Hall Effects on Unsteady Hydromagnetic Flow Induced by a Porous Plate

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ABSTRACT

Hall effects on the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid bounded by an infinite flat porous plate in the presence of a uniform transverse magnetic field has been analyzed. Initially ($t' = 0$) the fluid at infinity moves with uniform velocity U_0 . At time $t' > 0$, the plate suddenly moves with uniform velocity U_0 in the direction of the flow. The velocity field and the shear stress components at the plate are found exactly by using the Laplace transform technique. The solutions are also obtained for small as well as large times. It is observed that the primary velocity decreases whereas the secondary velocity increases with an increase in Hall parameter. The suction parameter is found to accelerate the primary velocity and it has a retarding influence on the secondary velocity. It is also found that the shear stress components decrease with an increase in time.

Key words: Hydromagnetic, Hall currents, inertial oscillations and porous plate

1. INTRODUCTION

In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and the magnetic fields. This phenomenon, well known in the literature, is called the Hall effect. The study of hydromagnetic viscous flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. The unsteady hydromagnetic flow of an incompressible electrically conducting viscous fluid induced by a porous plate is of considerable interest in the technical field due to its frequent occurrence in industrial and technological applications. Katagiri [1] have discussed the effects of Hall current on the boundary layer flow past a semi-infinite flat plate. Pop and Soundalgekar[2] have investigated the effects of Hall current on hydromagnetic flow near a porous plate. The hydromagnetic flow past a porous flat plate with Hall effects has been studied by Gupta [3]. Debnath et al.[4] have discussed the effects of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating system. Hossain [5] has studied the effects of Hall current on unsteady hydromagnetic free convection flow near an infinite vertical porous plate. The effects of Hall current on hydromagnetic free convection flow near an accelerated porous plate has been studied by Hossain and Mohammad [6]. Maji et al. [7] have studied the Hall effects on hydromagnetic flow on an oscillatory porous plate.

The aim of the present paper is to study the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid induced by an infinite porous flat plate in the presence of a uniform transverse magnetic field. Initially, at time $t' = 0$, the fluid at infinity moves with uniform velocity U_0 . At time $t' > 0$, the plate suddenly starts to move with uniform velocity U_0 in the direction of the flow. An exact solution of the governing equation has been obtained by using the Laplace transform technique. The solution for large and small times have also been obtained. It is observed that the primary velocity decreases whereas the secondary velocity increases with an increase in Hall parameter. It is also found that the shear stress components decrease with an increase in time.

2. MATHEMATICAL FORMULATION AND ITS SOLUTION

Consider the flow of a viscous incompressible electrically conducting fluid filling the semi-infinite space $z \ge 0$ in a cartesian coordinate system. Initially, at time $t' = 0$, the fluid flows past an infinitely long porous flat plate with free-stream velocity U_0 along x -axis. A uniform magnetic field B_0 is imposed along z -axis [See Fig.1] and the plate is taken electrically non-conducting. The y -axis is normal to the zx -plane. At time $t' > 0$, the plate suddenly starts to move with same uniform velocity as that of the free stream velocity U_0 .

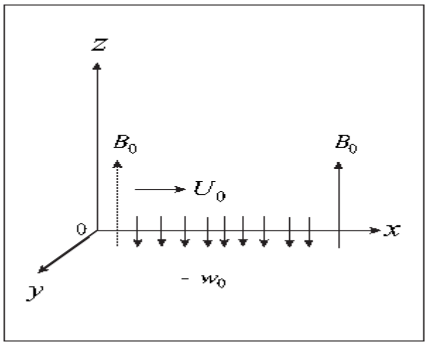

Fig.1: Geometry of the problem

At time $t' = 0$, the velocity components $\hat{u}, \hat{v}, \hat{w}$ in the directions of x , y and z -axes respectively, the equations of motion are

$$-w_0\frac{d\hat{u}}{dz}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{d^2\hat{u}}{dz^2}+\frac{B_0}{\rho}j_y, \quad (1)$$

$$-w_0\frac{d\hat{v}}{dz}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{d^2\hat{v}}{dz^2}-\frac{B_0}{\rho}j_x, \quad (2)$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial z}, \quad (3)$$

where p is the modified pressure including centrifugal force, ρ the density of the fluid and ν the kinematic coefficient of viscosity and $\vec{J}\equiv(J_x,J_y,J_z)$ the current density vector.

The boundary conditions are

$$\hat{u}=0,\ \hat{v}=0,\ \hat{w}=-w_0 \text{ at } z=0,$$

$$\hat{u}\to U_0,\ \hat{v}\to 0 \text{ as } z\to\infty \quad (4)$$

The generalized Ohm's law, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect, is (see Cowling [8])

$$\vec{J}+\frac{\omega_e\tau_e}{B_0}(\vec{J}\times\vec{B})=\sigma(\vec{E}+\vec{q}\times\vec{B}), \quad (5)$$

where $\vec{B}$ is the magnetic induction vector, $\vec{E}$ the electric field vector, ω_e the cyclotron frequency, τ_e the collision time of electron and σ the electrical conductivity.

We shall assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is generally very small for partially ionized gases. The solenoidal relation $\nabla.\vec{B}=0$ for the magnetic field gives $B_z=B_0=$ constant everywhere in the fluid where $\vec{B}\equiv(B_x,B_y,B_z)$. The equation of conservation of the charge $\nabla\cdot\vec{J}=0$ gives $J_z=$ constant. This constant is zero since $J_z=0$ at the plate which is electrically non-conducting. Thus $J_z=0$ everywhere in the flow. Since the induced magnetic field is neglected, the Maxwell's equation $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ becomes $\nabla\times\vec{E}=0$ which in turn gives $\frac{\partial E_x}{\partial z}=0$ and $\frac{\partial E_y}{\partial z}=0$. This implies that $E_x=$ constant and $E_y=$ constant everywhere in the flow.

In view of the above assumption, equation (5) gives

$$J_x+mJ_y=\sigma(E_x+\hat{v}B_0), \quad (6)$$

$$J_y-mJ_x=\sigma(E_y-\hat{u}B_0), \quad (7)$$

where $m=\omega_e\tau_e$ is the Hall parameter. Since the magnetic field is uniform in the free stream so that there is no current and hence, we have

$$J_x\to 0,\ J_y\to 0 \text{ as } z\to\infty. \quad (8)$$

On the use of (8) and the condition at infinity, equations (6) and (7) give

$$E_x=0,\ E_y=\sigma U_0B_0, \quad (9)$$

everywhere in the flow.

Substituting the above values of E_x and E_y in equations (6) and (7) and solving for J_x and J_y, we get

$$J_x=\frac{\sigma B_0}{1+m^2}\left[\hat{v}+m\left(\hat{u}-U_0\right)\right], \quad (10)$$

$$J_y=-\frac{\sigma B_0}{1+m^2}\left[\left(\hat{u}-U_0\right)-m\hat{v}\right]. \quad (11)$$

Under usual boundary layer approximations and on the use of (10) and (11), equations (1) and (2) become

$$-w_0\frac{d\hat{u}}{dz}=\nu\frac{d^2\hat{u}}{dz^2}-\frac{\sigma B_0^2}{\rho(1+m^2)}[(\hat{u}-U_0)-m\hat{v}], \quad (12)$$

$$-w_0\frac{d\hat{v}}{dz}=\nu\frac{d^2\hat{v}}{dz^2}-\frac{\sigma B_0^2}{\rho(1+m^2)}[\hat{v}+m(\hat{u}-U_0)]. \quad (13)$$

Introducing the non-dimensional variables

$$\eta=\frac{U_0z}{\nu},\ \hat{F}=(\hat{u}+i\hat{v})/U_0,\ i=\sqrt{-1}, \quad (14)$$

equations (12) and (13) become

$$-S\frac{\partial\hat{F}}{\partial\eta}=\frac{\partial^2\hat{F}}{\partial\eta^2}-\frac{M^2(1+im)}{1+m^2}(\hat{F}-1), \quad (15)$$

where $M^2=\frac{\sigma B_0^2\nu}{\rho U_0^2}$ is the magnetic parameter and $S=\frac{w_0}{U_0}$ the suction parameter .

The corresponding boundary conditions for $\hat{F}(\eta)$ are

$$\hat{F}=0 \text{ at } \eta=0 \text{ and } \hat{F}\to 1 \text{ as } \eta\to\infty. \quad (16)$$

The solution of (15) subject to the boundary conditions (16) can be obtained as

$$\frac{\hat{u}}{U_0}=1-e^{-\alpha\eta}\cos\beta\eta, \quad (17)$$

$$\frac{\hat{v}}{U_0}=e^{-\alpha\eta}\sin\beta\eta, \quad (18)$$

where

$$\alpha=\frac{S}{2}$$

$$+\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)^2+\left(\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}+\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$\beta = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)^2+\left(\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}-\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)\right]^{\frac{1}{2}}. \quad (19)$$

Solutions given by (17) and (18) is valid for both suction $(S > 0)$ and blowing $(S < 0)$ at the plate. Solutions given by equations (17) and (18) are identical with the equations (36) and (37) of Gupta [3].

At time $t' > 0$, the plate suddenly starts to move with uniform velocity U_0 along x-axis in the direction of flow. Assuming the velocity components $(u, v, -w_0)$ along the coordinate axes, we have the following equation of motion

$$\frac{\partial u}{\partial t'} - w_0\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[(u-U_0)-mv\right], \quad (20)$$

$$\frac{\partial v}{\partial t'} - w_0\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[v+m(u-U_0)\right], \quad (21)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}. \quad (22)$$

The initials and boundary conditions are

$$u = \hat{u},\ v = \hat{v} \text{ at } t' = 0 \text{ for } z \ge 0,$$
$$u = U_0,\ v = 0 \text{ at } z = 0,\ t' > 0, \quad (23)$$
$$u \to U_0,\ v \to 0 \text{ as } z \to \infty,\ t' > 0.$$

It is observed from equation (22) that p is independent of z. Further, equations (20) and (21) together with conditions $u \to U_0$ and $v \to 0$ as $z \to \infty$ yield

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \text{ and } -\frac{1}{\rho}\frac{\partial p}{\partial y} = 0. \quad (24)$$

On the use of (24), equations (20) and (21) become

$$\frac{\partial u}{\partial t'} - w_0\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}[(u-U_0)-mv], \quad (25)$$

$$\frac{\partial v}{\partial t'} - w_0\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}[v+m(u-U_0)]. \quad (26)$$

Equations (25) and (26) can be written in combined form as

$$\frac{\partial F}{\partial t'} - w_0\frac{\partial F}{\partial z} = \nu\frac{\partial^2 F}{\partial z^2} - \frac{\sigma(1+im)U_0B_0^2}{\rho(1+m^2)}F, \quad (27)$$

where

$$F = (u+iv)/U_0 - 1. \quad (28)$$

On the use of (14) together with $t = \frac{U_0^2 t'}{\nu}$, equation (27) yields

$$\frac{\partial F}{\partial t} - S\frac{\partial F}{\partial \eta} = \frac{\partial^2 F}{\partial \eta^2} - \frac{M^2(1+im)}{1+m^2}F. \quad (29)$$

The corresponding initial and the boundary conditions for $F(\eta,\tau)$ are

$$F(\eta,0) = \hat{F}(\eta),\ \eta \ge 0,$$
$$F(0,t) = 0 \text{ for } t > 0,\ F(\infty,t) = 0 \text{ for } t > 0, \quad (31)$$

where $\hat{F}(\eta)$ is given by (14).

To solve the equation (29), we assume

$$F(\eta,t) = H(\eta,t)e^{-\lambda t}, \quad (32)$$

where

$$\lambda = \frac{M^2(1+im)}{1+m^2}. \quad (33)$$

Using (32), the equation (29) becomes

$$\frac{\partial H}{\partial t} - S\frac{\partial H}{\partial \eta} = \frac{\partial^2 H}{\partial \eta^2}, \quad (34)$$

With the initial and the boundary conditions

$$H(\eta,0) = \hat{F}(\eta) \text{ for } \eta \ge 0, \quad (35)$$
$$H(0,t) = 0 \text{ for } t > 0,\ H(\infty,t) = 0 \text{ for } t > 0. \quad (36)$$

Taking the Laplace transform of the equation (34), we have

$$\frac{d^2\bar{H}}{d\eta^2} + S\frac{d\bar{H}}{d\eta} - S\bar{H} = e^{-(\alpha+i\beta)\eta}, \quad (37)$$

where

$$\bar{H}(\eta,s) = \int_0^\infty H(\eta,t)e^{-st}dt. \quad (38)$$

The boundary condition for $\bar{H}(\eta,s)$ are

$$\bar{H}(0,s) = 0 \text{ for } t > 0 \text{ and } \bar{H}(\infty,s) = 0 \text{ for } t > 0.$$
(39)

The solution of the equation (37) subject to the boundary conditions (39) is

$$\bar{H}(\eta,s) = \frac{e^{-\left(\frac{S}{2}+\sqrt{\frac{S^2}{4}+s}\right)\eta}}{s-\lambda} - \frac{e^{-(\alpha+i\beta)\eta}}{s-\lambda}. \quad (40)$$

The inverse Laplace transform of the equation (40) and on using (32) and (28), yields

$$\frac{u+iv}{U_0} = 1 - e^{-(\alpha+i\beta)\eta}$$

$$+\frac{1}{2}e^{-\frac{S}{2}\eta}\left[e^{(\alpha_1+i\beta_1)\eta}\operatorname{erfc}\left\{\frac{\eta}{2\sqrt{t}}+(\alpha_1+i\beta_1)\sqrt{t}\right\}\right.$$

$$\left.+e^{-(\alpha_1+i\beta_1)\eta}\operatorname{erfc}\left\{\frac{\eta}{2\sqrt{t}}-(\alpha_1+i\beta_1)\sqrt{t}\right\}\right], \qquad (41)$$

where

$$\alpha_1,\,\beta_1=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)^2+\left(\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}\pm\left(\frac{S^2}{4}+\frac{M^2}{1+m^2}\right)\right]^{\frac{1}{2}}. \qquad (42)$$

The solution given by (41) is valid for both suction $(S>0)$ and blowing $(S<0)$ at the plate.

3. RESULTS AND DISCUSSION

To study the flow situations due to the impulsive start of the porous plate for different values of Hall parameter m, suction parameter S and time t, the velocity are examined numerically and plotted in Figs. 2-7. It is seen from Figs.2 and 3 that the primary velocity $\frac{u}{U_0}$ decreases whereas the secondary velocity $\frac{v}{U_0}$ increases with an increase in Hall parameter m. This phenomenon is clearly supported by the physical reality. It is revealed from Figs.4 and 5 that the primary velocity $\frac{u}{U_0}$ increases whereas the secondary velocity $\frac{v}{U_0}$ decreases with an increase in suction parameter S. Figs.6 and 7 show that the primary velocity $\frac{u}{U_0}$ increases while the secondary velocity $\frac{v}{U_0}$ decreases with an increase in time t.

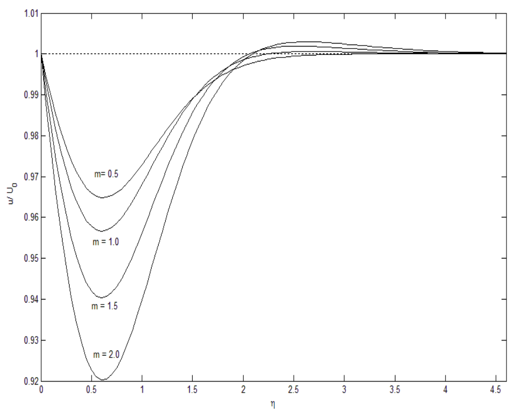

Fig.2: Primary velocity $\frac{v}{U_0}$ for m when $M^2=5$, $S=1$ and $\tau=0.2$

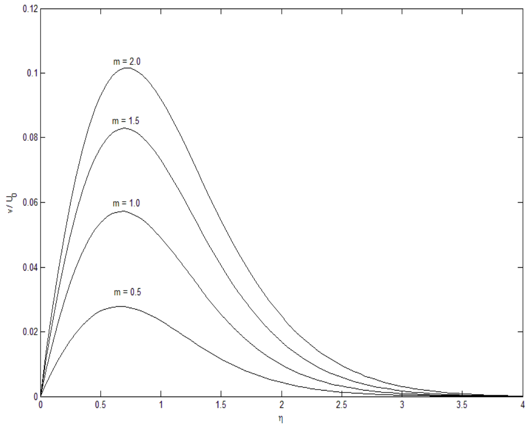

Fig.3: Secondary velocity $\frac{v}{U_0}$ for m when $M^2=5$, $S=1$ and $\tau=0.2$

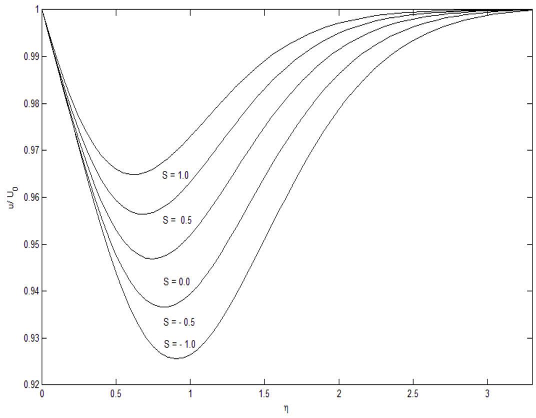

Fig.4: Primary velocity $\frac{u}{U_0}$ for S when $M^2=5$, $m=0.5$ and $\tau=0.2$

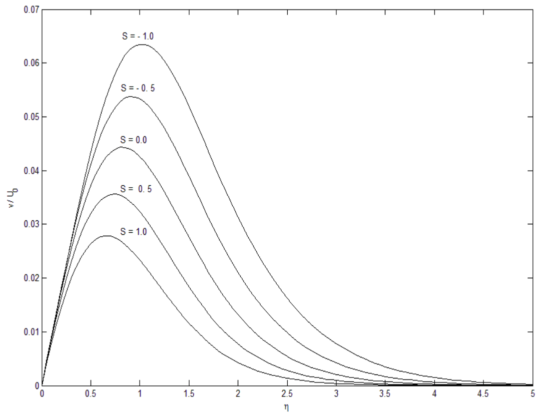

Fig.5: Secondary velocity $\frac{v}{U_0}$ for S when $M^2=5$, $m=0.5$ and $\tau=0.2$

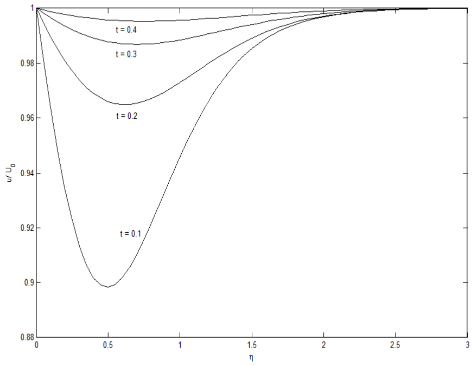

Fig.6: Primary velocity $\frac{u}{U_0}$ for τ when $M^2=5$, $m=0.5$ and $S=1$

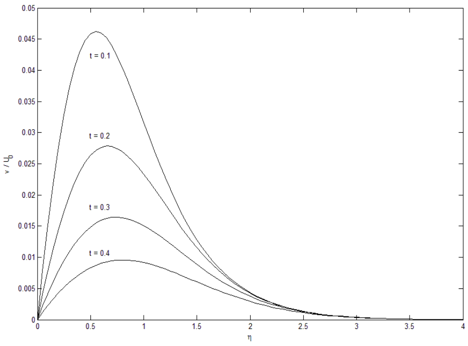

Fig.7: Secondary velocity $\frac{v}{U_0}$ for τ when $M^2=5$, $m=0.5$ and $S=1$

Now, we shall consider the case when t is small which correspond to large $s(\gg 1)$. For small times, method used by Carslaw and Jaegar [9] is used because it converges rapidly for small times. In this case, the inverse Laplace transform of the equation (40) yields

$$H(\eta,t)=e^{-\frac{S}{2}\eta-\frac{1}{4}S^2t}\sum_{n=0}^{\infty}\left(\frac{S^2}{4}+\lambda\right)^n(4t)^n i^{2n}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right)$$

$$-e^{-(\alpha+i\beta)\eta+\lambda t} \qquad (43)$$

On the use of (32), the equation (43) becomes

$$F(\eta,t)=e^{-\frac{S}{2}\eta-\frac{S^2}{4}t-\lambda t}\sum_{n=0}^{\infty}\left(\frac{S^2}{4}+\lambda\right)^n(4t)^n j^{2n}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right)$$

$$-e^{-(\alpha+i\beta)\eta},$$

where $j^n\operatorname{erfc}(.)$ denotes the repeated integrals of the complementary error function given by

$$j^n\operatorname{erfc}(x)=\int_x^{\infty}j^{n-1}\operatorname{erfc}(\xi)\,d\xi,\ n=0,1,2,\cdots,$$

$$j^0\operatorname{erfc}(x)=\operatorname{erfc}(x),\ j^{-1}\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}e^{-x^2}. \qquad (44)$$

Using (28) we have

$$\frac{u}{U_0}=1-e^{-\alpha\eta}\cos\beta\eta$$

$$+e^{-\left(\frac{S}{2}\eta+\alpha^*t\right)}\left[A(\eta,t)\cos\beta^*t+B(\eta,t)\sin\beta^*t\right], \qquad (45)$$

$$\frac{v}{U_0}=e^{-\alpha\eta}\sin\beta\eta$$

$$+e^{-\left(\frac{S}{2}\eta+\beta^*t\right)}\left[B(\eta,t)\cos\beta^*t-A(\eta,t)\sin\beta^*t\right], \qquad (46)$$

where

$$A(\eta,t)=T_0+\alpha^*(4t)T_2+\left(\alpha^{*2}-\beta^{*2}\right)(4t)^2T_4$$

$$+\left(\alpha^{*3}-3\alpha^*\beta^{*2}\right)(4t)^3T_6+\cdots,$$

$$B(\eta,t)=\beta^*(4t)T_2+2\alpha^*\beta^*(4t)^2T_4 \qquad (47)$$

$$+\left(3\alpha^{*2}\beta^*-\beta^{*3}\right)(4t)^3T_6+\cdots,$$

$$\alpha^*=\frac{S^2}{4}+\frac{M^2}{1+m^2},\ \beta^*=\frac{mM^2}{1+m^2}.$$

The above equations show that the Hall effects become important only when terms of order t is taken into account.

For small values of time, we have drawn the velocity components $\frac{u}{U_0}$ and $\frac{v}{U_0}$ from the exact solution given by equation (41) and the series solution given by equations (45) and (46) in Figs.8 and 9. It is seen that the series solution given by (45) and (46) converge more rapidly than the exact solution given by (41) for small times. Hence, we conclude that for small times, the numerical values of the velocities can be evaluated from the equations (45) and (46) instead of equation (41).

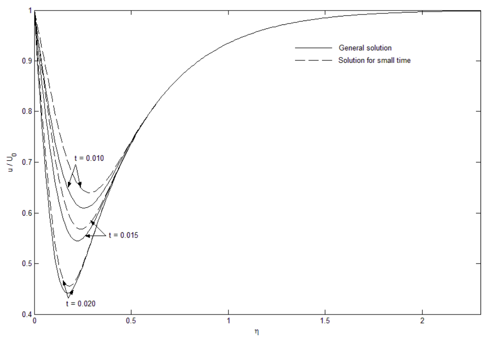

Fig.8: $\frac{u}{U_0}$ for general solution and solution for small times when $M^2=5$, $m=0.5$ and $S=1$

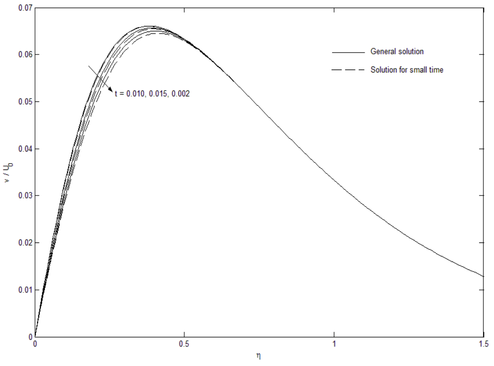

Fig.9: $\frac{v}{U_0}$ for general solution and solution for small times when $M^2 = 5$, $m = 0.5$ and $S = 1$

The shear stresses at the plate $\eta = 0$ due to the primary and secondary flow are given by [from equation (38)]

$$\tau_x + i\tau_y = \frac{u'(0) + iv'(0)}{U_0}$$

$$= \alpha + i\beta$$

$$-\left[\frac{S}{2} + (\alpha_1 + i\beta_1)\,\mathrm{erf}\,(\alpha_1 + i\beta_1)\sqrt{t} + \frac{1}{\sqrt{\pi t}} e^{-(\alpha_1 + i\beta_1)^2 t}\right]$$

(48)

The numerical results of the shear stress components τ_x and τ_y are shown in Figs.10 and 11 against Hall parameter m for different values of t with $M^2 = 5$ and $S = 1$. It is seen that the shear stress components τ_x and τ_y decrease with an increase in t. On the other hand, with an increase in Hall parameter m both τ_x and τ_y increase.

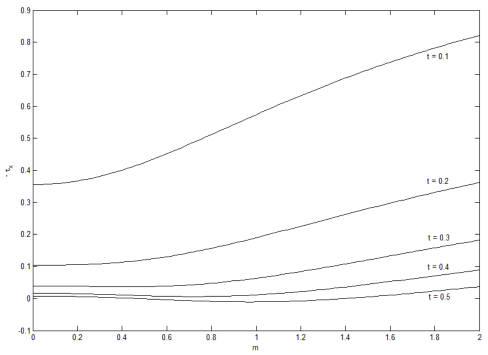

Fig.10: Shear stress τ_x for different time τ when $M^2 = 5$ and $S = 1$

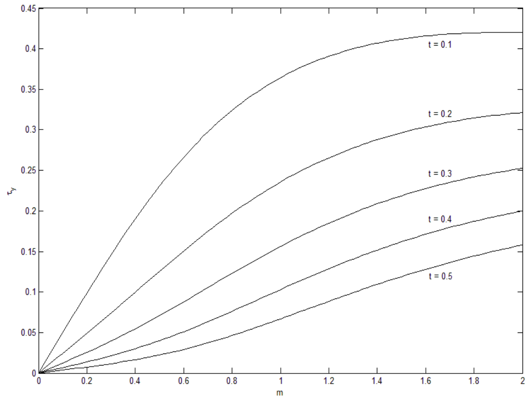

Fig.11: Shear stress τ_y for different time τ when $M^2 = 5$ and $S = 1$

By letting $t \to \infty$, the steady state shear stress components at the plate $\eta = 0$ are obtained as

$$\tau_x + i\tau_y = (\alpha + i\beta) - \frac{S}{2} + (\alpha_1 + i\beta_1). \qquad (49)$$

Estimation of the time which elapses from the starting of impulsive motion of the plate till the steady state is reached can be obtained as follows. It is observed from (48) that the steady state is reached after time t_0 when erf $(a + ib)\sqrt{t} = 1$. Since erf $(a + ib) = 1$ when $|a + ib|\sqrt{t_0} = 2$, it follows that

$$t_0 = 4\left[\left(\frac{S^2}{4} + \frac{M^2}{1+m^2}\right)^2 + \left(\frac{mM^2}{1+m^2}\right)^2\right]^{-\frac{1}{2}}. \qquad (50)$$

The above equation shows that for fixed m, the time t_0 to attain the steady state decreases with an increase in either suction parameter S or the magnetic parameter M^2. The values of the time t_0 are entered in the Table 1 for several values of Hall parameter m and suction parameter S for $M^2 = 5$. It is seen that for fixed M^2 and S, the time t_0 increases with an increase in Hall parameter m. This means that the system with Hall currents takes more time to reach the steady state than without Hall current.

Table 1. Values of time t_0 when $M^2 = 5$

m	$S = 0.0$	$S = 0.5$	$S = 1$
0.0	0.80000	0.79012	0.76190
0.5	0.89442	0.88337	0.85160
1.0	1.13137	1.11731	1.07628
1.5	1.44222	1.42171	1.37005

For small time, the shear stresses at the plate $\eta = 0$ due to primary and the secondary flows are given by

$$\tau_x = \frac{u'(0,t)}{U_0} = \alpha - \frac{1}{2} e^{-\alpha^* t} \Big[P(0,t)\cos\beta^* t + Q(0,t)\sin\beta^* t \Big], \tag{51}$$

$$\tau_y = \frac{v'(0,t)}{U_0} = \beta - \frac{1}{2} e^{-\alpha^* t} \Big[Q(0,t)\cos\beta^* t - P(0,t)\sin\beta^* t \Big], \tag{52}$$

where

$$P(\eta,t) = \left(ST_0 + \frac{Y_{-1}}{\sqrt{t}} \right) + \alpha^* (4t) \left(ST_2 + \frac{Y_1}{\sqrt{t}} \right)$$

$$+ \left(\alpha^{*2} - \beta^{*2} \right) (4t)^2 \left(ST_4 + \frac{Y_3}{\sqrt{t}} \right)$$

$$+ \left(\alpha^{*3} - 3\alpha^* \beta^{*2} \right) (4t)^3 \left(ST_6 + \frac{Y_5}{\sqrt{t}} \right) + \cdots, \tag{53}$$

$$Q(\eta,t) = \beta^* (4t) \left(ST_2 + \frac{Y_1}{\sqrt{t}} \right) + 2\alpha^* \beta^* (4t)^2 \left(ST_4 + \frac{Y_3}{\sqrt{t}} \right)$$

$$+ \left(3\alpha^{*2}\beta^* - \beta^{*3} \right) (4t)^3 \left(ST_6 + Y_5/\sqrt{t} \right) + \cdots, \tag{54}$$

with

$$\frac{dT_{2n}}{d\eta} = -\frac{Y_{2n-1}}{2\sqrt{t}}, \text{ where } Y_{2n-1} = j^{2n-1}\mathrm{erfc}\left(\frac{\eta}{2\sqrt{t}} \right). \tag{55}$$

Table 2. Shear stress due to primary flow when $M^2 = 5$, $S = 1$

	$-\tau_x$ (For General solution)			$-\tau_x$ (Solution for small time)		
$m \backslash t$	0.005	0.010	0.015	0.005	0.010	0.015
0.0	5.896093	3.644242	2.673381	5.896093	3.644238	2.673358
0.5	6.032860	3.765285	2.782767	6.032872	3.765358	2.782959
1.0	6.290388	3.998794	2.998242	6.290404	3.998886	2.998493
1.5	6.515985	4.208713	3.196242	6.515995	4.208762	3.196378

Table 3. Shear stress due to secondary flow when $M^2 = 5$, $S = 1$

	τ_y (For General solution)			τ_y (Solution for small time)		
$m \backslash t$	0.005	0.010	0.015	0.005	0.010	0.015
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.5	0.393570	0.361540	0.337486	0.393615	0.361790	0.338174
1.0	0.595888	0.555407	0.524776	0.595908	0.555517	0.525087
1.5	0.660252	0.622620	0.594005	0.660258	0.622650	0.594089

For small time, the numerical values of the shear stress components calculated from equations (48), (51) and (52) are entered in Tables 2 and 3 for different values of m and t. It is observed that for small times the shear stresses calculated from the equations (51) and (52) are greater than that calculated from equation (48). Hence, for small times, shear stresses should be calculated from equations (51) and (52) instead of the equation (48).

We may now write down the asymptotic form of the solution (38) by using asymptotic expansion of $\mathrm{erfc}(z)$ with complex argument in the form of

$$\mathrm{erfc}(z) \approx \frac{\exp(-z^2)}{z\sqrt{\pi}} \text{ as } |z| \to \infty, \tag{56}$$

together with the fact that $\mathrm{erfc}(-z) = 2 - \mathrm{erfc}(z)$. Therefore, the final solution is

$$\frac{u+iv}{U_0} = 1 + \frac{1}{2} e^{-\frac{S}{2}\eta} \left[e^{(\alpha_1 + i\beta_1)\eta} \mathrm{erfc}\left\{ (\alpha_1 + i\beta_1)\sqrt{t} + \frac{\eta}{2\sqrt{t}} \right\} \right.$$

$$\left. - e^{-(\alpha_1 + i\beta_1)\eta} \mathrm{erfc}\left\{ (\alpha_1 + i\beta_1)\sqrt{t} - \frac{\eta}{2\sqrt{t}} \right\} \right]. \tag{57}$$

Additionally, if $\eta \,\square\, 2\sqrt{t}$, $t \,\square\, 1$ then the solution becomes

$$\frac{u}{U_0}=1+\frac{e^{-\frac{S}{2}\eta-\left(\alpha_1^2-\beta_1^2\right)t}}{(\alpha_1^2+\beta_1^2)\sqrt{\pi t}}$$
$$\times[\left(\alpha_1\cos 2\alpha_1\beta_1 t-\beta_1\sin 2\alpha_1\beta_1 t\right)\sinh\alpha_1\eta\cos\beta_1\eta$$
$$+\left(\beta_1\cos 2\alpha_1\beta_1 t+\alpha_1\sin 2\alpha_1\beta_1 t\right)\cosh\alpha_1\eta\sin\beta_1\eta], \quad (58)$$

$$\frac{v}{U_0}=\frac{e^{-\frac{S}{2}\eta-(\alpha_1^2-\beta_1^2)t}}{(\alpha_1^2+\beta_1^2)\sqrt{\pi t}}$$
$$\times[\left(\alpha_1\cos 2\alpha_1\beta_1 t-\beta_1\sin 2\alpha_1\beta_1 t\right)\cosh\alpha_1\eta\sin\beta_1\eta$$
$$-\left(\beta_1\cos 2\alpha_1\beta_1 t+\alpha_1\sin 2\alpha_1\beta_1 t\right)\sinh\alpha_1\eta\cos\beta_1\eta]. \quad (59)$$

Equations (58) and (59) show the existence of inertial oscillations. The frequency of these oscillations is

$$\omega=2\alpha_1\beta_1=\frac{mM^2}{1+m^2}, \quad (60)$$

It is observed from equations (58) and (59) that the Hall parameter not only induced a cross flow but also occurs inertial oscillations of the fluid velocity. The frequency of these oscillations increases with an increase in M^2. On the other hand, with an increase m, the fequency ω first increases, reaches a maximum at $m=1$ and then decreases.

5. CONCLUSION

An analysis is made on the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid bounded by an infinitely long flat porous plate in the presence of a uniform transverse magnetic field. An analytical solution is obtained by using the Laplace Transform technique. It is found that Hall current has a retarding influence on the primary velocity and it accelerates the secondary velocity. It is also found that the primary velocity increases whereas the secondary velocity decreases with an increase in suction parameter. It is seen that the shear stress components decrease with an increase in either Hall parameter or time.

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