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Satellite Attitude Maneuver using Sliding Mode Control under Body Angular Velocity Constraints

Azza El-S. Ibrahim
Electronics Research Institute,
Computers & Systems dept.
Giza, Egypt

Ahamed M. Tobal
Electronics Research Institute,
Computers & Systems dept.
Giza, Egypt

Mohammad A. Sultan
Cairo University, Faculty of Engineering,
Electronics & Communications dept. Giza, Egypt

ABSTRACT
This paper presents a robust sliding-mode control technique to be applied to quaternion-based attitude control for rest-to-rest maneuvers with external disturbances. A sliding mode controller has been designed to force the state variables of the closed loop system to converge to the desired values. A control strategy is designed based on a novel mathematical rule that computes the discontinuous feedback gains. The proposed approach is defined in such a way that the selected controller parameters can drive the state to hit the sliding surface fast and then keep the state sliding along the surface with less chattering and tracking error. Moreover, the control parameters are adjusted to avoid the body angular velocity reached the upper limit during the maneuver. A simulation model of the controlled spacecraft system was developed in MATLAB-SIMULINK software. The phase portraits and the state plots prove the control technique power. The "chattering" problem of the sliding mode control has been adopted using variable thickness boundary layer technique. The second method of Lyapunov is used to guarantee the system stability under the proposed control laws action. Simulations have been carried out to demonstrate and verify the developed controller performance.

Keywords
Microsatellite, attitude control, sliding mode control,

1. INTRODUCTION
Satellites became an important application area of the new technological developments. They are used in many fields starting from telecommunications to defense technologies. For a successful operation the satellite should be stabilized at a given attitude. Thus attitude control is an important part of the space technology research. The low earth orbit satellites (LEO) are subject to some disturbances originating from earth and space environment. Because of that, the robustness characteristics of the stabilizing controller are one of the most important aspects of the attitude control approaches.

In general, the spacecraft motion is governed by the so-called kinematic and dynamic equations [6]. Actually, mathematical descriptions are highly nonlinear and thus, the conventional linear control techniques are not suitable for the controller design, especially when large-angle spacecraft maneuvers are required. Thus, the attitude control system must consider these nonlinearities. There are several methods trying to solve this problem [16, 17, and 20], where some research linearized the nonlinear attitude equations then applied different linear controls [7]. Doruk utilized integrator back - stepping method this, provides a recursive stabilization methodology [14]. Nadir-pointing control is achieved by a full-state feedback Linear Quadratic Regulator which drives the attitude quaternion and their respective rates of change into the desired reference [15]. A wide class of nonlinear control schemes has been proposed in details by "Slotine" [11].

Sliding Mode Control (SMC) has been recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic system operating under various conditions [2, 3, 5 and 15]. The main advantages of SMC, as a variable-structure-control (VSC) approach, are its fast dynamic response, robustness, simplicity in design and implementation [18]. Sliding mode control has been addressed in some previous studies for spacecraft attitude control problems [8, 12, 19 and 13]. The main drawback of SMC is the well known chattering phenomenon. Many methods have been proposed to solve this problem, such that the boundary layer technique [10, and 21].

This paper presents a control system design method for the three-axis-rotational attitude maneuver of a rigid spacecraft based on SMC. Determining the controller's parameters for nonlinear systems need a large computational time and effort to try and test the performance of the system. Most of the previous works done in this area did not target how the controller parameters can be chosen. Other works advise to compute them by trial and error method. This paper suggested a novel systematic mathematical rule to compute suitable and accurate controller parameters for any initial state which guarantees that the system reaches the desired state in finite time.

Moreover the developed control algorithm reduces the hitting time and attenuates the chattering using variable thickness boundary layer technique such that a high overall performance of small hitting time and small chattering can be achieved. In addition the developed algorithm ensures shortest angular path maneuvers.
The paper is organized as follows. In section 2, satellite attitude kinematics and dynamic equations are defined and modeled in state space form. The SMC technique is briefly reviewed in section 3; in section 4 we introduce a novel FRSMC based on a linear sliding manifold and discuss its benefits. Section 5 shows the utilized method to reduce the chattering. Section 6 shows the stability analysis of the controller. Numerical simulations and comparative study have been performed and presented in section7. Finally, the conclusions are outlined.

2. SYSTEM DESCRIPTION

The equations of motion of a spacecraft attitude dynamics can be divided into two sets, the kinematic equations of motion and dynamic equation of motion [7].

2.1 Kinematic Equations of Motion

The attitude kinematics part of a spacecraft can be represented by using various attitude parameters, but representation through quaternion parameter has the property of non-singularity and it is free from the trigonometric component. Therefore, this representation is widely used to study the attitude behavior of spacecraft. The kinematics of the satellite model is the part which expresses the relation between the attitude and angular velocities of the body. The kinematic equations through unit quaternion representation is given as:.

$$q^{\cdot} = \frac{1}{2}\Xi(q)\omega \quad (1)$$

Where, $$\Xi(q) = \begin{bmatrix} q_4 I_{3\times3} + [q_v \times] \\ -q_v^T q_v \end{bmatrix} \quad (2)$$

$I_{3\times3}$ is a 3x3 identity matrix and $[q_v \times]$ is a skew symmetric matrix expressed by:

$$[q_v \times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \quad (3)$$

ω is the body coordinate angular velocity vector represented in body frame with respect to the orbit frame.

2.2 Dynamic Equations of Motion

The attitude dynamic equations of a rigid spacecraft are given by:

$$I\omega^{\cdot} = -[\omega \times]I\omega + u + d \quad (4)$$

Where $\omega = [\omega_1\ \omega_2\ \omega_3]^T \in \mathbb{R}^{3\times1}$ is the angular rate of the spacecraft, $u = [u_1\ u_2\ u_3]^T \in \mathbb{R}^{3\times1}$ represents the control vector, $d = [d_1\ d_2\ d_3]^T \in \mathbb{R}^{3\times1}$ are bounded disturbances acting on the spacecraft body, and I is the inertia matrix.
For simplicity, let $f = -I^{-1}[[\omega \times]I\omega)]$, $b = I^{-1}$ therefore,

$$\dot{\omega} = f + bu + d \quad (5)$$

The nonlinear dynamic equations of the body angular velocity and the attitude kinematics differential equations are modeled and simulated via Matlab/Simulink software [22].

2.3 System Errors

Let $q_{e=}[q_{ve}\ \ q_{4e}]^T$denotes the relative attitude error from a desired reference frame to the body-fixed reference frame of the spacecraft, then one may have:

$$q_e = q \otimes q_d^{-1} \quad (6)$$

Where, q_d^{-1} is the inverse of the desired quaternion and $\otimes$ is the operator for quaternion multiplication [7]. For any given two groups of quaternion, the relative attitude error can be obtained by:

$$\begin{bmatrix} \dot{q}_{ve} \\ \dot{q}_{4e} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} q_{4e} I + [q_{ve} \times] \\ -q_{ve}^T \end{bmatrix} \omega_e(t) \quad (7)$$

Where,$\omega_e = \omega \ - \omega_d$,ω_d is the desired angular velocity of the body and equal to zero so $\omega_e = \omega$. Therefore, the error rate dynamic equation is the same as above in Eq. (5). Then the error state vector (q_e, ω) is fed to the controller module to compute the control signal command.

Controller objective is to drive the attitude states (q, ω) in system (1) and (5), from the initial states (q(0), ω(0)) to the desired states (q_d, ω_d), while the constraint $|\omega| \le \omega_{max}$ are met during the attitude maneuver.

Assumption 1: In spacecraft model equations (1) and (5), the unit-quaternion q and the body angular velocity ω are available in the feedback control design.

Assumption 2: The external disturbance d is assumed to be bounded and to satisfy the following condition: $\|d(t)\| < d_{max}$.

Assumption 3: initial angular velocity of the satellite body is zero.

3. ATTITUDE CONTROL DESIGN

The design of a sliding mode controller (SMC) involves designing of a sliding surface that represents the desired stable dynamics and a control law that makes the designed sliding surface attractive [1, 4]. The phase trajectory of a SMC can be investigated in two parts, representing two modes of the system. The trajectories starting from a given initial condition off the sliding surface tend towards the sliding surface. This is known as reaching or hitting phase and the system is sensitive to disturbances in this part of the phase trajectory. When the hitting to the sliding surface occurs, the sliding phase starts. In this phase the trajectories are insensitive to parameter variations and disturbances.

3.1 Sliding Surface Design

A linear sliding surface in vector form is defined as follows:

$$s = \omega_e + c\ q_{ev} \quad (8)$$

Where c is a strictly positive real constant determining the slope of the sliding surfaces $s(t) = [s_1\ s_2\ s_3] \in \mathbb{R}^{3\times1}$, and it divides the state space into two parts [1].

It is critical to note that for the quaternion parameterization of the spacecraft attitude $q_e(t)$ and $-q_e(t)$ represent identical physical tracking error rotations, although the former gives the shortest angular distance to the sliding manifold whilst the later gives the longest distance. Vadali and Crassidis et al.[8] proposed a modification to the sliding surface to ensure that the spacecraft follows the shortest angular path to the sliding manifold therefore reducing the required amount of control torque.

$$s = \omega + c\,\mathrm{sgn}(q_{e4})\, q_{ev} \quad (9)$$

The term $sgn(q_{e4})$ is used to drive the system to the desired trajectory in the shortest distance.

3.2 Control Law Design

The sliding mode control law divided into two main parts [4];

$$u(t) = u_{eq}(t) + u_d(t) \quad (10)$$

The first component of the proposed controller is $u_{eq}(t) = [u_{1eq}, u_{2eq}, u_{3eq}]^T$ which will make sliding surface s(t) invariant and it is calculated by setting $\dot{s}$ to zero considering s(t) to be zero. Second component $u_d(t) = [u_{1d}, u_{2d}, u_{3d}]^T$ is an extra control effort which forces the quaternion and angular velocity component to reach on sliding surface in finite time in spite of disturbances and it is computed according to constant reaching law [1, 11] as:

$$u_d = -k\ sign\ (s) \quad (11)$$

Where, k is a positive definite gain vector which will be derived in the next section

$$\mathrm{sign}(s) = \begin{cases} 1 & \text{for } s > 0 \\ -1 & \text{for } s < 0 \end{cases} \quad (12)$$

To obtain $u_{eq}(t)$,

$$\dot{s} = \dot{\omega} + c\,\mathrm{sgn}(q_{e4})\,\dot{q}_{ev} = 0 \quad (13)$$

Substituting by Eq. (5) into Eq. (13), therefore, a control input can be chosen as:

$$u(t) = \frac{1}{b}\left(-f - c\,sgn(q_{e4})\dot{q}_{ev} - k sign(s)\right) \quad (14)$$

The above control law has two design parameters (**c**, **k**) that should be selected to provide stability and better performance. The sliding surface slope, **c** is selected such that the system during the sliding mode is stable. In this work, c selection is depending on the body angular velocity constraint.

Most of the studies are getting the suitable values of them by try and error with many runs of the algorithm. And this process repeated every time the initial error is changed. For spacecraft attitude control these parameters should be known in real time. The proposed algorithm suggests a suitable geometric rule to compute the discontinuous feedback gain matrix for fast reaching sliding mode control (FRSMC).

4. FRSMC FEEDBACK GAIN VECTOR (K) SELECTION

In this section we propose a novel fast reaching sliding mode control (FSMC) technique based on a linear switching manifold. Actually the use of a large enough discontinuous signal, k is necessary to complete the reachability condition despite perturbations, but as small as possible in order to limit the chattering. The (ω_e, q_{ev}) error phase plane trajectories of the related controller for, different values of k are given in Figure 1. According to the value of k, the state trajectories reach a sliding surface at different points with different reaching time.

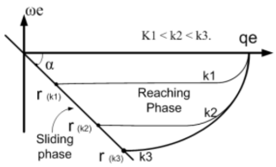

Figure 1: Error phase-plane at different values of k

As shown in Figure 2, for small value of k, the state trajectories take more time and long path to reach and vice-versa. Therefore for a pre-specified sliding surface slope there is a certain value of k which reduces the reaching phase, consequently decreases the reaching time.

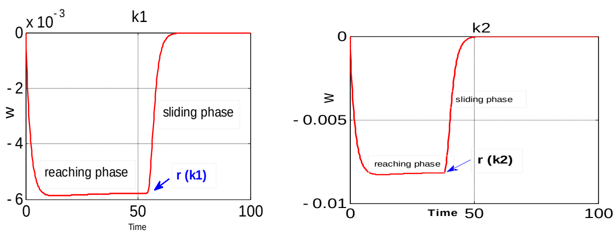

Figure 2: A trajectory of the body angular velocity for different values of k (k1 < K2)

This value of k is computed such that the extreme value of the angular velocity is achieved at the reaching point "r (k3)" on the sliding surface, Figure 3.

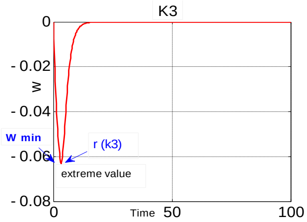

Figure 3: velocity trajectory in case of min. reaching time

Substitute with the control signal, u in Eq. (14) into the system dynamics therefore,

$$\dot{\omega} = -c\,sgn(q_{e4})\dot{q_{ev}} - k sign(s) + d \quad (15)$$

Firstly, to get k which make the extreme value of ω occurs at reaching point (the third case in Figure 3), put $\dot{\omega} = 0$, ω = ω_r, and $q_{ev} = q_r$ in Eq. (15) this lead to:

$$k\,sgn(s) = -\frac{1}{2}c\,sgn(q_{e4r})\{q_{4er}I + [q_r \times]\}\omega_r + d_{max} \quad (16)$$

Notice that, k is mainly depending on the position of reaching point (q_r, ω_r) and on the sliding line slope c, which is taken equal to 1.

Secondary, the reaching point "r" is selected as the intersection between the perpendicular line from the initial point and the sliding surface, to decrease the reaching trajectory path, as shown in Figure 4.

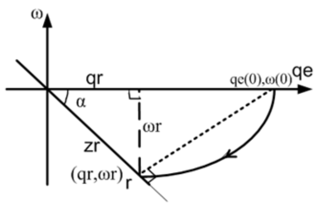

Figure 4: Selection of reaching point position

From the geometry of figure 4,

$$zr = q_e(0)\cos(\alpha), \quad where\ \alpha = \mathrm{atan}(c) \quad (17)$$

$$\omega_r = zr\sin(\alpha) = q_e(0)\cos(\alpha)\sin(\alpha) = \frac{q_e(0)}{2}\sin(2\alpha) \quad (18)$$

$$q_r = zr\cos(\alpha) = q_e(0)\cos^2(\alpha) \quad (19)$$

Hence, for every initial orientation of the satellite, the feedback gain vector k can be specified for lower reaching time to the switching surface, and more robust is the system. The previously stated rule represents a control action with growing magnitude of ω_r when the tracking error increases, i.e., when the system trajectory is far from the sliding manifold. Hence, if the initial operating point corresponds to a large error, then tumbling of the satellite may occur. Thus, the change in body angular velocity must be limited and don't exceed certain upper limit i.e. satisfy the following constraint.

$$|\omega| \leq \omega_{max} \quad (20)$$

Where, ω_{max} is the maximum admissible body angular velocity. According to Eq. (18), the value of ω_r is depending on the initial attitude error and on the angle α. it is clear that, the max value of ω_r, occur at $\alpha = 45\ deg$ (i.e. at c = 1). Let $\omega_{r\ max}$ is the max value of ω_r, as shown in flow chart in Figure 5, $\omega_{r\ max}$ is firstly calculated and then compared with

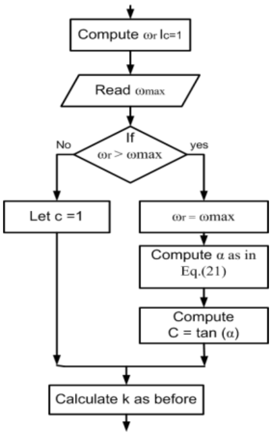

Figure 5: c selection to verify body angular velocity constraint

the allowable limit value of angular velocity (ω_{max}). If $\omega_{r\ max}$ is less than it put c=1 and continue as previous but if not put $\omega_r = \omega_{max}$ then compute α as in Eq. (21), c from Eq. (17).

$$\alpha = \frac{1}{2}\sin^{-1}\left(\frac{2\omega_{max}}{q_e(0)}\right) \quad (21)$$

ω_{max} can be known from detumbling controller design.

5. CHATTERING AVOIDANCE

The chattering phenomenon is generally perceived as motion which oscillates about the sliding manifold. This chattering control signal may cause possible damages to the actuators. Boundary layer method is used to reduce the chattering; the basic idea is to replace the sign function by a saturation function *sat*(*s/*ϵ) [10], defined as:

$$sat(s_i, \epsilon_i) = \begin{cases} 1 & for \quad s_i > \epsilon_i \\ \frac{s}{\epsilon} & for \quad |s_i| \le \epsilon_i \\ -1 & for \quad s_i < -\epsilon_i \end{cases} \qquad i = 1,2,3 \quad (22)$$

The larger the boundary layer width (ϵ_i), the smoother the control signal, however, it no longer drives the system to the origin, but to within the chosen boundary layer instead. Very small thickness layer introduces again the chattering phenomena. Chyun-Chau Fuh, [9] presented a method to suppress the chattering phenomenon while preserving the control accuracy and save the control energy. The developed control law uses the variable thicknesses boundary layer method as:

$$z = \sqrt{\omega^2 + (cq_{ve})^2} \quad (23)$$

Where z is called the error state vector, the boundary layer thickness is computed as,

$$\epsilon = s_{0+}\tan(\theta)\|z\| \quad (24)$$

Where (s_0, θ) are appropriate design positive constants. The higher the norm of the error state vector z, the greater the thickness of the boundary layer.

6. CONTROLLER STABILITY

Theorem 1: With sliding surface (9), and control law (14), the system reaches to sliding surface s = 0 in finite time.

Proof: Consider Lyapunov function, $V_1 = \frac{1}{2}s^T s$ Obtaining the time-derivative of V1

$$\dot{V}_1 = s^T\dot{s} = s^T[d - ksign(s)] \quad (25)$$

The derivative of Lyapunov function must be negative to make s reach zero therefore, $k \ge |d_{max}|$ i.e. k must be selected such that compensate for the upper bound of the disturbance which is satisfied by proposed rule of selecting k in Eq. (16).

Theorem 2: The trajectory in error state-space that slides on the sliding manifold can be shown to be asymptotically stable using Lyapunov's direct method. The following candidate Lyapunov function is proposed as:

$$V_2(q_{ve}) = \frac{1}{2}q_{ve}^T q_{ve} \quad (26)$$

Substitute Eq. (7) into Eq. (9) lead to the following kinematic equations for "ideal sliding motion on the sliding manifold (s=0).

$$\dot{q_{ve}} = -\frac{1}{2}c\,|q_{e4}|\,q_{ve} + \frac{1}{2}[q_{ve}\times](-k\,sgn\,(q_{e4})q_{ve}) \quad (27)$$

Substituting Eq. (27) into the derivative of the equation (26) leads to the following expression

$$\dot{V} = -\frac{1}{2}c\,|q_{e4}|\,q_{ve}^T\,q_{ve} \quad (28)$$

This is clearly negative definite provided c > 0. This results show that attitude tracking error $q_{ve} \to 0_{3\times1}$ ast $\to \infty$, and since the motion is on the sliding surface defined by $s(t) = 0_{3\times1}$ It follows that $\omega(t) \to 0_{3\times1}$ ast $\to \infty$.

7. SIMULATION RESULTS

To exhibit the results clearly a conversion between the real attitude vector (more truly speaking the roll, pitch and yaw angles) and the quaternion are performed in the simulator. The attitude control problem of a rigid body spacecraft system is simulated and results are presented in this section to illustrate the performances of the sliding mode control law proposed in this paper.

Consider a rigid spacecraft with the nominal inertia matrix I = [14.28 0 0; 0 15.74 0; 0 0 12.5] kg.m^2. The initial attitude orientation of the unit-quaternion is q (0) = [.1603, -.1431, .06252, .9746]^T (which is equivalent to initial Euler angles E(0) = [20 -15 10] deg.) , and the initial value of the angular velocity is ω(0) = [0, 0, 0]^T rad/s. The disturbances are assumed to be bounded by $d_{max} = [.0001\ .0001\ .0001]$ N.m.

The FRSMC algorithm computes the suitable feedback gain, k = [.03992 .03564 .01563] required to steer the satellite from initial attitude condition to zero states in 18 sec. with steady state error = [8.038(10)$^{-5}$ 7.27(10)$^{-5}$ 9.174(10)$^{-5}$] and slope, c=[1 1 1]. The effect of boundary layer thickness is shown in Figure 9. A chattering in control signal is appeared in Figure (9-a) such that the thickness of the boundary layer is selected very small. If the thickness is appropriately selected, the control signals have no chattering as in Figure (9-b). In case of using variable thickness boundary layer instead of fixed one the signal become smoother and its extreme values are lower as shown in Figure (9-c). The design parameters of the boundary layer are $\theta = \frac{\pi}{10}$ and $s_0 = 0.1k$.

7.1 Comparative Study

To validate the developed control law, the results are compared with a conventional SMC controller. A simulator of a classical sliding mode controller is also built and the controller parameters are selected such that the slope of the sliding surface is same as in the proposed algorithm. The value of feedback gain vector is selected by try and error such that gives same steady state error. The time taken by

conventional SMC is 41 sec. for k = [0.003992 0.003564 0.001563].

Simulation studies have been performed to test both controllers. Figure 6, and Figure 7 clearly show the performance of the conventional SMC and FRSMC controllers. From figures, the FRSMC technique follows the reference angles with very fast response. Finally, Figure 8 shows the phase portraits of the error state-space in FRSMC & SMC controllers.

8. CONCLUSION

In this paper a nonlinear controller for attitude maneuvers of low earth orbit satellites subject to velocity constraint is developed. The quaternion vector is adopted as the attitude parameters for global representation without singularities or trigonometric functions calculation.

The main contribution of this work is the procedure for the selection of controller parameters. The proposed algorithm is able to choose the sliding surface and tune the discontinues feedback gains according to how far the initial states of the system from the desired states such that the reaching time and tracking error in the approaching phase can be significantly reduced.

In this procedure, the overall system performance has improved with respect to the classical sliding mode controller. In addition the computation of controller parameters is simple and its tuning is straightforward. Moreover, the proposed control strategy can guarantee no chattering occurrence such that it uses variable thickness boundary layer. The feasibility and effectiveness of the variable thickness boundary layer have been demonstrated.

The attitude variables achieved are shown to be globally asymptotically stable through the standard Lyapunov technique. Satellite attitude simulator with a developed control algorithm was built in a previous work using MATLAB /SIMULINK software.

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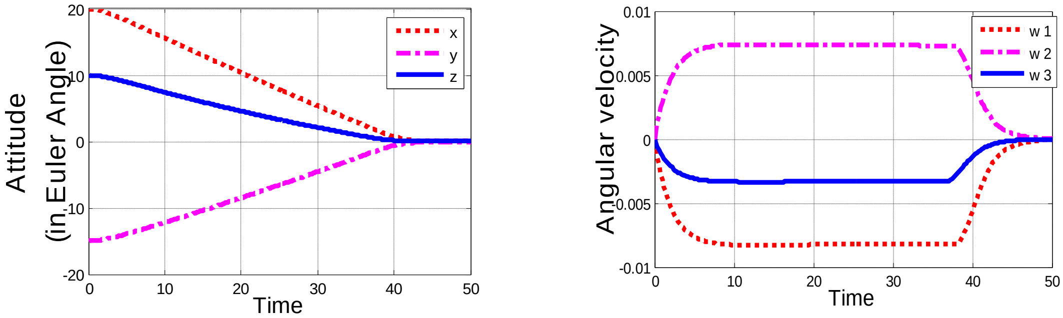


Figure 6: Attitude and body angular velocity at conv. SMC

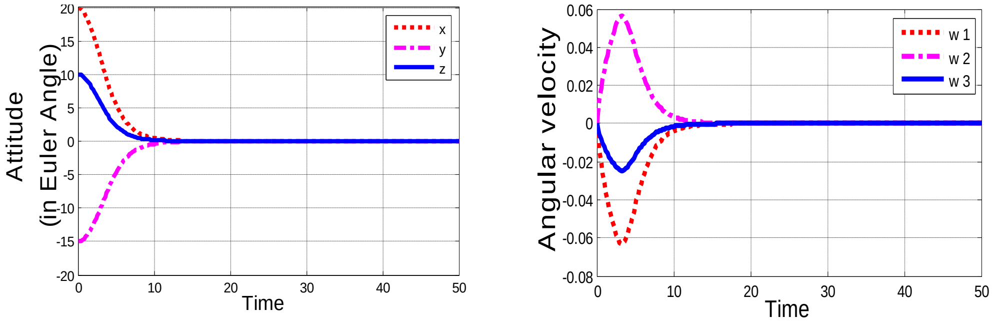
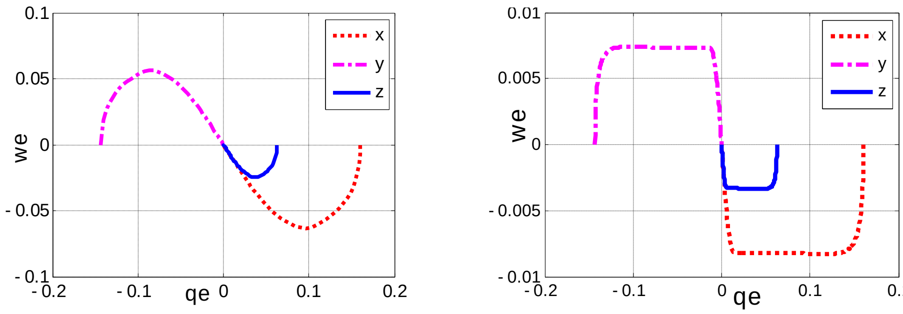
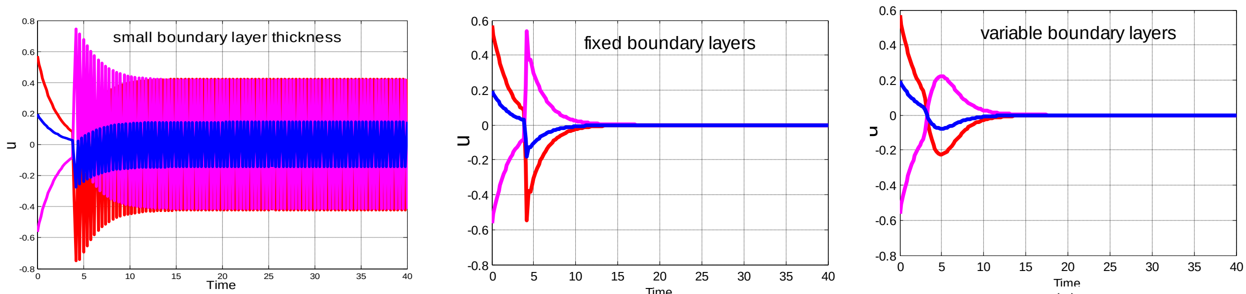


Figure 7: Attitude and body angular velocity at FRSMC



(a) FRSMC **(b) conv. SMC**

Figure 8: A phase plane portrait of the system in (a) FRSMC and (b) conv. SMC



(a) **(b)** **(c)**

Figure 9: Control signal trajectory in three different cases of boundary layer thickness