 3 1 2 1 3 2 1 2 1 3 2 1 1 0 2 1 2 0 1 0 0 2 1 3 3 0 2 0 0 1 0 3 3 3 3 1 0 2 3 0 3 2 1 1 1 0 2 3 3 0 1 0 2 0 1 0 0 1 3 1 1 0 2 0 3 2 3 0 2 0 2 1 3 2 2 2 0 2 3 2 0 2 2 2 0 3 1 1 3 3 3 1 2 1 0 1 0 1 2 2 3 1 2 2 1 1 3 2
1 1 1 2 1 3 1 2 1 0 0 1 2 3 1 0 0 2 1 3 2 3 1 3 1 0 1 1 2 3 0 3 3 2 3 3 0 0 1 2 0 3 0 0 0 3 1 1 0 2 0 2 1 2 3 3 3 3 3 2 2 3 0 3 2 3 1 1 0 2 3 3 2 3 2 0 2 3 2 0 3 3 3 1 2 2 0 1 0 3 0 0 2 0 1 1 0 0 2 2 2 3 0 0 0 0 2 0 3 3 3 3 1 1 3 0 1 2 3 0 3 2 0 1 1 0 0
1 1 1 2 3 3 3 0 3 2 2 1 2 1 1 2 0 0 3 1 2 3 3 1 1 2 1 1 2 1 0 3 1 0 1 1 0 0 1 0 0 1 0 0 0 3 3 3 0 2 0 2 1 0 3 3 3 3 3 2 0 1 2 1 0 1 3 1 2 2 1 3 0 3 2 2 0 3 2 2 1 1 3 3 0 0 0 3 0 3 2 0 0 2 1 1 0 2 0 2 0 1 2 2 0 2 0 0 1 3 1 1 1 3 1 0 1 0 3 0 1 2 0 1 3 2 2
1 1 1 3 1 3 1 0 3 2 3 0 1 2 0 3 3 2 2 1 1 1 0 1 1 1 2 1 0 2 3 2 3 1 0 0 0 1 3 2 3 1 3 3 2 3 3 0 1 2 3 1 2 1 3 2 0 1 0 2 3 0 2 2 1 1 3 2 0 0 0 1 2 1 3 1 1 3 2 2 1 2 3 0 1 3 3 0 0 1 3 0 3 0 2 2 1 0 0 3 1 3 1 0 2 0 1 2 1 2 0 2 0 1 3 0 2 0 0 0 1 0 0 2 0 0 0
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1 1 2 1 3 2 1 0 3 1 0 3 3 2 2 3 2 2 1 2 0 2 3 1 3 0 0 2 2 1 2 1 1 3 1 1 0 0 3 2 1 2 1 1 3 2 2 1 1 2 3 0 0 2 1 2 0 3 1 1 3 2 2 0 1 2 3 2 2 0 2 3 1 0 2 2 0 2 1 2 2 0 0 0 2 3 1 3 3 1 3 3 0 3 0 1 2 3 2 0 3 1 2 0 1 1 1 0 3 3 1 2 1 0 2 1 2 3 1 0 0 0 3 1 0 1 3
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1 1 2 2 3 3 3 0 3 3 3 2 3 0 2 1 2 1 3 0 0 0 3 1 2 1 3 1 1 2 1 1 0 3 2 1 3 2 3 1 2 0 3 0 2 1 0 2 2 3 3 1 2 0 0 0 3 2 3 1 3 3 1 2 2 3 2 1 0 3 1 1 0 0 3 1 0 1 0 2 2 3 0 0 1 3 1 3 0 2 0 3 2 3 2 0 2 0 3 1 0 2 0 0 0 3 0 0 2 0 0 0 3 3 3 1 3 1 3 0 1 2 1 0 3 2 0
1 1 2 3 1 1 3 2 1 1 2 3 2 3 1 0 1 1 2 0 1 0 0 1 0 2 2 1 1 1 2 0 0 0 1 2 3 1 3 3 1 0 0 1 0 3 2 1 1 3 0 2 3 3 0 1 2 0 2 3 2 2 3 3 3 1 2 2 2 3 2 1 2 2 0 0 3 1 2 0 0 2 2 1 2 0 0 2 0 2 3 1 1 1 1 3 3 0 3 2 1 2 3 2 0 3 3 0 2 3 3 3 2 3 1 1 0 1 2 2 1 0 1 1 0 2 2
1 1 2 3 3 1 1 0 3 3 0 3 2 1 1 2 1 3 0 2 1 0 2 3 0 0 2 1 1 3 2 0 2 2 3 0 3 1 3 1 1 2 0 1 0 3 0 3 1 3 0 2 3 1 0 1 2 0 2 3 0 0 1 1 1 3 0 2 0 3 0 1 0 2 0 2 1 1 2 2 2 0 2 3 0 2 0 0 0 2 1 1 3 3 1 3 3 2 1 2 3 0 1 0 0 1 1 0 0 3 1 1 2 1 3 1 0 3 2 2 3 0 1 1 2 0 0
1 1 3 1 2 1 3 2 3 0 3 1 0 3 3 0 0 1 0 0 3 0 0 0 3 3 3 2 0 2 0 3 2 1 3 1 3 1 2 2 1 2 3 2 1 3 3 0 0 1 1 2 1 2 0 0 3 0 2 3 3 3 3 0 2 2 3 2 1 0 2 0 0 3 1 2 2 2 0 0 3 0 2 0 0 0 0 3 3 1 1 3 1 3 0 1 0 3 2 1 0 2 3 1 0 2 1 3 1 0 3 1 1 2 3 2 0 3 2 3 1 2 0 2 3 1 0
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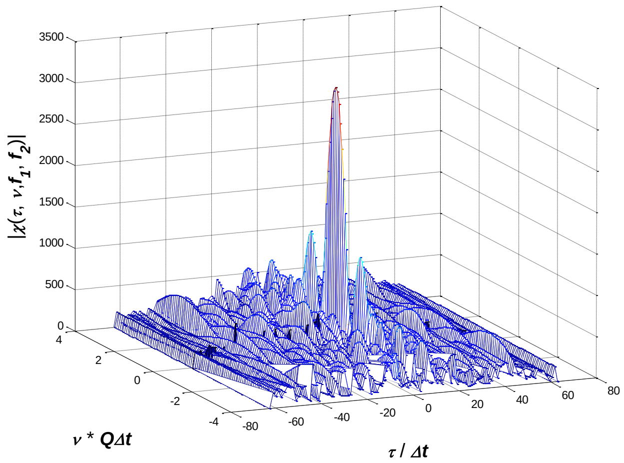

Figure 3: MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-1 with $M = 5, Q = 63, K = 4, \ f_1 = 0, f_2 = 0$

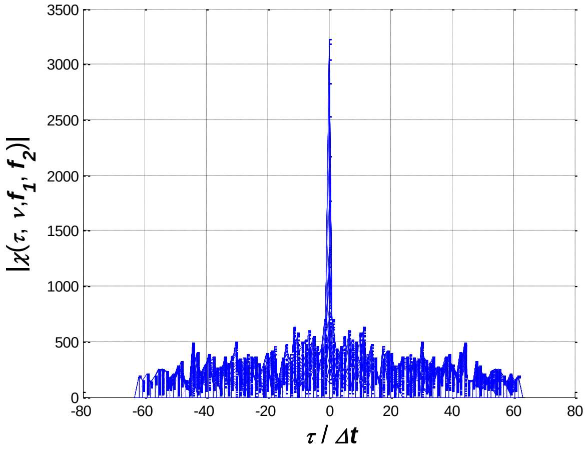

Figure 4: Delay cut of the MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-1 with $M = 5, Q = 63, K = 4, f_1 = 0, f_2 = 0$

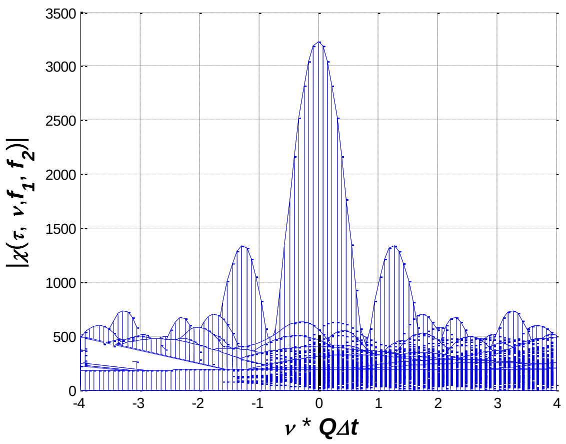

Figure 5: Doppler cut of the MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-1 with $M = 5, Q = 63, K = 4, f_1 = 0, f_2 = 0$

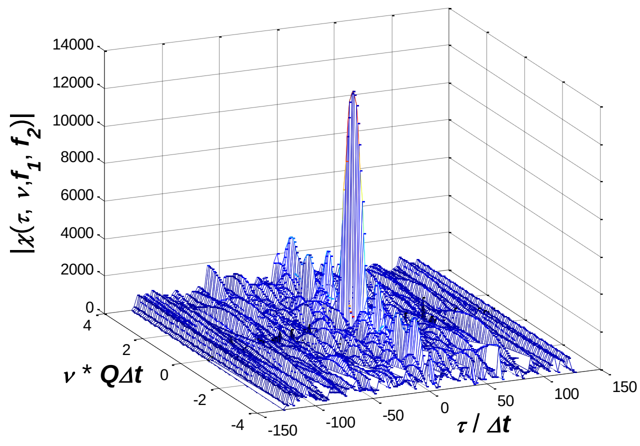

Figure 6: MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-2 with $M = 16, Q = 127, K = 4, \ f_1 = 0, f_2 = 0$

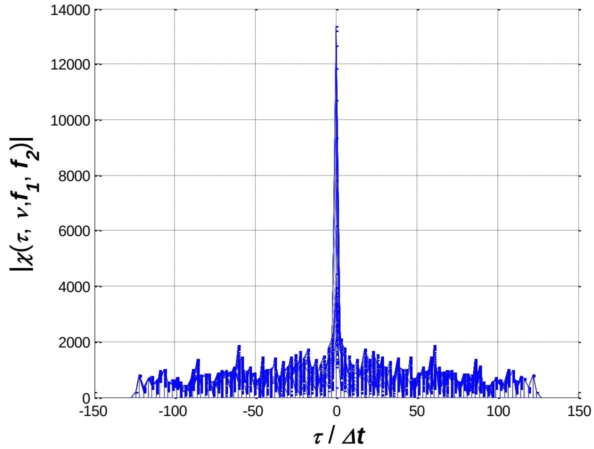

Figure 7: Delay cut of the MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-2 with $M = 16, Q = 127, K = 4, \ f_1 = 0, f_2 = 0$

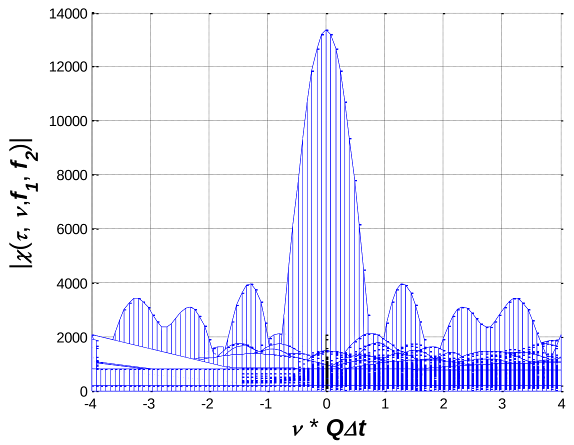

Figure 8: Doppler cut of the MIMO radar ambiguity function of Phase Coded waveforms generated using sequences designed in Table-2 with $M = 16, Q = 127, K = 4, \ f_1 = 0, f_2 = 0$

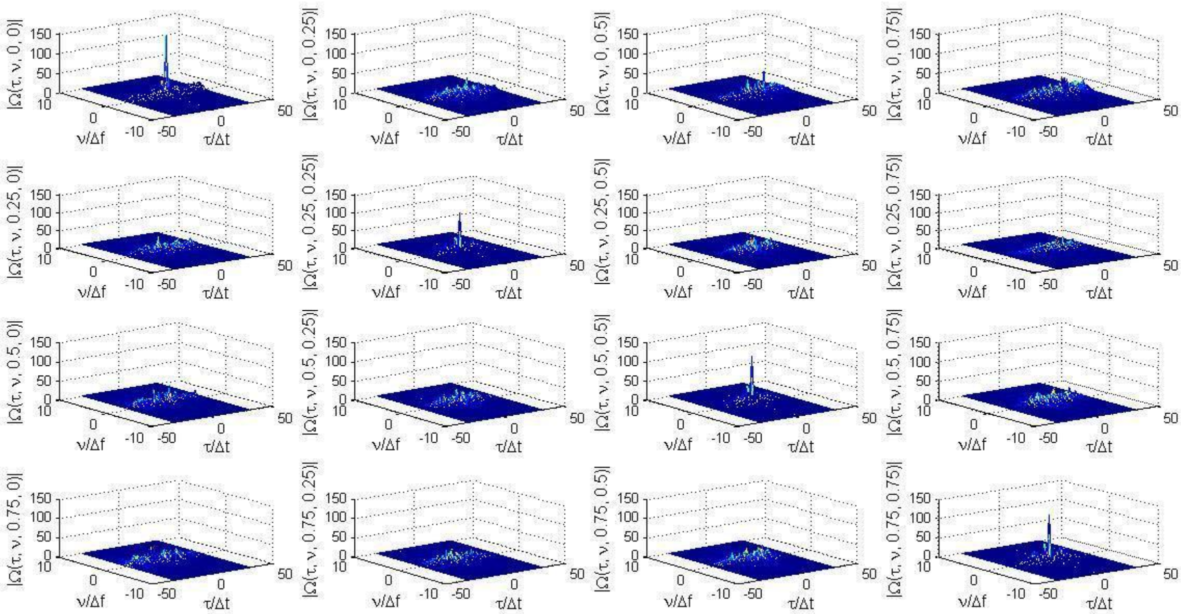

Figure 9: Plots of MIMO radar ambiguity function at various values of f_1 and f_2, with M=5, Q=63 and K=4

6. CONCLUSIONS

Considering the many advantages that phase coded waveforms offer over frequency hopping waveforms in terms of better resolution properties, this paper derived the MIMO ambiguity function of phase coded pulse waveforms. Further, a numerical optimization algorithm based on simulated annealing is proposed for designing parameters of the phase coded pulse waveforms that minimize the peak of the ambiguity function at all mismatched values of delay, Doppler and angular dimensions. Phase coded waveforms with good delay, Doppler and spatial resolution properties have been designed using the proposed algorithm. It is seen that that phase coded waveforms offer improved resolution performance over frequency hopping waveforms (in delay, Doppler and spatial dimensions) even when large number of transmit antennas (and hence large number of waveforms) are employed. The choice of phase coded or frequency hopping waveforms to use depends on channel characteristics and signal processing complexity. In MIMO sonar applications, where the phase information of the received echoes is severely distorted due to reverberation effects of the channel, frequency hopping waveforms are a better choice. Whereas for most of the radar applications, phase coded waveforms are a better choice, due to their constant modulus property, easy generation and processing.

7. REFERENCES

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