Power Summation- A Computer Dimension

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ABSTRACT
Series which can be quite enigmatic and mesmerizing is dealt with a different prospective in this paper. Power summation of a series will be encountered in a different light with a touch of computer. A computer program is made and discussed through series power summation.

1. INTRODUCTION
A series is a summation of the terms of a sequence, which is an ordered set of numbers that most often follows some rule or pattern to determine next term in the order. The Greek letter sigma ∑ is used to represent the summation of terms of a sequence of numbers. Series are typically written in the following form :

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_{n-1} + a_n$$

Where index of summation, i takes consecutive integer values from the lower limit, 1 to upper limit ,n.

A finite series is a summation of a finite number of terms. An infinite series has an infinite number of terms and an upper limit of infinity.

There are two main types of sequences. An arithmetic sequence is one in which successive terms differ by same amount, as (3, 6 , 9, 12,…). A geometric series is one in which quotient of any two successive terms is a constant, as (3, 9, 27, 81, ……). Similarly, there are also arithmetic and geometric series, which are simply summations of arithmetic and geometric sequences, respectively.

Sum of first natural numbers is

$$1+2+3+................+n-1+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Sum of squares of first n natural number is

$$1^2+2^2+3^2+.................+(n-1)^2+n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

And similarly with powers 3, 4, 5…………up to 10 is summarized in the table.

Table 1. Summation [2]

Summation	Expansion	Equivalent Value
$\sum_{k=1}^{n} k$	= 1 + 2 + 3 + 4 + .. + n	**= (n^2 + n) / 2** = (1/2)n^2 + (1/2)n
$\sum_{k=1}^{n} k^2$	= 1 + 4 + 9 + 16 + .. + n^2	**= (1/6)n(n+1)(2n+1)** = (1/3)n^3 + (1/2)n^2 + (1/6)n
$\sum_{k=1}^{n} k^3$	= 1 + 8 + 27 + 64 + .. + n^3	= (1/4)n^4 + (1/2)n^3 + (1/4)n^2
$\sum_{k=1}^{n} k^4$	= 1 + 16 + 81 + 256 + .. + n^4	= (1/5)n^5 + (1/2)n^4 + (1/3)n^3 - (1/30)n
$\sum_{k=1}^{n} k^5$	= 1 + 32 + 243 + 1024 + .. + n^5	= (1/6)n^6 + (1/2)n^5 + (5/12)n^4 - (1/12)n^2
$\sum_{k=1}^{n} k^6$	= 1 + 64 + 729 + 4096 + .. + n^6	= (1/7)n^7 + (1/2)n^6 + (1/2)n^5 - (1/6)n^3 + (1/42)n
$\sum_{k=1}^{n} k^7$	= 1 + 128 + 2187 + 16384 + .. + n^7	= (1/8)n^8 + (1/2)n^7 + (7/12)n^6 - (7/24)n^4 + (1/12)n^2
$\sum_{k=1}^{n} k^8$	= 1 + 256 + 6561 + 65536 + .. + n^8	= (1/9)n^9 + (1/2)n^8 + (2/3)n^7 - (7/15)n^5 + (2/9)n^3 - (1/30)n
$\sum_{k=1}^{n} k^9$	= 1 + 512 + 19683 + 262144 + .. + n^9	= (1/10)n^{10} + (1/2)n^9 + (3/4)n^8 - (7/10)n^6 + (1/2)n^4 - (3/20)n^2
$\sum_{k=1}^{n} k^{10}$	= 1 + 1024 + 59049 + 1048576 + .. + n^{10}	= (1/11)n^{11} + (1/2)n^{10} + (5/6)n^9 - n^7 + n^5 - (1/2)n^3 + (5/66)n

It's very difficult to predict the equivalent value as summation is not following some pattern. To overcome this difficulty we are writing the summation in a different way.

$${}^2C_1\sum n = n^2 + {}^2C_2\sum n^0 = n^2 + n \quad \text{Or} \quad \sum n = \frac{n(n+1)}{2}$$

$${}^3C_1\sum n^2 = n^3 + {}^3C_2\sum n - {}^3C_3\sum n^0 = n^3 + 3\frac{n(n+1)}{2} - n$$

Or $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$${}^4C_1\sum n^3 = n^4 + {}^4C_2\sum n^2 - {}^4C_3\sum n + {}^4C_4\sum n^0$$
$$= n^4 + 6\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$$
$$or\sum n^3 = \frac{(n(n+1))^2}{4}$$

Similarly

$${}^5C_1\sum n^4 = n^5 + {}^5C_2\sum n^3 - {}^5C_3\sum n^2 + {}^5C_4\sum n - {}^5C_5\sum n^0$$

,

$${}^6C_1\sum n^5 = n^6 + {}^6C_2\sum n^4 - {}^6C_3\sum n^3 + {}^6C_4\sum n^2 - {}^6C_5\sum n + {}^6C_6\sum n^0$$

Generalizing

$${}^{r+1}C_1\sum n^r =$$
$$n^{r+1} + (-1)^2({}^{r+1}C_2\sum n^{r-1}) + (-1)^3({}^{r+1}C_3\sum n^{r-2}) + ---$$
$$-- + (-1)^r({}^{r+1}C_r\sum n^1) + (-1)^{r+1}({}^{r+1}C_{r+1}\sum n^0).$$

To find $\sum n^r$ one has to know $\sum n^{r-1}$

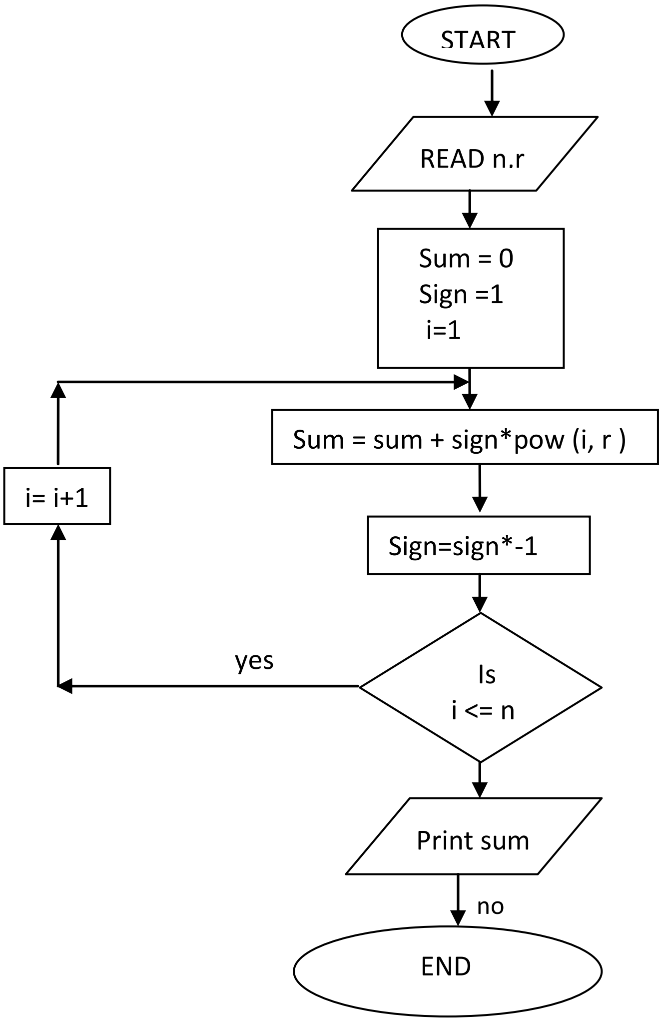
So one cannot directly calculate power summation of a series, to do this we need to apply a computer program to execute the required series.

Algorithm

Sum-series:

1. Begin
2. sum=0, sign =1
3. Read n, r
4. for i = 1 to n do step 5,6
5. sum = sum + sign * pow(i,r)
6. sign= sign * -1
7. Print sum
8. End submission

FLOWCHART



PROGAM

```
#include <stdio.h>
#include <conio.h>
#include <math.h>

Void main ()
{
    int n, r;
    int sum = 0;
    int i;
    int sign=1;
```

```
            printf("Enter the value of n:");
            scanf("%d", &n);
            printf("Enter the value of r:");
            scanf("%d", & r);
            for(i+1;i< =n;i++)
            {
            Sum=sum+ sign* pow (i,r);
            sign = sign * -1;
            }
            printf("\n Required sum of the series is %d",sum);
            getch();
}
```

Since general equation is

$$ {}^{r+1}C_1 \sum n^r = $$
$$ n^{r+1} + (-1)^2({}^{r+1}C_2 \sum n^{r-1}) + (-1)^3({}^{r+1}C_3 \sum n^{r-2}) + --- $$
$$ -- + (-1)^r({}^{r+1}C_r \sum n^1) + (-1)^{r+1}({}^{r+1}C_{r+1} \sum n^0). $$

Putting r = 5
Program generates

$$ {}^{6}C_1 \sum n^5 = n^6 + 3n^5 + \frac{5}{2}n^4 - \frac{n^2}{2} \quad \text{or} $$
$$ \sum n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5}{12}n^4 - \frac{n^2}{12} $$

And for r= 9 than
Program generates

$$ {}^{10}C_1 \sum n^9 = n^{10} + 5n^9 + \frac{15}{2}n^8 - 7n^6 + 5n^4 $$

Or

$$ \sum n^9 = \frac{n^{10}}{10} + \frac{n^9}{2} + \frac{3}{4}n^8 - \frac{7}{10}n^6 + \frac{n^4}{2} $$

Similarly putting r = 1 to 10 we can easily generate above table.

2. Conclusion

As we can see from above table and general equation, to find summation of cubes of n, one need to know summation of squares of n. But with the help of computer program it become easy for us to calculate the required result. Computer method is quite simple, effective and finally less time consuming. Further work can be done where one does not need help of system to find the required result. Power Summation becomes easy and understandable as one uses computer. This study will be very helpful for researchers and intellectuals to easy understanding and practicing of power summation in the field of computer science and technology.

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