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Design of IIR Digital Highpass Butterworth Filter using Analog to Digital Mapping Technique

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ABSTRACT
Infinite Impulse Response (IIR) filter is of recursive type filter. The present output sample of an IIR filter depends on the present input samples, past input samples and past output samples. There are a number of techniques available to determine the digital IIR Filters. This paper is based on the computer based approach to design the digital IIR filter along with the calculation of filter coefficients using the analog to digital mapping technique. The program based on the proposed algorithm is simulated in Matlab and found the result is satisfying.

Keywords: IIR filter, Digital filters, Butterworth filter, High pass filter, coefficient, analog to digital mapping

1. INTRODUCTION
Filters play a very important role in signal processing. In this paper the IIR digital filter is discussed which is very essential in Digital Signal Processing (DSP).In DSP, there are two type of systems. The first type of system performs signal filtering in time domain. They are known as Digital filters [1][9]. Another type of system provide signal representation in frequency domain. They are known to as Spectrum analyzer [1][3]. The term IIR comes from infinite impulse response meaning that the impulse response of filter is of infinite duration whereas the impulse response of a FIR (Finite Impulse Response) filter is of finite duration.
IIR filter processes certain properties such as width of the pass-band, width of the stop-band, maximum allowable ripple at pass-band and maximum allowable ripple at stop-band[1][2][11]. A preferred design of IIR filter can be done with help of those properties [4]. There are various process to design IIR Digital filter. Basically IIR digital filter is designed from an analog filter. Then using analog to digital mapping technique or frequency transformation an IIR Digital filter can be designed suitably[2][4].
An analog filter generally constructed by resistors, capacitors and op amps to produce the required filtering effect. Those filter circuits can be widely used in reduction of noise, video signal enhancement, graphic equalizers in hi-fi systems, and many other areas. Those analog filters are actually designed as per specific requirement for producing satisfying filtering output. Digital filters perform many filtering works by replacing the analog filters. The digital filters have many features for which we can replace the analog filters and use the digital filters and the features are high accuracy and reliability, small physical size and reduced sensitivity to component tolerances or drift[5][13].
Analog filter can be designed by either active element or passive element. When the design of an active filter is completed, the cutoff frequency can be calculated. When the cutoff frequency is obtained, the pass-band or stop-band allowable frequency can easily be obtained. Now the IIR digital filter can be implemented directly, such as for example if a analog Low Pass Filter is designed and the designer wants to design a High Pass Filter, the designer can apply mapping technique and frequency transformation to transform from Low Pass Filter to High Pass Filter or the designer can design from analog High Pass Filter to High Pass Filter using frequency transformation technique.[2]
There are two transformation techniques available that are widely used are- Impulse invariant method and bilinear transformation [2][4].

2. DIGITAL IIR FILTER DESIGN
An IIR filter, as discussed in the introductory part, can be designed from active or passive element. When a voltage source is applied across the input terminal, the filter becomes an active filter. Now there are many type of IIR filters such as Butterworth filter, Chebyshev filter, Elliptic filter etc. A Butterworth filter designed by Opamp is shown in fig.1[5]

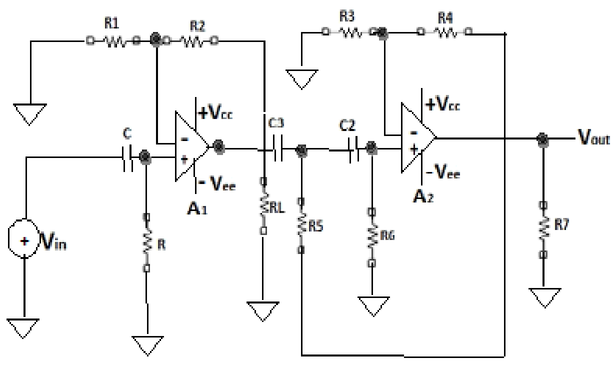

Fig.1 3rd Order IIR Highpass Butterworth Filter

In signal processing, the order of the FIR filter is always higher than that of the IIR filter when we basically view the same magnitude response. So in that case, the group delay of FIR filter is large enough compared to IIR filter. So, in order to process a signal processing with high-speed and with high-precision, it is very important to design the IIR filters [12].

Now from the Fig.1 [5], a digital IIR Filter can be designed. The left hand section is a 1st Order section and the right hand section is a 2nd order section. This is because the general available designs for filters are the 1st Order filter and the 2nd Order filter. Therefore by means of cascading those two a 3rd Order filter can be designed [5].

A digital filter means its transfer function must be in the z-plane, i.e. H(z). The impulse response h (n) for a realizable filter is,

$$h(n) = 0 \quad \text{for } n \leq 0 \qquad \dots\dots(1.1)$$

A stable filter must satisfy the condition,

$$\sum_{n=0}^{\infty} |\, h(n) \,| < \infty \qquad \dots\dots(1.2)$$

Now the generalized transfer function [1][2][10] of an IIR Digital filter is,

$$H(z) = \frac{\sum_{n=0}^{M} b(n) z^{-n}}{1 + \sum_{n=1}^{N} a(n) z^{-n}} \qquad \dots\dots(1.3)$$

$$= \frac{B(z)}{A(z)} = \frac{b(0) + b(1)z^{-1} + b(2)z^{-2} + \dots\dots + b(M)z^{-M}}{1 + a(1)z^{-1} + a(2)z^{-2} + \dots\dots\dots\dots + a(N)z^{-N}} \qquad \dots\dots(1.3a)$$

Where,

b(n) = Numerator coefficient of the filter
a(n) = Denominator coefficient of the filter

IIR filters have many advantages as follows[1]:-

- It requires less number of arithmetic operations.
- There are shorter time delays in these filters.
- IIR Filters have similarities with analog filters.
- These filters depend not only upon the input but also upon previous output values.
- They are more susceptible to noises.

A suitable digital filter can design by calculating the numerator coefficient and denominator coefficient. There are various techniques available for the design and calculation of those coefficients. The algorithm proposed for this purpose is smartly eligible to determine the filter coefficients and hence helps to design the IIR Digital filter with correct specification provided to it. This algorithm also provide with the transfer function in the digital domain. The flowchart of the proposed algorithm is shown in fig.2

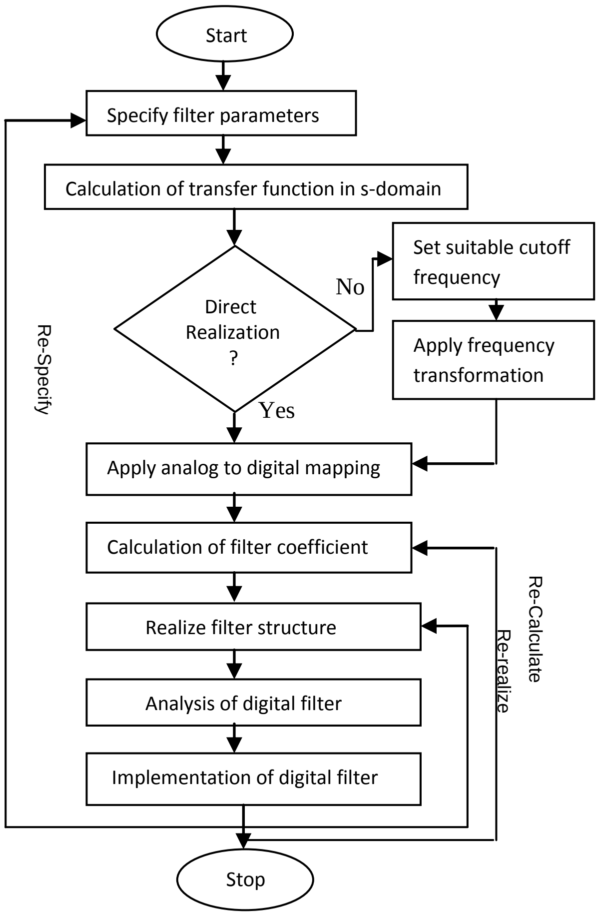

Fig.2 Proposed algorithm

Now, with help of this algorithm, the designer can specify the necessary parameters for a filter directly or the transformation from Low pass filter to High pass filter or Low pass filter to Low pass filter or High pass filter to Low pass filter or High pass to High pass filter are also allowed.

In the proposed algorithm, the direct and indirect realization of a digital filter can be performed. Let, the parameters of an analog filter is specified. Then after calculation the transfer function of the analog filter i.e. in s-domain is obtained. Then it's the designer choice whether he is interested in direct realization or in indirect realization. If the realization is the indirect one, the suitable cutoff frequency must be specified and on that the frequency transformation will be done. For direct realization, the analog to digital mapping will be performed on the calculated transfer function in s-domain so that it produces the final transfer function in z-domain which indicates the transfer function of a digital filter. Then by calculation, the filter coefficient can be determined .The stability of the digital filter is determined by the pole-zero plot. So by applying the algorithm, the digital stable filter can be determined.

3. ANALOG TO DIGITAL MAPPING

Analog to Digital mapping means, simply, transformation of the transfer function of a specified circuit from the s-domain to z-domain. If a filter i.e. its transfer function is designed in s-domain, it is called the analog filter. After mapping, when the transfer function is finally designed in z-domain, this is called a digital filter.

Now to perform mapping, let we consider the impulse response of the filter in time domain is h(t). So the transfer function corresponding to h(t) can be obtained by the Laplace transform[6],i.e.

$$H(s) = L\{h(t)\} = \int_0^{\infty} h(t).e^{-st}dt \qquad \ldots \ldots (1.4)$$

Where,

s = complex variable
= σ + jω

Here h(t) is continuous. To obtain the discrete format of h(t) i.e. h(n), substitute

$$t = nT \qquad \ldots \ldots (1.5)$$

where,

T = sampling time

So, by substituting t = nT, we get h(nT) from h(t). If the sampling time T = 1 sec, then we obtain the discrete form of h(t) as h(n). Now from h(n) we can easily design H(z), i.e.,

$$H(z) = Z\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \qquad \ldots \ldots (1.6)$$

So, in this way, the transfer function of the digital filter can be obtained and moreover the stable poles of s-plane are mapped inside the unit circle of the z-plane[2]. Fig.3 shows the concept of mapping from s-plane to z-plane and vice versa.

The Butterworth High pass filter response increases logarithmically with increase in frequency and provides with monotonic amplitude frequency response at 0db. The magnitude curve can be obtained from the software program by providing the coefficients to it and the coefficient is obtained by the mapping technique.

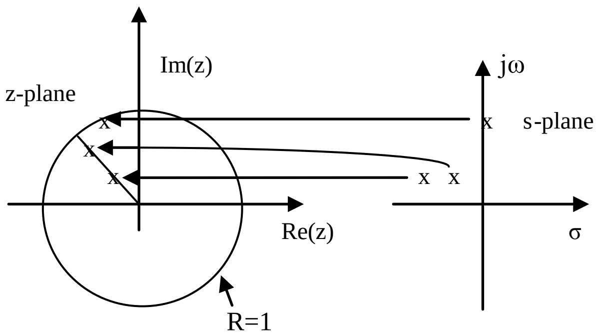

Fig.3 Mapping of poles

Basically the practical interest goes to the point that to determine the filter coefficients properly that in case truly help the designer to construct a filter of interest. By using the proposed algorithm, the filter coefficient can be determined.

4. SIMULATION RESULT AND DISCUSSION

The program for the design of IIR Butterworth High-pass filter is simulated in MATLAB7 by choosing the proper specifications such as Pass-band frequency, Stop-band frequency, Maximum allowable Pass-band and Stop-band ripples, so that the designed High-pass filter will be perfect. The output graphs are shown from Fig.4 to Fig.11.

Table 1 gives the results of the coefficients of High-pass Butterworth filter for order = 3 and order = 5.

Table.1 Result for coefficients

Filter name	Filter order	Numerator coefficient	Denominator coefficient
Butterworth Highpass Filter	3	-0.1247, -0.4272, -0.4776	-0.3616, 0.3602, -0.2123
	5	-0.5807,-0.424, -1.548,-2.826, -2.064	-1.471,1.912, -1.855,1.355, -0.8383

IIR Butterworth High-pass Filter(Order = 3)

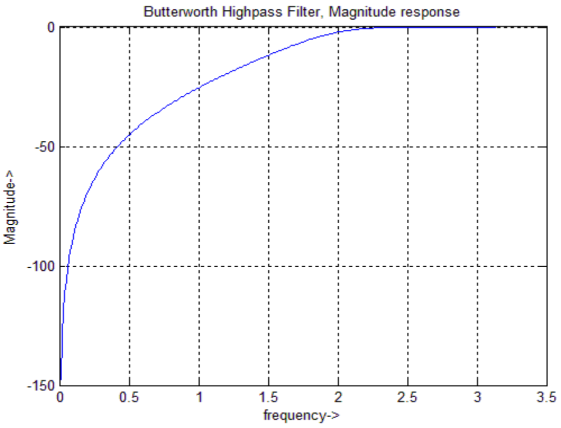

Fig.4 Magnitude response(Order=3)

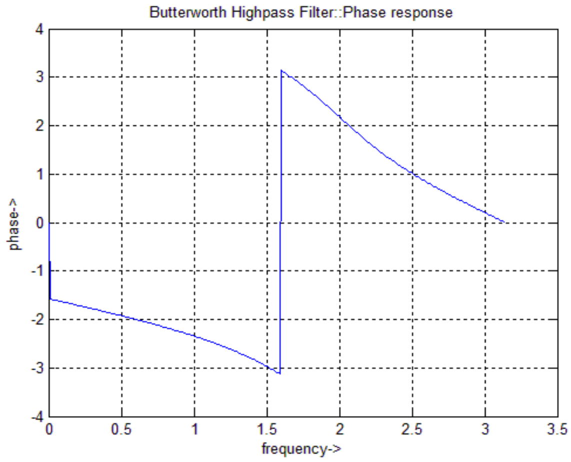

Fig.6 Phase response(Order=3)

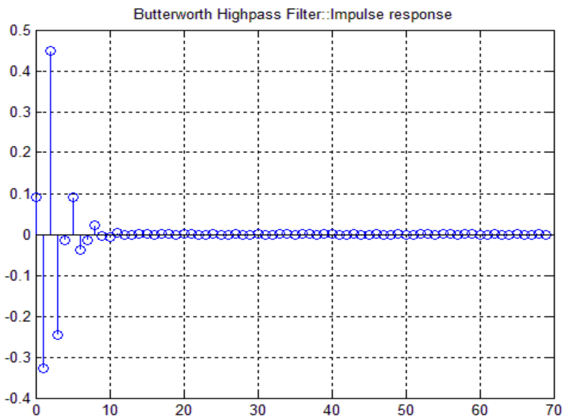

Fig.8 Impulse response(Order=3)

IIR Butterworth High-pass Filter(Order = 5)

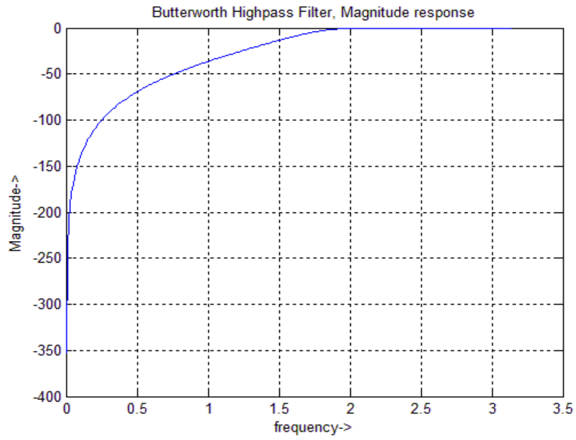

Fig.5 Magnitude response(Order=5)

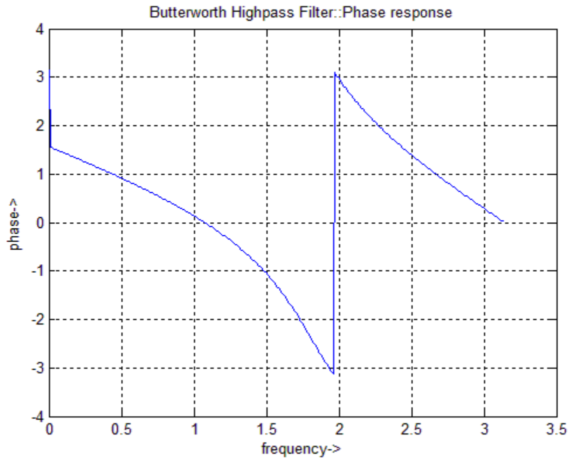

Fig.7 Phase response(Order=5)

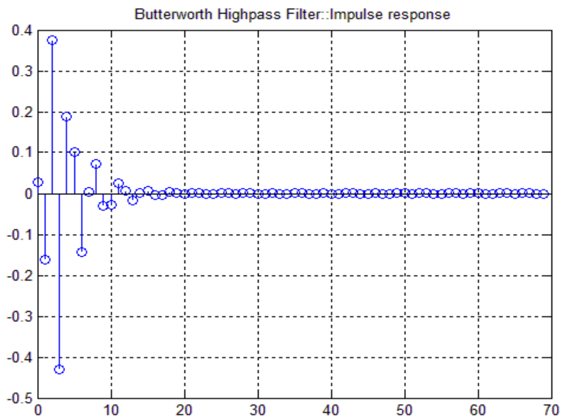

Fig.9 Impulse response(Order=5)

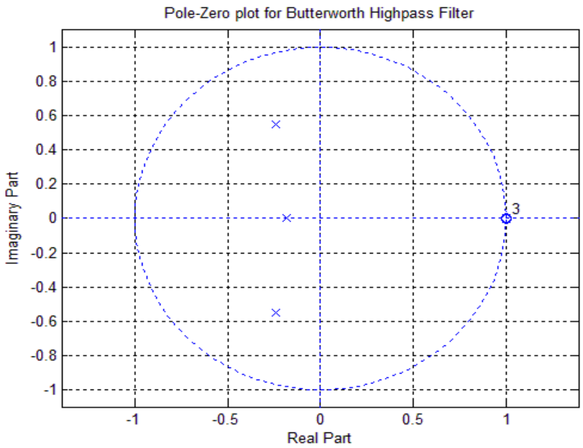

Fig.10 Pole-Zero plot(Order=3)

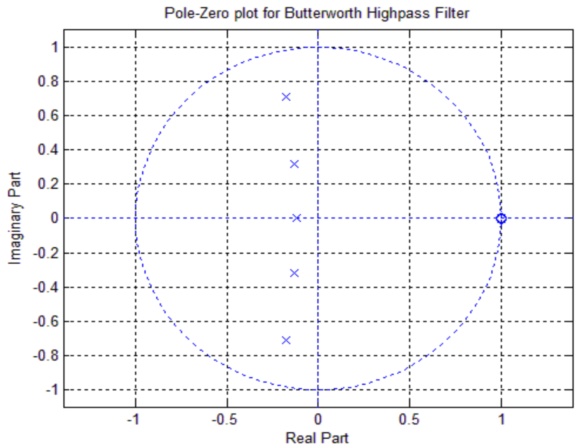

Fig.11 Pole-Zero plot(Order=5)

5. CONCLUSION

In this paper, the calculation and result for the coefficients of IIR Butterworth High-pass filter using the analog to digital mapping is shown. Those coefficients, that are determined by the proposed algorithm, are truly necessary for designing the filter. Now, if we look on the pole-zero plot of the filter, we can see that the filters are stable. So, we can design an analog filter to a stable digital filter by applying analog to digital mapping technique and with help of suitable proposed algorithm by which a stable filter as well as the optimum values of its coefficients is obtained and the output figures are presented in above figures, generated by Matlab 7.

6. REFERENCES

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