A New type of Generalized Separation Axioms

M.LellisThivagar
School of Mathematics, M.K.U,
Madurai-25021,TamilNadu,India.

M.Anbuchelvi
V.V.Vanniaperumal College for
Women,Virudhunar,T.N,

ABSTRACT

In this paper we introduce and investigate some weak separation axioms associated with $\hat{\Omega}$-closed sets and $\hat{\Omega}$-closure operators.Also we find some of their applications.

Keywords and Phrases:- $\hat{\Omega}$ -closed sets, $\hat{\Omega}$ -closure, $D_{\hat{\Omega}}$-sets, $\hat{\Omega}$ - D_i -sets i = 0 ,1 ,2; $\hat{\Omega}$ - R_0 -spaces.,

AMS subject classification 2010 :57D05

1. INTRODUCTION

In 1943,the separation axioms R_0 and R_1 were introduced and investigated by N.A.Shanin [13]. In 1963,they were developed more by A.S.Davis [3].Also their properties were found by K.K.Dube [4].In recent years,this idea of separation axioms were introduced and investigated through the sets δ-open set, b -open set, δ -semi open set etc.In this paper we introduce and investigate some weak separation axioms namely $\hat{\Omega}$ - D_i ; $\hat{\Omega}$ - T_i -spaces i = 0 ,1 , 2; $\hat{\Omega}$ - R_0 -spaces and $\hat{\Omega}$ - R_1 -spaces by utilizing $\hat{\Omega}$ -closed sets and it's closure operator and kernel.Also we find some of their applications.

2. PRELIMINARIES

Throughout this paper (X,τ) (or briefly X) represent a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of X , $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A respectively.The family of all $\hat{\Omega}$-open (resp. $\hat{\Omega}$ -closed) subsets of X is denoted by $\hat{\Omega}O(X)$. (resp. $\hat{\Omega}C(X)$).

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1 A subset A of a topological space (X,τ) is called

i)Semi-open [9]if $A \subseteq cl(int(A))$.

ii) pre-open [10] if $A \subseteq int(cl(A))$.

iii) δ -pre open [12] if $A \subseteq int(\delta cl(A))$.

iv) $\hat{\Omega}$ -open[7] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X,τ).

Definition 2.2 A topological space (X,τ) *is*

i) pre-T_0 [[6],[11]] (resp. (δ, p)-T_0 [1], δ-T_0 [5]) if for any distinct pair of points x and y of X, there exists a pre-open (resp. δ -pre open, δ -open) set U of X containing x but not y (or) containing y but not x. ii) pre-T_1 [[6],[11]] (resp. (δ, p)-T_1 [1], δ-T_1 [5]) if for any distinct pair of points x and y of X, there exists a pre-open (resp. δ -pre open, δ -open) set U of X containing x but not y and a pre-open (resp. δ -pre open) set V of X containing y but not x.

iii) pre-T_2 [[6],[11]] (resp. (δ, p)-T_2 [1], δ-T_2 [5]) if for any distinct pair of points x and y of X, there exists disjoint pre-open (resp. δ -pre open, δ -open) sets U and V of X containing x and y respectively.

Definition 2.3 *[14]* A subset A of X is called δ -closed in a topological space (X,τ) if $A = \delta cl(A)$, where $\delta cl(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in \tau, x \in U\}$. The complement of δ -closed set in (X,τ) is called δ-open set in (X,τ).

Definition 2.4 *[2]* A subset A of a space X is said to be δD -set if there are two δ -open sets U and V such that $U \neq X$ and $A = U \setminus V$.

Definition 2.5 *[8]*A function $f : X \to Y$ is said to be $\hat{\Omega}$-irresolute if the inverse image of every $\hat{\Omega}$ -open set in Y is $\hat{\Omega}$ -open in X .

3. $\hat{\Omega}$-T_i-SPACES, $i = 0,1,2$.

Definition 3.1 Let (X,τ) be a space and $A \subseteq X$. Then the $\hat{\Omega}$ -kernel of A, denoted by $Ker_{\hat{\Omega}}(A)$ is defined as $Ker_{\hat{\Omega}}(A) = \bigcap\{G \in \hat{\Omega}O(X) : A \subseteq G\}$.

Lemma 3.2 Let A be a subset of (X,τ),then $Ker_{\hat{\Omega}}(A)=\{x\in X:\hat{\Omega}cl(\{x\})\cap A\neq\emptyset.\}$

Proof. Let $x\in Ker_{\hat{\Omega}}(A)$ and suppose that $\hat{\Omega}cl(\{x\})\cap A=\emptyset$.Then $A\subseteq X\setminus\hat{\Omega}cl(\{x\})$. If we define $V=X\setminus\hat{\Omega}cl(\{x\})$,then by[7] remark 5.2,$V$ is a $\hat{\Omega}$-open set such that $A\subseteq V$ and $x\notin V.$By the definition of $ker_{\hat{\Omega}}(A)$, $x\notin Ker_{\hat{\Omega}}(A),$ a contradiction. Therefore, $Ker_{\hat{\Omega}}(A)\subseteq\{x\in X:\hat{\Omega}cl(\{x\})\cap A\neq\emptyset\}$.To prove the reversible inclusion,if $x\in X$ such that $\hat{\Omega}cl(\{x\})\cap A\neq\emptyset$ and suppose that $x\notin Ker_{\hat{\Omega}}(A),$ then there exists a $\hat{\Omega}$-open set U such that $A\subseteq U$ and $x\notin U.$ Since $\hat{\Omega}cl(\{x\})\cap A\neq\emptyset$, we can choose $y\in\hat{\Omega}cl(\{x\})\cap A.$ Then $y\in U$ and hence U is an $\hat{\Omega}$-open set containing y but not $x.$By [7] theorem 5.11, $y\notin\hat{\Omega}cl(\{x\}),$ a contradiction. Thus, $x\in ker_{\hat{\Omega}}(A)$.

Lemma 3.3 *Let* (X,τ) *be a space and* $x\in X$ *.Then* $y\in Ker_{\hat{\Omega}}(\{x\})$ *if and only if* $x\in\hat{\Omega}cl(\{y\})$.

Proof. **Necessity**- Assume that $y\notin Ker_{\hat{\Omega}}(\{x\}).$ By the definition of kernel,there exists a $\hat{\Omega}$-open set U containing x such that $y\notin U.$ By [7] theorem 5.11, $x\notin\hat{\Omega}cl(\{y\})$.

Sufficiency- Assume that $x\notin\hat{\Omega}cl(\{y\}).$ By [7] theorem 5.11,there exists a $\hat{\Omega}$-open set U containing x such that $y\notin U.$ By the definition of kernel, $y\notin Ker_{\hat{\Omega}}(\{x\}).$

Lemma 3.4 *For any points* x *and* y *in a space* (X,τ) *the following are equivalent.* $Ker_{\hat{\Omega}}(\{x\})\neq Ker_{\hat{\Omega}}(\{y\})$.

$\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$

Proof. $(i)\Rightarrow(ii)$ Suppose $Ker_{\hat{\Omega}}(\{x\})\neq Ker_{\hat{\Omega}}(\{y\}).$

Then, there exists a point $z\in X$ such that $z\in Ker_{\hat{\Omega}}(\{x\})$ but not in $Ker_{\hat{\Omega}}(\{y\})$. By lemma 3.2, $\{x\}\cap\hat{\Omega}cl(\{z\})\neq\emptyset$ and hence $x\in\hat{\Omega}cl(\{z\})$

.By[7], $\hat{\Omega}cl(\{x\})\subseteq\hat{\Omega}cl(\hat{\Omega}cl(\{z\}))=\hat{\Omega}cl(\{z\}).$ Again by lemma 3.2, $\{y\}\cap\hat{\Omega}cl(\hat{\Omega}(\{z\}))=\emptyset$ and hence $y\notin\hat{\Omega}cl(\{z\}).$ Therefore, $y\notin\hat{\Omega}cl(\{x\}).$ Thus, $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$.

$(ii)\Rightarrow(i)$ Suppose that $\hat{\Omega}cl(\{x\})\neq\delta\omega cl(\{y\})$ .Then there exists a point $z\in X$ such that $z\in\hat{\Omega}cl(\{x\})$ and $z\notin\hat{\Omega}cl(\{y\}).$ By[7] theorem 5.11,there exists a $\hat{\Omega}$-open set U containing z such that $U\cap(\{y\})=\emptyset$ and $U\cap(\{x\})\neq\emptyset.$ Now U is a $\hat{\Omega}$-open set containing x but not $y.$ By [7] theorem 5.11, $x\notin\hat{\Omega}cl(\{y\})$.By lemma 3.3, $y\notin ker_{\hat{\Omega}}(\{x\}).$ Thus $ker_{\hat{\Omega}}(\{x\})\neq ker_{\hat{\Omega}}(\{y\}).$

Lemma 3.5 Let A and B be any two subsets in a opological space X .If $A\subseteq B$,then $Ker_{\hat{\Omega}}(A)\subseteq Ker_{\hat{\Omega}}(B)$.

Proof. Suppose that $A\subseteq B$ and if $x\notin Ker_{\hat{\Omega}}(B)$ .By the definition of $Ker_{\hat{\Omega}}$,there exists a $\hat{\Omega}$-open set U such that $B\subseteq U$ and $x\notin U$ .Since $A\subseteq B\subseteq U$,again by the definition of $Ker_{\hat{\Omega}}$, $x\notin Ker_{\hat{\Omega}}(A)$ .Therefore, $Ker_{\hat{\Omega}}(A)\subseteq Ker_{\hat{\Omega}}(B)$.

Definition 3.6 *A space* (X,τ) is said to be

i) $\hat{\Omega}$-T_0 if for any distinct pair of points x and y of X, there exists a $\hat{\Omega}$-open set U of X containing x but not y (or) containing y but not $x.$

ii) $\hat{\Omega}$-T_1 if for any distinct pair of points x and y of X, there exists a $\hat{\Omega}$-open set U of X containing x but not y and a $\hat{\Omega}$-open set V of X containing y but not $x.$

iii) $\hat{\Omega}$-T_2 if for any distinct pair of points x and y of X, there exists disjoint $\hat{\Omega}$-open sets U and V of X containing x and y respectively.

Example 3.7

$X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},\{a,c\},X\}$. Then $\hat{\Omega}O(X)=\tau$. It is $\hat{\Omega}$-T_0 but not $\hat{\Omega}$-T_1 because for the pair of distinct points a and c, there is no $\hat{\Omega}$-open set containing c but not $a.$

Example 3.8 $X=\{a,b,c\}$ $\tau=\{\emptyset,\{a\},\{b,c\},X\}$. Then, $\hat{\Omega}O(X)=\Pi(X)$. It is $\hat{\Omega}$-T_1 as well as $\hat{\Omega}$-$T_2.$

Remark 3.9 *If* (X,τ) *is* $\hat{\Omega}$-$T_i,$ *then it is* $\hat{\Omega}$-$T_{i-1}, i=1,2.$

Theorem 3.10 *Every* δ-T_i *space is* $\hat{\Omega}$-$T_i.$ (i = 0,1,2.)

Proof. It follows from [7]theorem 3.2, the fact that every δ -open set is $\hat{\Omega}$ -open subset of X.

Theorem 3.11 Every $\hat{\Omega}\text{-}T_i$. space is pre--T_i. (i=0,1,2)

Proof. It follows from [7] because every $\hat{\Omega}$ -open set in X is pre open in X.

Remark 3.12. The converse of the above is not true in general from the following examples.

Example 3.13 .X = {a,b,c} and $\tau = \{\varnothing, \{a\}, X\}$ then PO(X) = { Φ ,{a},{a,b},{a,c},X } and

$\hat{\Omega}$ O(X) $= \{\varnothing, \{a\}, X\}$.It is pre-$\mathbf{T_o}$ but not $\hat{\Omega}$ -$\mathbf{T_o}$.

Example 3.14 X = {a,b,c} and τ ={ Φ ,{a,b},X} then

PO(X) ={ Φ ,{a},{b},{a,b},{a,c},{b,c},X} and

$\hat{\Omega}$ O(X) $= \{\varnothing, \{a\}, \{b\}, \{a,b\}, X\}$ .It is pre-T_1 as well as pre- T_2 but not $\hat{\Omega}\text{-}T_1$ or $\hat{\Omega}\text{-}T_2$.

4. CHARACTERIZATIONS OF $\hat{\Omega}\text{-}T_i$-SPACES.

Theorem 4.1 A space (X, τ) is $\hat{\Omega}$ -T_o if and only if

$\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ for every pair of distinct points x,y,in X.

Proof.**Necessity**- Suppose that X is $\hat{\Omega}$ -$\boldsymbol{T_o}$.-space and x,y are a pair of distinct points in X.By the definition of $\hat{\Omega}$ -$\boldsymbol{T_o}$.-space,there exists $\hat{\Omega}$ -open set U containing any one of these two points,Without loss of generality,take x in U and y not in U.Therefore X\U is a $\hat{\Omega}$ -closed set in X containing y but not x.By [7] remark 5.2, $\hat{\Omega}$ cl({y}) is the smallest $\hat{\Omega}$ -closed set containing {y} and hence $\hat{\Omega}$ cl({y}) $\subseteq$ X\U. Therefore, x $\notin$ $\hat{\Omega}$ cl({y}).

Sufficiency- Let x,y be any two points in X such that $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ .Choose z in X such that z is in $\hat{\Omega}$ cl({x}) and z does not belongs to $\hat{\Omega}$ cl({y}).If x $\in$ $\hat{\Omega}$ cl({y}),then by [7] remark 5.2, $\hat{\Omega}$ cl({x}) $\subseteq$ $\hat{\Omega}$ cl({y}).Therefore,z $\in$ $\hat{\Omega}$ cl({y}),a contradiction.Thus x $\notin$ $\hat{\Omega}$ cl({y}).Then, X\$\hat{\Omega}$ cl({y}) is a $\hat{\Omega}$ -open set in X such that x $\in$ X\$\hat{\Omega}$ cl({y}) and y $\notin$ X\$\hat{\Omega}$ cl({y}).Thus, (X,τ) *is* $\hat{\Omega}$ -T_o-space.

Corollary 4.2 A space X is $\hat{\Omega}$ -T_o if and only if each pair of distinct points x,y of X,either $y \notin$ $\hat{\Omega}$ cl({x})or $x \notin$ $\hat{\Omega}$ cl({y}).

Theorem 4.3 A space (X,τ) is $\hat{\Omega}\text{-}T_0$ if and only if for each pair of distinct points x, y in X, $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$.

Proof. Necessity- Suppose that X is $\hat{\Omega}\text{-}T_0$ -space and x, y are two distinct points in X. By theorem 3.16, $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$. By lemma 3.4, $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$.

Sufficiency- Let x, y be two distinct points in X such that $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$. By lemma 3.4, $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$. By theorem 3.16, (X,τ) is a $\hat{\Omega}$ -T_0-space.

Theorem 4.4 The following are equivalent in in a topological space (X,τ).

i) (X,τ) *is* $\hat{\Omega}$ -T_1-space.

ii) For every x $\in$ X,{x} = $\hat{\Omega}$ cl({x}).

iii) {x} = $\bigcap\{U : U \in \hat{\Omega}O(X,x)\} = \ker_{\hat{\Omega}}(\{x\})$ for each x in X.

Proof.

i) $\Longrightarrow$ ii) Suppose that X is $\hat{\Omega}$ -T_1-space and x is any point of X.Assume that y $\in$ $\hat{\Omega}$ cl({x}) and y $\neq$ x.Since X is $\hat{\Omega}$ -T_1-space,there exists a $\hat{\Omega}$ -open set U in X such that y $\in$ U and x $\notin$ U.By [7]theorem 5.11,$U \cap \{x\} \neq \Phi$. Therefore, x $\in$ U, a contradiction. Thus,{x} = $\hat{\Omega}$ cl({x}).

ii) $\Longrightarrow$ i) Suppose that {x} = $\hat{\Omega}$ cl({x}) for every x $\in$ X.Let x,y be any two distinct points in X.By hypothesis,y $\notin$ $Ker_{\hat{\Omega}}$ ({x}).By the definition of $\hat{\Omega}$ -kernal,there exists a $\hat{\Omega}$ -open set U containing x but noy y. Similarly if x $\notin$ $Ker_{\hat{\Omega}}$ ({y}),then there exists a $\hat{\Omega}$ -open set V containing y but noy x.Thus, (X,τ) *is* $\hat{\Omega}$ -T_1-space.

Theorem 4.5 (X,τ) *is* $\hat{\Omega}\text{-}T_1$, *if and only if* $\{x\}$ *is* $\hat{\Omega}$ -*closed for each* $x \in X$.

Proof. By theorem 3.19, is $\hat{\Omega}\text{-}T_1$, if and only if $\{x\} = \hat{\Omega}cl(\{x\})$ for every $x \in X$. By[?] remark 5.2, $\delta\omega cl(\{x\})$ is a $\hat{\Omega}$ closed set and hence $\{x\}$ is $\hat{\Omega}$ closed set for every $x \in X$.

Theorem 4.6 The following statements are equivalent in a topological space (X,τ)

(i) (X,τ) is $\hat{\Omega}$-T_2-space.

(ii) If $x \in X,$ then for each $y \neq x$, there exists a $\hat{\Omega}$-open set U containing x such that $y \notin \hat{\Omega}cl(U).$

(iii) $\{x\} = \bigcap\{\hat{\Omega}cl(U) : U \in \hat{\Omega}O(X,x)\}$ for every .

Proof. i) $\Rightarrow$ ii) Suppose that X is $\hat{\Omega}$-T_2-space and x,y are any two distinct points of X.Then there exists disjoint $\hat{\Omega}$-open sets U and V such that $x \in U, y \in V$. If F = X\V, then F is $\hat{\Omega}$-closed set such that $U \subseteq F$ and $y \notin F$.By the definition of $\hat{\Omega}$-closure,$y \notin \hat{\Omega}$ cl(U).

.$(ii) \Rightarrow (iii)$. Suppose that $x \in X$. By hypothesis,for any distinct point $y \in X,$ there exists a $\hat{\Omega}$-open set U containing x such that $y \notin \hat{\Omega}cl(U).$ Therefore, $y \notin \bigcap\{\hat{\Omega}cl(U) : U \in \hat{\Omega}O(X,x)\}$. Thus, $\{x\} = \bigcap\{\hat{\Omega}cl(U) : U \in \hat{\Omega}O(X,x)\}.$

iii) $\Rightarrow$ i) Suppose that x and y are any two distinct points in X. By hypothesis, there exists $U \in \hat{\Omega}O(X,x)$ such that $y \notin \hat{\Omega}$ cl(U).Therefore,U and X\$\hat{\Omega}$ cl(U) are two distict $\hat{\Omega}$-open sets containing x and y respectively.

5. $D_{\hat{\Omega}}$-SET AND $\hat{\Omega}$-D_I – SPACES I= 0,1,2.

Definition 5.1 A subset A in a space X is said to be $D_{\hat{\Omega}}$-*set if there are two* $\hat{\Omega}$-open sets U and V such that U $\neq$ *X and A =U \V* such that U $\neq$ X and A U\V.

Theorem 5.2 Every δD set is $D_{\hat{\Omega}}$ set.

Proof. By [7] theorem 3.2,every δ-open set is $\hat{\Omega}$-open set and hence it follows.

Remark 5.3 Every proper $\hat{\Omega}$-open set U is $D_{\hat{\Omega}}$-set because A = U and V = Φ.But the converse is not true in general from the following example.

Example 5.4
X={a,b,c,d}, $\tau = \{\Phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$

,then $\hat{\Omega}O(X) = \tau$ .By taking two $\hat{\Omega}$-open sets U = {a,b,c} $\neq$ X and V – {a,b} it is known that A = U\V = {c} which is $D_{\hat{\Omega}}$-set but not $\hat{\Omega}$-open set in X.

Definition 5.5 A space X is said to be

i) $\hat{\Omega}$-D_o if for any distinct pair of points x and y of X, there exists **a** $D_{\hat{\Omega}}$-set of X containing x but not y or a $D_{\hat{\Omega}}$-set of X containing y but not x.

ii) $\hat{\Omega}$-D_1 if for any distinct pair of points x and y of X, there exists **a** $D_{\hat{\Omega}}$-set of X containing x but not y and a $D_{\hat{\Omega}}$-set of X containing y but not x.

iii) $\hat{\Omega}$-D_2 if for any distinct pair of points x and y of X, there exists disjoint pair of $D_{\hat{\Omega}}$-sets U and V of X containing x y and respectively.

Example 5.6 X = {a,b,c,} , $\tau = \{\Phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$.

Then, $\hat{\Omega}O(X) = \tau$ .It is $\hat{\Omega}$-D_1as well as $\hat{\Omega}$-D_2.

Remark 5.8. i) If X is $\hat{\Omega}$-T_i ,then it is $\hat{\Omega}$-D_i ,i = 0,1,2.

ii) If X is $\hat{\Omega}$-D_i,then it is $\hat{\Omega}$-D_{i-1} ,i = 0,1,2.

Theorem 5.9 Let X be a topological space. Then,

i) X is $\hat{\Omega}$-D_1if and only if $\hat{\Omega}$-D_2.

ii) X is $\hat{\Omega}$-$\mathbf{T_o}$ if and only if $\hat{\Omega}$-D_o.

proof i) Necessity-Suppose that X is $\hat{\Omega}$-D_1 .Then for every pair of distinct points x, y in X, there exists $D_{\hat{\Omega}}$ sets U and V such that $x \in U$ and $y \notin U$ and $y \in V, x \notin V$. By the definition of $D_{\hat{\Omega}}$ set ,U = U_1\$U_2$, V= V_1\$V_2$,where U_i ,V_i $\in$ $\hat{\Omega}$ O(X) i = 1,2 and $U_1 \neq X$ $V_1 \neq X$.Since x $\notin V$ = V_1\$V_2$,there are two cases such that either x $\notin V_1$ or $x \in V_2$.

Case i) x $\notin V_1$.Since y $\notin U$,either y$\notin U_1$ or y $\in U_1$ and y$\in U_2$.Subcase(a) Suppose $y \notin U_1$. Since $x \in U = U_1 \setminus U_2,$ and $y \in V = V_1 \setminus V_2,$ we have $x \in U_1 \setminus (U_2 \cup V_1)$ and $y \in V_1 \setminus (U_1 \cup V_2).$ Also, $(U_1 \setminus (U_2 \cup V_1)) \cap V_1 \setminus (U_1 \cup V_2) = \emptyset.$ BY[7] theorem 4.16,$U_2 \cup V_1$ and $U_1 \cup V_2$ are $\hat{\Omega}$-open sets.

Sub case (b).Suppose $y \in U_1$ and $y \in U_2$. In this case,the disjoint pair of $D_{\hat{\Omega}}$ sets $U_1 \setminus U_2$ and U_2 satisfy our requirement.

Case(ii).Suppose $x \in V_1$ and $x \in V_2$. Here,our required sets are $V_1 \setminus V_2$ and V_2 because $y \in V_1 \setminus V_2$ and $x \in V_2$ and $(V_1 \setminus V_2) \cap V_2 = \emptyset.$ Therefore, (X,τ) is $\hat{\Omega}$-D_2 in the above all cases.

Sufficiency- Follows from the remark 4.8.

(ii) **Necessity**: Follows from the remark 4.8.

Sufficiency : Let (X,τ) be a $\hat{\Omega}$-D_0. Then for each pair of distinct points $x, y \in X,$ there exists $D\,\hat{\Omega}$ set U of X containing x but not y. Suppose $U = U_1 \setminus U_2$ and

$U_1 \neq X$ and $U_1, U_2 \in \hat{\Omega}O(X)$. Then we have either $y \notin U_1$ or $y \in U_1$ and $y \in U_2$. Therefore,we have the following two cases.

Case (i).Suppose $y \notin U_1$. Since $x \in U_1$ and $y \notin U_1, U_1$ is our required $\hat{\Omega}$ -open set.

Case (ii). Suppose $y \in U_1$ and $y \in U_2$. Then, U_2 is a $\hat{\Omega}$ -open set which contains y but not x. Thus in both cases, (X,τ) is $\hat{\Omega}$ - T_0.

Definition 5.10 A point x of a topological space (X,τ) which has X as the only $\hat{\Omega}$ -open set containing x is known as $\hat{\Omega}$ -neat point.

Example5.11 $X = \{a,b,c,d\}$ $\tau = \{\varnothing, \{a\}\{a,b\}, X\}$ .Then $\hat{\Omega}O(X) = \{\varnothing, \{a\}, X\}$. Here the points b, c and d are $\hat{\Omega}$ -neat points.

Theorem 5.12 In a $\hat{\Omega}$ - T_0 space,the following are equivalent.

i) (X,τ) is $\hat{\Omega}$ - D_1.

ii) (X,τ) has no $\hat{\Omega}$ -neat point.

Proof. $(i) \Rightarrow (ii)$ Since it is $\hat{\Omega}$ - D_1, every $x \in X$ is contained in some $D_{\hat{\Omega}}$ -set $U \setminus V, U \neq X, U$ and V are $\hat{\Omega}$ -open sets in X. Since $U \neq X$ and U is $\hat{\Omega}$ -open set containing x and since $x \in X$ is arbitrary, every point of X belongs to some $\hat{\Omega}$ -open set other than X. Therefore,any $x \in X$ is not a $\hat{\Omega}$ -neat point. $(ii) \Rightarrow (i)$ Since X is $\hat{\Omega}$ - T_0, for every disjoint pair of points $x, y \in X$, there exists a $\hat{\Omega}$ -open set U containing x but not y. Since every proper $\hat{\Omega}$ -open set is $D_{\hat{\Omega}}$ -set,we have U is $D_{\hat{\Omega}}$ -set.By hypothesis, y is not a $\hat{\Omega}$ -neat point.Therefore,there exists $\hat{\Omega}$ -open set V containing y such that $V \neq X$. Hence $V \setminus U$ is a $D_{\hat{\Omega}}$ -set containing y but not x .Therefore, (X,τ) is $\delta\omega$ - D_1.

Theorem 5.13 If $f:(X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$ -irresolute and surjective,then $f^{-1}(A)$ *is* $D_{\hat{\Omega}}$ -set in X whenever A is $D_{\hat{\Omega}}$ -set in Y.

Proof. If A is $D_{\hat{\Omega}}$ -set in Y ,then by the definition of $D_{\hat{\Omega}}$ -set,there exists $\hat{\Omega}$ -open sets U and V in Y such that $A = U \setminus V$ and $U \neq Y$. Since f is $\hat{\Omega}$ -irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are $\hat{\Omega}$ -open sets in X. Since $U \neq Y$ and f is surjective, $X = f^{-1}(Y) \neq f^{-1}(U)$. Also, $f^{-1}(A) = f^{-1}(U) \setminus f^{-1}(V)$ .Therefore, $f^{-1}(A)$ is $D_{\hat{\Omega}}$ -set in X.

Theorem 5.14 Let $f:(X,\tau) \to (Y,\sigma)$ be a $\hat{\Omega}$ -irresolute and bijective mapping.If (Y,σ) *is* $\hat{\Omega}$ - D_1 ,then (X,τ) is $\hat{\Omega}$ - D_1.

Proof. Assume that Y is $\hat{\Omega}$ - D_1 space and f is a $\hat{\Omega}$ -irresolute and bijective mapping.Suppose that x and y are any pair of distinct points in X .Since f is injective, $f(x) \neq f(y)$ in a $\hat{\Omega}$ - D_1 space Y .Therefore,there exists $\hat{\Omega}$ -open sets U and V in Y such that $f(x) \in U, f(y) \notin U$ and $f(y) \in V, f(x) \notin V$ .By theorem 5.13, $f^{-1}(U)$ and $f^{-1}(V)$ are $D_{\hat{\Omega}}$ -set in X such that $x \in f^{-1}(U), y \notin f^{-1}(U)$ and $y \in f^{-1}(V), x \notin f^{-1}(V)$ .Therefore, X is $\hat{\Omega}$ - D_1 space.

Definition 5.15 A topological space (X,τ) is said to be $D_{\hat{\Omega}}$ compact if every cover of X by $D_{\hat{\Omega}}$ sets has finite sub cover.

Theorem 5.16 Let $f:(X,\tau) \to (Y,\sigma)$ be a $\hat{\Omega}$ -irresolute and surjective function.If (X,τ) is $D_{\hat{\Omega}}$ compact,then (Y,σ) is $D_{\hat{\Omega}}$ compact.

Proof. Suppose that $\{G_\alpha : \alpha \in J\}$ is any indexed family of $D_{\hat{\Omega}}$ sets in X such that $X = \cup_{\alpha \in J} G_\alpha$. By theorem 5.13,each $f^{-1}(G_\alpha)$ is $D_{\hat{\Omega}}$ set in X .Since f is surjective, $X = f^{-1}(Y) = \cup_{\alpha \in J} f^{-1}(G_\alpha)$ .Since X is $D_{\hat{\Omega}}$ compact,there exist a finite set J_0 such that $X = \cup_{\alpha \in J_0} f^{-1}(G_\alpha)$ .Then

$$Y = f(f^{-1}(Y)) = \cup_{\alpha \in J_0} f(f^{-1}(G_\alpha)) = \cup_{\alpha \in J_0} G_\alpha$$

.Therefore, Y is $D_{\hat{\Omega}}$ compact.

Definition 5.17 A topological space (X,τ) is said to be $D_{\hat{\Omega}}$ connected it can not be expressed as the union of two disjoint non empty $D_{\hat{\Omega}}$ sets in X.

Theorem 5.18 Let $f:(X,\tau)\to(Y,\sigma)$ be a $\hat{\Omega}$-irresolute and surjective function.If (X,τ) is $D_{\hat{\Omega}}$ connected,then (Y,σ) is $D_{\hat{\Omega}}$ connected.

Proof. If $Y=U\cup V$ where U and V are disjoint non empty $D_{\hat{\Omega}}$ sets in Y ,then by theorem 5.13, $f^{-1}(U)$ and $f^{-1}(V)$ are $D_{\hat{\Omega}}$ sets in X .Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are two non empty disjoint $D_{\hat{\Omega}}$ sets whose union is X ,a contradiction.Therefore, Y is $D_{\hat{\Omega}}$ connected.

6. $\hat{\Omega}$-R_0 AND $\hat{\Omega}$-R_1 SPACES.

Definition 6.1 A topological space (X,τ) is $\hat{\Omega}$-R_0 if every $\hat{\Omega}$-open set contains the $\hat{\Omega}cl$ of each of its singletons. That is,for any $\hat{\Omega}$-open set U in X we have $\hat{\Omega}cl(\{x\})\subseteq U$ for every $x\in U$.

Definition 6.2 A topological space (X,τ) is $\hat{\Omega}$-R_1 if for any $x,y\in X$ such that $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$, there exists disjoint $\hat{\Omega}$-open sets U and V such that $\hat{\Omega}cl(\{x\})\subseteq U$ and $\hat{\Omega}cl(\{y\})\subseteq V.$

Example6.3 $X=\{a,b,c,d\}$, $\tau=\{\varnothing,\{a\},\{b,c,d\},X\}$

Then $\hat{\Omega}$-open sets are $P(X)$ Therefore it is $\hat{\Omega}$-R_0 as well as $\hat{\Omega}$-R_1.

Theorem 6.4 A topological space (X,τ) is $\hat{\Omega}$-R_0 if and only if for any $x,y\in X$,

$$\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})\Rightarrow\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{y\})=\varnothing$$

Proof. Necessity-Suppose that (X,τ) is $\hat{\Omega}$-R_0 and $x,y\in X$ are such that $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$. Then there exists some $z\in\hat{\Omega}cl(\{x\})$ and $z\notin\hat{\Omega}cl(\{y\})$ By[7] theorem 5.11,there exists $\hat{\Omega}$-open set U containing z but not y. Since $z\in\hat{\Omega}cl(\{x\})$, by [7] theorem 5.11, $x\in U$ and $x\notin\hat{\Omega}cl(\{y\})$. Therefore, $X\setminus\hat{\Omega}cl(\{y\})$ is a $\hat{\Omega}$-open set containing x. By hypothesis, $\hat{\Omega}cl(\{x\})\subseteq X\setminus\hat{\Omega}cl(\{y\})$ and hence $\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{y\})=\varnothing.$

Sufficiency- Suppose that U is any $\hat{\Omega}$-open subset of X and let $x\in U$ be arbitrary point. If $y\notin U$, then $y\in X\setminus U$ and hence $x\neq y$ .By[7] theorem 5.11, $x\notin\hat{\Omega}cl(\{y\})$. Then $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$. By hypothesis, $\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{y\})=\varnothing.$ Therefore, $y\notin\hat{\Omega}cl(\{x\})$ and hence $\hat{\Omega}cl(\{x\})\subseteq U.$ Therefore, every $\hat{\Omega}$-open set contains $\hat{\Omega}$-closure of it's every singletons.

Theorem 6.5 A topological space (X,τ) is a $\hat{\Omega}$-R_0 iff $ker_{\hat{\Omega}}(\{x\})\neq ker_{\hat{\Omega}}(\{y\})\Rightarrow ker_{\hat{\Omega}}(\{x\})\cap ker_{\hat{\Omega}}(\{y\})=\varnothing$ for every $x,y\in X$.

Proof. Necessity- Suppose that (X,τ) is $\hat{\Omega}$-R_0 and $x,y\in X$ are such that $ker_{\hat{\Omega}}(\{x\})\neq ker_{\hat{\Omega}}(\{y\})$. Then by lemma 3.4, $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$. Assume that $ker_{\hat{\Omega}}(\{x\})\cap ker_{\hat{\Omega}}(\{y\})\neq\varnothing$.Then there exists $z\in X$ such that $z\in ker_{\hat{\Omega}}(\{x\})$ and $z\in ker_{\hat{\Omega}}(\{y\})$. Then by lemma 3.3, $z\in ker_{\hat{\Omega}}(\{x\})$ implies that $x\in\hat{\Omega}cl(\{z\})$ .Therefore, $x\in\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{z\})$ and hence by theorem 6.4, $\hat{\Omega}cl(\{x\})=\hat{\Omega}cl(\{z\})$. Similarly by lemma 3.3, $z\in ker_{\hat{\Omega}}(\{y\})$ implies that $y\in\hat{\Omega}cl\{z\})$ Therefore $y\in\hat{\Omega}cl(\{y\})\cap\hat{\Omega}cl(\{z\})$ and hence by 6.4, $\hat{\Omega}cl(\{y\})=\hat{\Omega}cl(\{z\})$. Therefore, $\hat{\Omega}cl(\{x\})=\hat{\Omega}cl(\{y\})=\hat{\Omega}cl(\{z\})$, a contradiction.Hence, $ker_{\hat{\Omega}}(\{x\})\cap ker_{\hat{\Omega}}(\{y\})=\varnothing.$

Sufficiency-Let $x,y\in X$ be $\hat{\Omega}cl(\{x\})\neq\hat{\Omega}cl(\{y\})$. By lemma 3.4, $ker_{\hat{\Omega}}(\{x\})\neq ker_{\hat{\Omega}(\{y\})}.$

By hypothesis, $ker_{\hat{\Omega}}(\{x\})\cap ker_{\hat{\Omega}}(\{y\})=\varnothing.$

If $\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{y\})\neq\varnothing$, then there exists $z\in X$ such that $z\in\hat{\Omega}cl(\{x\})$ and $z\in\hat{\Omega}cl(\{y\})$. By lemma 3.3, $x\in ker_{\hat{\Omega}}(\{z\})$ and $y\in ker_{\hat{\Omega}}(\{z\})$. Therefore, $x\in ker_{\hat{\Omega}}(\{x\})\cap ker_{\hat{\Omega}}(\{z\})$ and $y\in ker_{\hat{\Omega}}(\{y\})\cap ker_{\hat{\Omega}}(\{z\})$. By hypothesis, $ker_{\hat{\Omega}}(\{x\})=ker_{\hat{\Omega}}(\{y\})=ker_{\hat{\Omega}}(\{z\})$, a contradiction.Therefore $\hat{\Omega}cl(\{x\})\cap\hat{\Omega}cl(\{y\})=\varnothing.$ By theorem 6.4, (X,τ) is a $\hat{\Omega}$-R_0.

Theorem 6.6 In a topological space (X,τ) , the following statements are equivalent.

i) (X,τ) is a $\hat{\Omega}$-R_0 space.

ii)For any non-empty set A and $U \in \hat{\Omega}O(X)$ such that $A \cap U \neq \emptyset,$ there exists $F \in \hat{\Omega}C(X)$ such that $A \cap F \neq \emptyset.$ and $F \subseteq U.$ iii) $U = \bigcup\{F \in \hat{\Omega}C(X) : F \subseteq U\}$ for every $U \in \hat{\Omega}O(X)$. iv) $F = \bigcap\{U \in \hat{\Omega}O(X) : F \subseteq U.\} = ker_{\hat{\Omega}}(F)$ for every $F \in \hat{\Omega}C(X)$.

v) $\hat{\Omega}cl(\{x\}) \subseteq ker_{\hat{\Omega}}(\{x\})$ for any $x \in X$.

Proof. $(i) \Rightarrow (ii)$ Let A be any non-empty set and $U \in \hat{\Omega}O(X)$ be such that $A \cap U \neq \emptyset.$ Choose $x \in A \cap U,$ by hypothesis, $\hat{\Omega}cl(\{x\}) \subseteq U.$ If $F = \hat{\Omega}cl(\{x\}),$ then by[7] remark 5.2, $F \in \hat{\Omega}C(X)$ such that $A \cap F \neq \emptyset$ and $F \subseteq U.$

$(ii) \Rightarrow (iii)$ Suppose that $U \in \hat{\Omega}O(X)$. Then for every $x \in F \in \hat{\Omega}C(X), F \subseteq U, x \in U$ and hence $\bigcup\{F \in \hat{\Omega}C(X) : F \subseteq U.\} \subseteq U.$ On the other hand,suppose that x is an arbitrary point of U .If we define $A = \{x\}$,then by hypothesis, there exists $F \in \hat{\Omega}C(X),$ such that $x \in F$ and $F \subseteq U.$ Therefore, $U \subseteq \bigcup\{F \in \hat{\Omega}C(X) : F \subseteq U.\}$

(iii) $\Rightarrow$ (iv) It is obvious

$(iv) \Rightarrow (v)$ Let $x \in X$ be arbitrary and $y \notin ker_{\hat{\Omega}}(\{x\})$ .Then there exists $U \in \hat{\Omega}O(X)$ containing $\{x\}$ but not $y.$ Therefore $\{y\} \subseteq U^c$ and U^c is a $\hat{\Omega}$-closed set in X .By[7] theorem 5.3, $\hat{\Omega}cl(\{y\}) \subseteq \hat{\Omega}cl(U^c) = U^c$ and hence $\hat{\Omega}cl(\{y\}) \cap U = \emptyset.$ By hypothesis, $\hat{\Omega}cl(\{y\}) = ker_{\hat{\Omega}}(\hat{\Omega}cl(\{y\}))$ and hence $ker_{\hat{\Omega}}(\hat{\Omega}cl(\{y\})) \cap U = \emptyset.$ Since $x \in U, x \notin ker_{\hat{\Omega}}(\hat{\Omega}cl(\{y\})).$ Therefore, there exists $G \in \hat{\Omega}O(X)$ containing $\hat{\Omega}cl(\{y\})$ but not $x.$ Hence $\{x\} \subseteq G^c$ and G^c is a $\hat{\Omega}$-closed set in X .By[7] remark 5.2, $\hat{\Omega}cl(\{x\}) \subseteq \hat{\Omega}cl(G^c) = G^c.$ That is, $\hat{\Omega}cl(\{x\}) \cap G = \emptyset.$ By[7] theorem 5.11, $y \notin \hat{\Omega}cl(\hat{\Omega}cl(\{x\})) = \hat{\Omega}cl(\{x\}).$ $\hat{\Omega}cl(\{x\}) \subseteq ker_{\hat{\Omega}}(\{x\}).$

$(v) \Rightarrow (i)$ Let U be any $\hat{\Omega}$-open set and $x \in U$ be arbitrary point.By v), $\hat{\Omega}cl(\{x\}) \subseteq ker_{\hat{\Omega}}(\{x\}$ and hence $\hat{\Omega}cl(\{x\}) \subseteq U.$ Thus, U contains $\hat{\Omega}$-closure of each of it's singletons.Therefore, (X, τ) is a $\hat{\Omega}$-R_0 space.

Corollary 6.7 In a topological space (X, τ) , the following statements are equivalent.

i) (X, τ) is a $\hat{\Omega}$-R_0 space.

ii) $\hat{\Omega}cl(\{x\}) = ker_{\hat{\Omega}}(\{x\})$ for all $x \in X.$

Proof. $(i) \Rightarrow (ii)$ Suppose that (X, τ) is a $\hat{\Omega}$-R_0 space.By theorem 6.6, $\hat{\Omega}cl(\{x\}) \subseteq ker_{\hat{\Omega}}(\{x\})$ for all $x \in X.$ On the other hand,let $y \in ker_{\hat{\Omega}}(\{x\}),$ then by lemma3.3, $x \in \hat{\Omega}cl(\{y\})$ .Therefore, $x \in \hat{\Omega}cl(\{x\}) \cap \hat{\Omega}cl(\{y\})$. .By theorem 6.4, $\hat{\Omega}cl(\{x\}) = \hat{\Omega}cl(\{y\})$ and hence $y \in \hat{\Omega}cl(\{x\})$ .Therefore, $ker_{\hat{\Omega}}(\{x\}) \subseteq \hat{\Omega}cl(\{x\}).$ $(ii) \Rightarrow (i)$ It follows from theorem 6.6.

Definition 6.8 A topological space (X, τ) is $\hat{\Omega}$-symmetric if for each $x, y \in X, x \in \hat{\Omega}cl(\{y\}$ implies that $y \in \hat{\Omega}cl(\{x\}.$

Theorem 4.9 A topological space (X, τ) is $\hat{\Omega}$-R_0 if and only if it is $\hat{\Omega}$-symmetric.

Proof. Necessity- Suppose that (X, τ) is $\hat{\Omega}$-R_0 and $x \in \hat{\Omega}cl(\{y\}.$ Let U be any $\hat{\Omega}$-open set such that $y \in U.$ By hypothesis, $\hat{\Omega}cl(\{y\} \subseteq U$ and hence $x \in U.$ By[7] theorem 5.11, $y \in \hat{\Omega}cl(\{x\}.$

Sufficiency-Let U be any $\hat{\Omega}$-open set and $x \in U$ be arbitrary point.If $y \notin U,$ then by[7] theorem 5.11, $x \notin \hat{\Omega}cl(\{y\}).$ By hypothesis, $y \notin \hat{\Omega}cl(\{x\}.$ Therefore, $\hat{\Omega}cl(\{x\} \subseteq U$ and hence (X, τ) is $\hat{\Omega}$-$R_0.$

Theorem 6.10 In a topological space (X, τ) , the following statements are equivalent.

i) (X, τ) is a $\hat{\Omega}$-R_0 space.

ii)For any $F \in \hat{\Omega}C(X)$ and $x \in F,$ $ker_{\hat{\Omega}}(\{x\} \subseteq F.$

iii) For $x \in X, ker_{\hat{\Omega}}(\{x\}) \subseteq \hat{\Omega}cl(\{x\})$.

Proof. $(i) \Rightarrow (ii)$ Let $F \in \hat{\Omega}C(X)$ and $x \in F$. By lemma 3.5, and by theorem 6.6 (iv), $ker_{\hat{\Omega}}(\{x\}) \subseteq ker_{\hat{\Omega}}(F) = F$. So, $ker_{\hat{\Omega}}(\{x\} \subseteq F$.

$(ii) \Rightarrow (iii)$ *By [7]* remark 5.2, $\hat{\Omega}cl(\{x\}$ is $\hat{\Omega}$-closed set containing x.By hypothesis, $ker_{\hat{\Omega}}(\{x\} \subseteq \hat{\Omega}cl(\{x\}$.

(iii) $\Rightarrow$ (i)Let x $\in$ $\hat{\Omega}$ cl({y}).By 3.3 $y \in ker_{\hat{\Omega}}(\{x\})$ .Therefore,X is $\hat{\Omega}$-symmetric.By theorem 6.9,X is $\hat{\Omega}$ - R_0 .

Theorem 6.11 (X, τ) is $\hat{\Omega}$-T_1 if and only is it is $\hat{\Omega}$-$\mathbf{T_o}$ and $\hat{\Omega}$-$R_\mathbf{o}$

Proof.Necessity- Suppose that X is $\hat{\Omega}$ -T_1.By remark 3.9,X is $\hat{\Omega}$ -$\mathbf{T_o}$-space.By theorem 4.4,{x} = $\hat{\Omega}$ cl({x}) = $ker_{\hat{\Omega}}$ ({x}) for all x in X. Therefore by theorem 6.10,X is $\hat{\Omega}$ -$R_\mathbf{o}$.

Sufficiency-Let x and y be any two distinct points in X. Since X is $\hat{\Omega}$ -T_o,there exists $U \in \hat{\Omega}O(X)$ such that x $\in$ Ubut not y.By [7] theorem 5.11, $x \notin \hat{\Omega}cl(\{y\})$.By theorem 6.9, $y \notin \hat{\Omega}cl(\{x\}$. Agai [7] by theorem 5.11,there exists $\hat{\Omega}$ -open set V containing y which does not intersect {x}.Therefore, X is $\hat{\Omega}$ -T_1 .

Theorem 6.12 Every $\hat{\Omega}$ - R_1-space is $\hat{\Omega}$ - R_0 .

Proof. Suppose that X is $\hat{\Omega}$ - R_1 and G is any $\hat{\Omega}$ -open set in X.Let x be any point of X.Then for each y $\in$ X\G, x $\neq$ y. By hypothesis, there exists disjoint $\hat{\Omega}$ -open sets U_x and V_y such that $\hat{\Omega}cl(\{x\}) \subset U_x$ and $\hat{\Omega}cl(\{y\}) \subset U_y$ .If $V = \{V_y : y \in X \setminus G\}$,then V is a $\hat{\Omega}$ -open set in X and hence X\V is $\hat{\Omega}$ -cloed set in X containing x ecause $x \notin V_y$ for every y in X\G.By [7] remark 5.2, $\hat{\Omega}cl$ ({x}) is the smallest $\hat{\Omega}$ -closed set containing x.Therefore, $\hat{\Omega}cl(\{x\}) \subseteq X \setminus V \subseteq G$ .Thus, X is $\hat{\Omega}$ -$R_\mathbf{o}$-space.

Theorem 6.13.If $\hat{\Omega}$ -$R_\mathbf{o}$ space is such that for each pair of points x and y with $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$,then there exists $\hat{\Omega}$ -open sets U and V containing x and y respectively.

The next theorem states the characterization of $\hat{\Omega}$ - R_1 - *space.*

Theorem 6.14 A topological space X is $\hat{\Omega}$ - R_1 if and only if for each pair of x and y with $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$,there exists $\hat{\Omega}$ -open sets U and V such that $\hat{\Omega}cl(\{x\}) \subseteq U$ and $\hat{\Omega}cl(\{y\}) \subseteq V$.

Proof.Necessity-Suppose that X is $\hat{\Omega}$ - R_1 and x and y are any two points suchthat $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$ .By lemma 3.4, $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ .By the definition of $\hat{\Omega}$ - R_1 space ,there exists disjoint $\hat{\Omega}$ -open sets U and V such that $\hat{\Omega}cl(\{x\}) \subseteq U$ and $\hat{\Omega}cl(\{y\}) \subseteq V$.

Sufficiency-Let x and y be any two points of X such that $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ .By lemma 3.4, $ker_{\hat{\Omega}}(\{x\}) \neq ker_{\hat{\Omega}}(\{y\})$ .By hypothesis, there exists disjoint $\hat{\Omega}$ -open sets U and V such that $\hat{\Omega}cl(\{x\}) \subseteq U$ and $\hat{\Omega}cl(\{y\}) \subseteq V$.

Theorem 6.15 The following statements are equivalent in a topological space X.

i) X is $\hat{\Omega}$ -T_2-space.

ii) X is $\hat{\Omega}$ - R_1 and $\hat{\Omega}$ -T_1.

iii) X is $\hat{\Omega}$ - R_1 and $\hat{\Omega}$ -T_o.

proof. i) $\Rightarrow$ ii) Suppose that X is $\hat{\Omega}$ -T_2-space.By remark 3.9,it is $\hat{\Omega}$ -T_1-space.Therefore,it is enough to show that $\hat{\Omega}$ - R_1 space.Let x and y are any two distinct p[oints in X such that $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ .By theorem 4.4, $\{x\} = \hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\}) = \{y\}$ .Since X is $\hat{\Omega}$ -T_2 , there exists disjoint $\hat{\Omega}$ -open sets U and V such that $\hat{\Omega}cl(\{x\}) \subseteq U$ and $\hat{\Omega}cl(\{y\}) \subseteq V$ .Hence, X is $\hat{\Omega}$ - R_1 -space.

ii) $\Rightarrow$ iii) It follows from definition.

iii) $\Rightarrow$ i) Suppose that X is $\hat{\Omega}$ - R_1 and $\hat{\Omega}$ -T_o.

Let x and y be any two distinct points in X.By theorem 4.1, $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$ .Since X is $\hat{\Omega}$ - R_1,

there exists disjoint $\hat{\Omega}$ -open sets U and V *such that* $\hat{\Omega}cl(\{x\}) \subseteq U$ *and* $\hat{\Omega}cl(\{y\}) \subseteq V$ .By theorem 6.11,X is $\hat{\Omega}$ -T_1.By theorem 4.4, $x = \hat{\Omega}cl(\{x\})$ and $y = \hat{\Omega}cl(\{y\})$ .Therefore, $\hat{\Omega}$ -T_2 axiom is satisfied and hence X is $\hat{\Omega}$ -T_2.

Theorem 6.16 A topological space (X, τ) *is* $\hat{\Omega}$ - R_1 if and only if for each pair of points x and y with $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{x\})$, there exists two $\hat{\Omega}$ -closed sets P and Q such that $x \in P, y \notin P$ and $y \in Q, x \notin Q$ and $X = P \cup Q$.

Proof. Necessity- Suppose that (X,τ) is $\hat{\Omega}$-R_1-space. Let x and y be any two points such that $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{x\})$. By hypothesis, there exists two disjoint $\hat{\Omega}$-open sets U and V such that $\hat{\Omega}cl(\{x\}) \subseteq U$ and $\hat{\Omega}cl(\{y\}) \subseteq V$. If $P = X \setminus V$ and $Q = X \setminus U$, then P and Q are $\hat{\Omega}$-closed sets such that $x \in P, y \notin P$ and $y \in Q, x \notin Q$. Since every $x \in X$ belongs to either P or $Q, X = P \cup Q$.

Sufficiency- Let x, y be any two points in X such that $\hat{\Omega}cl(\{x\}) \neq \hat{\Omega}cl(\{y\})$. By hypothesis,there exists two $\hat{\Omega}$-closed sets P and Q such that $x \in P, y \notin P$ and $y \in Q, x \notin Q$ and $X = P \cup Q$. If $U = X \setminus Q$ and $V = X \setminus P$, then U and V are disjoint $\hat{\Omega}$-open sets such that $x \in U$ and $y \in V$. Therefore, (X,τ) is $\hat{\Omega}$-T_2-space.By theorem 6.15 it is $\hat{\Omega}$-R_1.

7. CONCLUSION

In this paper some properties of weaker seperation axioms for $\hat{\Omega}$-closed set are derived and their characterizations are proved.

8. REFERENCES

[1] Caldas,M.,Fukutake,T.,Jafari,S.,Noiri,T."Some applications of δ-pre open sets in topological spaces," Bull.Inst.Math.Aca.Sinica,33 ,(3),2005, 267-276.

[2] Caldas,M.,Jafari,S. "On δ D-sets and Associated weak Separation Axioms," Bulletin of the Malaysian Mathematical Sciences Society(second series)25,(2002)173-185.

[3] Davis,A..S."Indexed systems of neighbourhoods for general Topological Spaces,"American Mathematical Monthly,68 (1961).

[4] Dube, K.K."A note on R_0 Topological Spaces," Math.Vesnik,11 ,(1974) 203-208.

[5] Jain ,R.C."The role of regularity open sets in general topology," Ph.D.Thesis,Meerut University,Institute os Advaced Studies,Meerut,India.

[6] Kar ,A.,Bhattacharyya, P."Some weak separation axioms" Bull. Calcutta Math.Soc,82,1990,415-422.

[7] Lellis Thivagar, M., Anbuchelvi ,M."Note on $\hat{\Omega}$ sets in topological spaces" (Accepted).

[8] Lellis Thivagar, M.Anbuchelvi,M. " New spaces and continuity via $\hat{\Omega}$ -closed sets,"(Communicated).

[9] Levin ,N. "Semi open stets and semi continuity in topological spaces",Amer.math.monthly ,70,1963,36-41.

[10] Mashhour, AS.ME Abd El-Monsef and SN El-Deep .(1982) "On pre continuous and weak pre continuous functions," Proc.Math.Phys.Soc.Egypt, 53,47-53.

[11] Nour ,T.M.J. "Contributions to the theory of bitopological spaces ," Ph.D.thesis University of Delhi.1989.

[12] Raychaudhuri ,S.,Mukherjee ,M.N."On δ-pre open sets," Bull.Inst.Math.Aca.Sinica-21,1993, 357-366.

[13] Shanin ,N.A. "On Separation in Topological Spaces," Dokl.Akad.Nbd SSSR 38,(1943),110-113.

[14] Velico,N,V." H closed topologicalspaces," Amer.math.soc.Trans,78,1968,103-18.