Semi Analytical Formulation of Dispersion Characteristics for a Step and Parabolic–Index Optical Fiber: Introduction of Nelder-Mead Simplex Method for Nonlinear Unconstrained Minimization

Raja Roy Choudhury
Associate professor,
Applied Electronics and Instrumentation Department,
Sikkim Manipal Institute of Technology,
Majitar-737136. Sikkim, INDIA

Arundhati Roy Choudhury
Associate professor,
Physics Department,
Sikkim Manipal Institute of Technology,
Majitar-737136. Sikkim, INDIA

Mrinal Kanti Ghose
Professor,
Computer Science Department.
Sikkim Manipal Institute of Technology,
Majitar-737136. Sikkim, INDIA,

ABSTRACT
Variational principle has been incorporated in the proposed semi analytical formulation to characterize graded index optical fiber and optimization process is being carried out by Nelder-Mead Simplex method for nonlinear unconstrained minimization. By introducing increased optimizing parameters in the formulation of fundamental modal field, more flexibility to refine a design and thus achieving a greater accuracy for a particular requirement, has been presented. Employing variational technique, effective index, normalized propagation constant, normalized group delay and modal dispersion parameter of an optical fiber have been evaluated analytically, by using the proposed formulation of fundamental field. The calculations show excellent agreement to the exact results, over a wide range of normalized frequency. Optimized values of three parameters, incorporated in the fundamental modal field have been presented for various values of normalized frequencies. This approximation can be used in the analysis of doped and nonlinear fiber amplifier, or in the case of dispersion–shifted fiber with large effective area.

Key Words
Single mode fiber, Nelder-Mead, Dispersion.

1. INTRODUCTION
To meet ever increasing demand for larger capacity of signal transmission, characterization of single mode fiber plays a crucial role in optical communication system. For analysis of the behavior of such a fiber, expression of fundamental modal field appears within integrals. Hence, an accurate choice of fundamental modal field leads to exact description of propagation characteristics of an optical fiber. Initially, Gaussian approximation [1] was used to describe fundamental field, to provide simple analytical expressions for modeling an optical fiber. But, Gaussian expression fails to give accurate result for lower values of normalized frequency. Also, it is not useful to estimate evanescent field exactly. Many attempts have been made to achieve higher accuracy than the Gaussian field, such as, Gaussian exponential expression [2], Gaussian–Hankel[3], the extended Gaussian[4], the generalized Gaussian[5] and the Laguerre–Gauss/Bessel expansion approximation [6]. These modified approximations are much more accurate than the Gaussian field. Among these, Gaussian exponential expression[2] provides good result near the cut off frequency of next higher mode, but does not give accurate result for lower range of normalized frequencies[3]. An analytical expression without any requirement of optimization has also been reported [7]. But, such an expression may not be capable to meet all specifications of an optical fiber. Moreover, efficient optimizing software package is readily available in the market. So, optimization can be preferred as an appropriate process, capable to satisfy variety of designer's demands.

All approximations for fundamental modal field reported so far, are limited to one or two variational parameters. But, recent advent of fiber manufacturing technology is associated with various design specifications. Thus, to accommodate all kinds of design, more flexibility in terms of three parameter approximation in formulating various propagation parameters of graded–index fibers has been introduced. It can combat with ever increasing demand of fibers in diverse application area. Implementing variational technique, which is considered to be the most appropriate technique [8] for such kind of calculations, it has been demonstrated that the proposed approximation provides much more accurate analytical expressions to characterize variety of fiber behaviors. Again, the optimization process requires accurate analytical expression for propagation constant β, and core parameter U , which involve various fiber parameters, such as, core radius (a), wavelength ($\lambda=2\pi/ k$), refractive indices of core and cladding (n_{co} and n_{cl}). Hence, any desired specification can be substantiated by varying these fiber parameters.

Single mode fiber design criteria considerably depend on its dispersion characteristics. However, broadening of pulse due to dispersion can be controlled by using dispersion–shifted fiber. Again, dependence of refractive index profile on intensity of propagating wave can cause severe signal distortion in a non–soliton regime. This non linear behavior of refractive index is inversely proportional to the effective area of the fiber. Hence, the trade-off between dispersion and effective area is very important for fiber design. A single mode fiber with a large effective area can effectively reduce signal distortion due to non linear effect. Hence, accurate computation of effective area and dispersion characteristics with a flexible formulation of fundamental modal field, can lead to an exact description of dispersion–flattened or dispersion–shifted fiber with large effective area, to be used in long–haul networks for high bit rate applications, with variety of index profile. Even, a flexible formulation of fundamental mode can be used for study of fiber polarization mode dispersion (PMD), which is one of the crucial aspects in fiber design to achieve high capacity over long distances.

To highlight that the proposed approximation is capable to estimate the fiber effective area and dispersion characteristics over a wide range of normalized frequency, evaluation of effective index, normalized propagation constant, normalized group delay and modal dispersion parameter of a step and parabolic–index optical fiber is done with the proposed formulation of fundamental field. All results from the calculations are in excellent agreement with the exact numerical result.

For optimization purpose Nelder-Mead Simplex method for nonlinear unconstrained minimization is used as Nelder-Mead Simplex method is a direct search method which does not require any derivative information, so it can optimize non-stationary functions, as needed for the problems under study [9, 10, 11]. Nelder-Mead Simplex method is also widely used for solving parameter estimation and similar statistical problems, where the function values are uncertain, noisy or even discontinuous [12, 13]. The Nelder-Mead simplex method gained popularity very quickly. At earlier time, due to its simplicity and low storage requirements, it was ideally suited for use on minicomputers especially in laboratories i.e. it is effective and computationally compact. Although the method is relatively old and considering recent advances in direct search methods, the Nelder-Mead method is still among the most popular direct search methods in practice [14, 15].

2. THEORY

2.1. Basic formulations

For a weakly guiding fiber, the refractive index profile is given by,

$$n^2(R) = n_2^{\,2} + (n_1^{\,2} - n_2^{\,2})\, f_1 = n_{f1} \qquad \text{for, } R \le 1$$

$$n^2(R) = n_2^{\,2} + (n_1^{\,2} - n_2^{\,2})\, f_2 = n_{f2} \qquad \text{for, } R > 1 \tag{1}$$

Here the normalized profile function is given by,

f_1=1 for, step–index profile,
f_1=1 – R^2 for, parabolic–index profile,
and, f_2=0

Here, R (= r/a) is the normalized radius, a is the core radius, n_1 and n_2 being the refractive indices of core axis and cladding respectively.

The following three parameter approximation for the fundamental mode as the trial field has been proposed

$$\psi = [1 + \{\alpha(R/R_0)^2\}/\mu]^{-(\mu+1)/2} \tag{2}$$

To employ variational technique, first we consider the expression of propagation constant β,

$$\beta = \frac{1}{\langle \psi^2 \rangle}\Big[\int_0^\infty k^2 n^2(R)\,|\psi|^2\, RdR - \frac{1}{a^2}\langle \psi'^2 \rangle\Big]$$

where, $$\langle \psi^2 \rangle = \int_0^\infty |\psi|^2\, RdR \tag{3}$$

and, $$\langle \psi'^2 \rangle = \int_0^\infty \left|\frac{d\psi}{dR}\right|^2 RdR \tag{4}$$

Applying equation (1), we obtain,

$$\beta = \frac{1}{\langle \psi^2 \rangle}\Big[\int_0^1 k^2 n_{f1}\,|\psi|^2\, RdR + \int_1^\infty k^2 n_{f2}\,|\psi|^2\, RdR - \frac{1}{a^2}\langle \psi'^2 \rangle\Big] \tag{5}$$

Here, k is the free space wave number. Now, the core parameter U is given by,

$$U^2 = a^2(k^2 n^2 - \beta^2) \tag{6}$$

For a fixed value of normalized frequency V, U is minimized with respect to the variational parameters α, R_0 and μ. With the optimized values of these parameters, propagation constant β and other characteristic parameters of an optical fiber can be evaluated.

2.1.1. Effective Index (n_{eff}):

The Effective index of an optical fiber is defined as,

$$n_{eff} = \frac{\beta}{k} \tag{7}$$

Once propagation constant β is found, effective index can be obtained from the above equation.

2.1.2. Normalized Propagation Constant (b):

The well known expression for the normalized propagation constant is given by,

$$b = \{(\beta^2/k^2) - n_{cl}^{\,2}\}/\{n_{co}^{\,2} - n_{cl}^{\,2}\} \tag{8}$$

Putting (7) in (8), normalized propagation constant can be evaluated.

2.1.3. Normalized Group Delay (d):

The normalized group delay (d) of an optical fiber can be expressed in terms of normalized propagation constant b and is given by,

$$d(V) = \frac{d(Vb)}{dV} = b + \frac{2\langle \psi'^2 \rangle}{V^2 \langle \psi^2 \rangle} \tag{9}$$

Where, V is the normalized frequency.

2.1.4. Dispersion parameter (g):

The modal dispersion parameter (g) is expressed as,

$$g(V) = V\frac{d^2(Vb)}{dV^2} = 2\frac{d}{dV}\left[\frac{1}{V}\frac{\langle \psi'^2 \rangle}{\langle \psi^2 \rangle}\right] \tag{10}$$

2.2. Evaluation of integrals

Let, $e_1 = \{(10^4 R_0^{\,2}\mu + \alpha)/(R_0^{\,2}\mu)\}^{\mu}$, $e_2 = \{(\mu+\alpha)/\mu\}^{\mu}$,

$e_3 = \{(R_0^{\,2}\mu + 10^6\alpha)/(R_0^{\,2}\mu)\}^{\mu}$,

$e_4 = \{(R_0^{\,2}\mu + \alpha)/(R_0^{\,2}\mu)\}^{\mu}$,

$e_5 = 10^{4\mu} e_2$, $e_6 = \{(R_0^{\,2}\mu + 25\alpha)/(R_0^{\,2}\mu)\}^{\mu}$,

$e_7 = (10^4 R_0^{\,2}\mu + \alpha)^2$ and $e_8 = (R_0^{\,2}\mu + 25\alpha)^2$.

2.2.1. Analytical expression for propagation constant (β):

$$\left\langle \psi^2 \right\rangle = \frac{1}{2} R_0^{\,2} \left(\frac{(-e_1 + 10^{4\mu} e_2)}{\alpha e_1 e_2} - \frac{(e_2 + e_3)}{\alpha e_2 e_3} \right)$$

(11)

$$\left\langle \psi'^2 \right\rangle = [\mu / \{2a^2(\mu+2)e_2(\mu+\alpha)^2\}][e_1(\alpha^3\mu^2 + \alpha^2\mu^2 + 2\alpha^2\mu^2 + 2\alpha^3 + \alpha^2\mu + 3\alpha^3\mu) + 10^8 R_0^{\,4} e_1 (3\mu^4 + 3\alpha\mu^3 + 2\alpha\mu^2 + \alpha\mu^4 + \mu^3) + 2\times10^4 R_0^{\,2} e_1 (\alpha^2\mu^3 - 6\alpha^3\mu^2 + 2\alpha^2\mu + \alpha\mu^3 + \alpha\mu^2 + 3\alpha^2\mu^2) - e_5 10^4 R_0^{\,2} (6\alpha^2\mu^2 - 3R_0^{\,2}\alpha^3\mu - 2\alpha\mu^2 - 4\alpha^2\mu - 2\alpha^2\mu^3 - 2\alpha^3 - 3\alpha\mu^3 - \alpha\mu^4) - 10^8 R_0^{\,4} e_5 (\mu^4 + 2\alpha\mu^2 + \mu^3 + \alpha^2\mu + 2\alpha^2\mu^2) - 10^4 R_0^{\,2} e_2 \alpha^3 \mu^2 \} / e_1 e_7] + [e_3\{10^{12}(\mu^2\alpha^2 + \mu^2\alpha^3 + 3\mu\alpha^3 + \mu\alpha^2 + 2\alpha^3) + \mu^4 R_0^{\,4} + 2\mathrm{x}10^6 R_0^{\,2}(\mu^2\alpha + 3\mu^2\alpha^2 + 2\mu\alpha^2 + R_0^{\,2}\mu^3\alpha + \mu^3\alpha^2) + R_0^{\,4}(2\mu^2\alpha + \mu^4\alpha + \mu^3 + 3\mu^3\alpha)\} - e_2\{10^6 R_0^{\,2}(2\alpha^3 + \mu^4\alpha + \mu^2\alpha^3 + 2\mu^3\alpha^2 + 3\mu^3\alpha + 3\mu\alpha^3 + 2\mu^2\alpha + 4\mu\alpha^2 + 6\mu^2\alpha^2) + R_0^{\,4}(\mu\alpha^2 + 2\mu^2\alpha + 2\mu^3\alpha + \mu^3 + \mu^2\alpha^2 + \mu^4)\}] / \{e_3(R_0^{\,2}\mu + 10^6\alpha)^2\}]$$

(12)

$$\int_0^1 k^2 n_{f1} |\psi|^2 \, RdR = \{R_0^{\,2} k^2 n_1^{\,2} / (2\alpha e_2)\}[\{(e_5 - e_1)/e_1\} - \{(e_2 - e_4)/e_4\}]$$

for, step–index profile

(13a)

$$\int_0^1 k^2 n_{f1} |\psi|^2 \, RdR = [\{5\times10^{-5} R_0^{\,2} k^2\} / e_1 e_4 \alpha^2 (\mu - 1)] [10^4 e_1 (\alpha(n_1^{\,2} - \mu n_2^{\,2}) + R_0^{\,2}\mu(n_1^{\,2} - n_2^{\,2})) + 10^{4\mu} e_4 \{\alpha\mu(n_2^{\,2} + 9999 n_1^{\,2}) - 10^4(\alpha n_1^{\,2} + R_0^{\,2}\mu(n_1^{\,2} - n_2^{\,2}))\}]$$

for, parabolic–index profile

(13b)

$$\int_1^\infty k^2 n_{f2} |\psi|^2 \, RdR = -R_0^{\,2} k^2 n_2^{\,2} \{(e_4 - e_3)/(2\alpha e_3 e_4)\}$$

(13c)

$$\int_1^{R_0} k^2 n_{f2} |\psi|^2 \, RdR = \mathrm{R}_0^{\,2} k^2 n_2^{\,2} (\mathrm{e}_2 - \mathrm{e}_4)/(2\alpha \mathrm{e}_2 \mathrm{e}_4)$$

(13d)

$$\int_{R_0}^\infty k^2 n_{f2} |\psi|^2 \, RdR = -R_0^{\,2} k^2 n_2^{\,2} \{(e_2 - e_3)/(2\alpha e_2 e_3)\}$$

(13e)

Putting (12) – (13) in (5), we get analytical expression for propagation constant β, and hence, expression for *U* can be obtained from (6).

2.2.2. Analytical expressions for Effective Index (n_{eff}):

Once propagation constant β is obtained, effective index can be estimated from (7).

2.2.3. Analytical expressions for Normalized Propagation Constant (b):

Using expression for effective index in (8), normalized propagation constant can be evaluated for different values of normalized frequency.

2.2.4. Analytical expressions for Normalized Group Delay (d):

Using the expression of normalized propagation constant *b*, and substituting equation (11) and (12) in (9), the analytical expression of normalized group delay can be found out.

2.2.5. Analytical expressions for Modal Dispersion Parameter (g):

Putting (11) and (12) along with the expression of normalized propagation constant in equation (10), modal dispersion parameter (*g*) of an optical fiber can be evaluated.

2.3. Nelder-Mead Simplex Method for Nonlinear Unconstrained Optimization [12]

The general Nelder-Mead Simplex Method can be given as:

1) Construct the initial working simplex *S*.
2) Repeat the following steps until the termination test is satisfied:
    a) calculate the termination test information;
    b) If the termination test is not satisfied then transform the working simplex.
3) Return the best vertex of the current simplex *S* and the associated function value.

Even though the method is quite simple, it can be implemented in many different ways. Apart from some minor computational details in the basic algorithm, the main difference between various implementations lies in the construction of the initial simplex, and in the selection of convergence or termination tests used to end the iteration process, as stated hereunder.

2.3.1 Initial simplex

The initial simplex *S* is usually constructed by generating $(n+1)$ vertices $x_0, x_1, \ldots\ldots x_n$ around a given input point $x_{in} \in \mathbf{R}^n$. In practice, the most frequent choice is $x_0 = x_{in}$ to allow proper restarts of the algorithm. The remaining *n* vertices are then generated to obtain one of two standard shapes of *S*:

- *S* is right-angled at x_0, based on coordinate axes, or
  $x_j := x_0 + h_i e_j \,, \quad j = 1, \ldots.., n,$
  where h_j is a step size in the direction of unit vector $\boldsymbol{e_j}$ in $\mathbf{R}^n$

- *S* is a regular simplex, where all edges have the same specified length.

2.3.2 Simplex transformation algorithm

One iteration of the Nelder-Mead method consists of the following three steps.

1. **Ordering**: Determine the indices h, s, l of the *worst*, *second worst* and the *best* vertex, respectively, in the current working simplex *S*

   $$f_h = \max_j f_j, \quad f_s = \max_{j \neq h} f_j, \quad f_l = \min_{j \neq h} f_j.$$

   In some implementations, the vertices of *S* are ordered with respect to the function values, to satisfy $f_0 \le f_1 \le .... \le f_{n-1} \le f_n$. Then $l = 0, s = n - 1,$ and $h = n$. Consistent tie-breaking rules for this ordering were given by [9].

2. **Centroid**: Calculate the *centroid* ***c*** of the *best* side—this is the one opposite the *worst* vertex x_h

   $$c := \frac{1}{n}\sum_{j \neq h} x_j.$$

3. **Transformation**: Compute the new working simplex from the current one.

   - First, try to replace only the *worst* vertex x_h with a better point by using reflection, expansion or contraction with respect to the best side.
   - All test points lie on the line defined by x_h and *c*, and at most two of them are computed in one iteration.
   - If this succeeds, the *accepted* point becomes the new vertex of the working simplex.
   - If this fails, shrink the simplex towards the *best* vertex x_l. In this case, *n* new vertices are computed.

Simplex transformations in the Nelder-Mead method are controlled by four parameters: α for reflection, β for contraction, γ for expansion and δ for shrinkage. These should satisfy the following constraints $\alpha > 0, 0 < \beta < 1, \gamma > 1, \gamma > \alpha, 0 < \delta < 1.$

The standard values, used in most implementations, are $\alpha = 1, \beta = \frac{1}{2}, \gamma = 2, \delta = \frac{1}{2}.$

A slightly different choice was suggested by Parkinson and Hutchinson [10]. An alternate notation for these four parameters: $\rho, \gamma, \chi, \sigma,$ respectively, is used in [9] and Matlab implementation.

The following algorithm describes the working of simplex transformations in step 3 [11], and the effects of various transformations are shown in the corresponding figures.

Reflect: Compute the *reflection* point $x_r := c + \alpha(c - x_h)$ and $f_r := f(x_r)$. If $f_l \le f_r < f_s$, accept x_r and terminate the iteration.

- **Expand**: If $f_r < f_l$, compute the *expansion* point $x_e := c + \gamma(x_r - c)$ and $f_e := f(x_e)$. If $f_e < f_r$, accept x_e and terminate the iteration. Otherwise (if $f_e \ge f_r$), accept x_r and terminate the iteration.
  This "greedy minimization" approach includes the better of the two points x_r, x_e in the new simplex, and the simplex is expanded only if $f_e < f_r < f_l$. It is used in most implementations, and in theory [9]. The original Nelder-Mead paper [12] uses "greedy expansion", where x_e is accepted if $f_e < f_l$ and $f_r < f_l$, regardless of the relationship between f_r and f_e. It may happen that $f_r < f_e$, so x_r would be a better new point than x_e, and x_e is still accepted for the new simplex. The working simplex is kept as large as possible, to avoid premature termination of iterations, which is sometimes useful for non-smooth functions [13].

- **Contract**: If $f_r \ge f_s$, compute the *contraction* point x_c by using the better of the two points x_h and x_r.
  - **Outside**: If $f_s \le f_r < f_h$, compute $x_c := c + \beta(x_r - c)$ and $f_c := f(x_c)$. If $f_c \le f$, accept x_c and terminate the iteration.
    Otherwise, perform a shrink transformation.
  - **Inside**: If $f_r \ge f_h$, compute $x_c := c + \beta(x_h - c)$ and $f_c := f(x_c)$. If $f_c < f_h$, accept x_c and terminate the iteration.
    Otherwise, perform a shrink transformation.

- **Shrink**: Compute *n* new vertices $x_j := x_l + \delta(x_j - x_l)$ and $f_j := f(x_j)$, for $j = 0, \ldots\ldots, n,$ with $j \neq l$.

The shrink transformation was introduced to prevent the algorithm from failing in the following case:

A failed contraction is much rarer, but can occur when a valley is curved and one point of the simplex is much farther from the valley bottom than the others; contraction may then cause the reflected point to move away from the valley bottom instead of towards it. Further contractions are then useless. The action proposed contracts the simplex towards the lowest point, and will eventually bring all points into the valley.

2.3.3 Termination tests

A practical implementation of the Nelder-Mead method must include a test that ensures termination in a finite amount of time. The termination test is often composed of three different parts: ***term_x*, *term_f*** and ***fail*.**

- ***term_x*** is the *domain* convergence or termination test. It becomes *true* when the working simplex *S* is sufficiently *small* in some sense (some or all vertices x_j are close enough).
- ***term_f*** is the *function-value* convergence test. It becomes *true* when (some or all) function values f_j are *close* enough in some sense.
- ***fail*** is the *no-convergence* test. It becomes *true* if the number of iterations or function evaluations exceeds some prescribed maximum allowed value.

The algorithm terminates as soon as at least one of these tests becomes *true*.

Various forms of ***term_x*** and ***term_f*** tests have been used in practice, and some common implementations of the algorithm have only one of these two tests [14, 15].

If the algorithm is expected to work for discontinuous functions ***f***, then it must have some form of a ***term_x*** test. This test is also useful for continuous functions, when a reasonably accurate minimizing point is required, in addition to the minimal function value. In such cases, a ***term_f*** test is only a safeguard for "flat" functions.

Table 1: Values of optimizing parameters in different normalized frequency for Step index profile

α	R_0	μ	V
1.068963	1.190014	2.033124	1.4000
0.810013	0.894742	3.086655	1.6000
2.139003	1.311694	4.595406	1.8000
1.708235	1.086141	6.939758	2.0000
1.533399	0.970482	11.088520	2.2000
1.142917	0.799907	20.448212	2.4000
0.033122	0.131163	61.545382	2.6000
1.839455	0.941374	61.719697	2.8000
0.726407	0.572523	61.543884	3.0000
0.000570	0.015588	62.724726	3.2000

Table 2: Values of optimizing parameters with different normalized frequency for parabolic–index profile.

α	R_0	μ	V
1.356165	1.258932	1.314966	1.7500
1.451176	1.070906	1.925299	2.0000
1.245270	0.864681	2.651519	2.2500
1.019513	0.705841	3.518595	2.5000
0.988457	0.641018	4.557522	2.7500
1.582103	0.759293	5.806230	3.0000
1.050838	0.585569	7.310931	3.2500
1.619058	0.693173	9.128715	3.5000
0.560740	0.391322	11.329089	3.7500
0.854820	0.465577	13.997434	4.0000

3. RESULT AND DISCUSSION

To illustrate accuracy of the proposed approximation, detailed comparison between the proposed formulation and exact results have been carried out. Optimized values of three variational parameters, α, R_0 and μ, over a wide range of normalized frequency have been presented in Table I and II, for step and parabolic–index fiber respectively. The range of frequency chosen corresponds to single mode fiber operation. However, optimized values of these parameters for other normalized frequencies can be found out, if required. In order to demonstrate the applicability and accuracy of the approximation, the proposed approximation is employed to find the effective index of graded index fibers. Variation of effective index as a function of normalized frequency *V* has been presented in Fig. 1 and 2 for step and parabolic–index fibers respectively, and simultaneously compared with the available exact numerical results.

To verify the applicability of the proposed approximation in dispersion related devices, we have estimated normalized propagation constant, normalized group delay and modal dispersion parameter of graded–index fiber, with the proposed field. Variations of these parameters with normalized frequency are depicted in Fig. 3 and 4, for step and parabolic–index profile respectively. Again, the results are in excellent agreement with the exact available results. The Gaussian approximation fails to provide accurate result, particularly in the lower normalized frequency region. Accurate estimation of dispersion characteristic indicates that the proposed approximation can be used successfully to find the analytical description of dispersion shifted fiber with large effective area, useful for long haul communication. Also, it can be used to describe non linear or doped fiber amplifier.

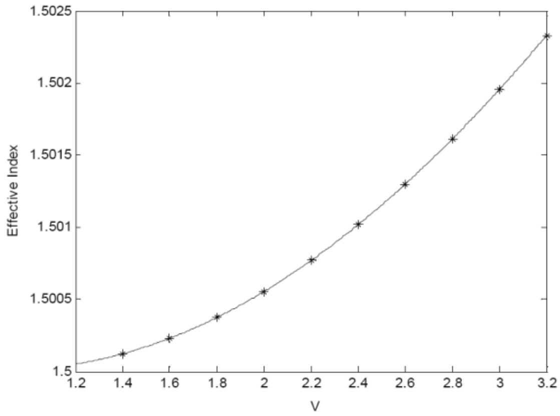

Fig1. Variation of Effective Index with normalized frequency (*V*) for step–index profile (—— exact results; * results by our approximation.).**

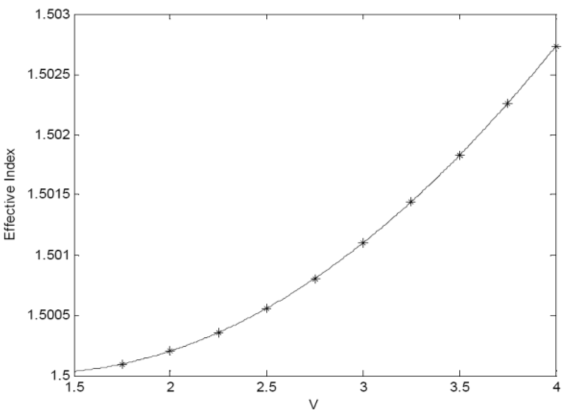

Fig.2. Variation of Effective Index with normalized frequency (*V*) for parabolic–index profile (—— exact results; ∗∗∗ results by our approximation.).

4. CONCLUSIONS

An accurate approximation of fundamental modal field involving three parameters have been presented, which can be successfully used to estimate effective index, normalized propagation constant, normalized group delay and modal dispersion parameter of a graded–index fiber. Since, accurate predictions of dispersion characteristics have immense importance in current trend of designing high capacity optical fiber in long haul transmission system, the proposed model can be used to obtain dispersion and propagation properties of graded–index fiber. Incorporating three optimized parameters, the modal field approximation has advantage of greater flexibility to meet any designer specifications. This model can also be implemented in the analysis of non linear and doped fiber amplifier.

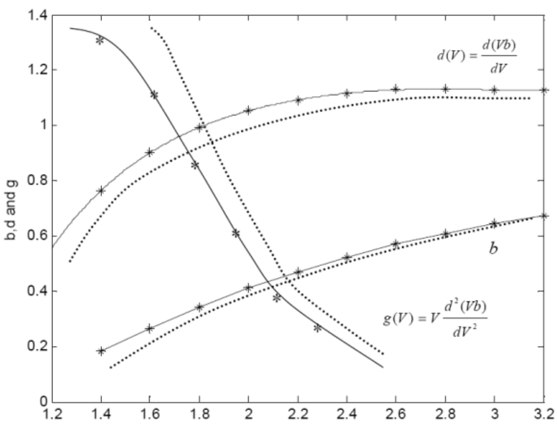

Fig.3. Variation of Normalized Propagation constant (*b*), Normalized Group Delay (*d*) and Modal Dispersion Parameter (*g*) as a function of Normalized Frequency (*V*) for step–index fiber (— exact results; ∗∗∗ results by our approximation; ----- results based on Gaussian Approximation.).

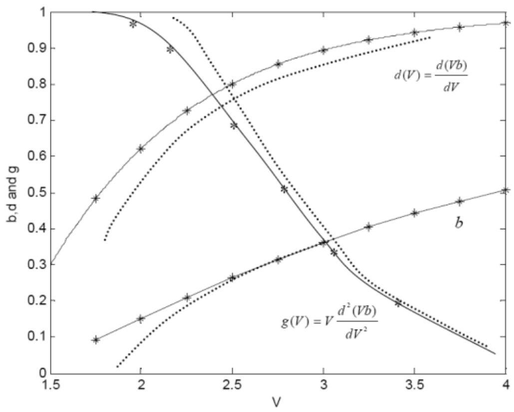

Fig.4. Variation of Normalized Propagation constant (*b*), Normalized Group Delay (*d*) and Modal Dispersion Parameter (*g*) as a function of Normalized Frequency (*V*) for parabolic–index fiber (— exact results; ∗∗∗ results by our approximation; ----- results based on Gaussian Approximation.).

5. REFERENCES

[1] D. Marcuse, "Gaussian approximation of the fundamental mode of graded-index fibers," *J. Opt. Soc. Am.* 68, 103-109 (1978).

[2] A. Sharma and A. K. Ghatak, "A variational analysis of single mode graded-index fibers," *Opt. Comm.* 36, 22-24 (1981).

[3] A. Sharma, S. I. Hossain and A. K. Ghatak, "The fundamental mode of graded-index fibers: Simple and accurate variational methods," *Opt. and Quantum Elect.* 14, 7-15 (1982).

[4] S. C. Chao, W. H. Tasi and M. S. Wu, "Extended Gaussian approximation for single-mode graded-index fibers," *J. Lightwave Tech*. 12, 392-395 (1994).

[5] A. Ankiewicz and G. D. Peng, "Generalized Gaussian approximation for single-mode fibers" *J. Lightwave Tech.* 10 (1), 22-27 (1992).

[6] G. De Angelis, G. Panariello and A. Scaglione, "A variational method to approximate the field of weakly guiding optical fibers by Laguerre-Gauss/Bessel expansion," *J. Lightwave Tech.* 17, 2665-2674 (1999).

[7] Q. Cao and S. Chi, "Approximate analytical description for fundamental-mode fields of graded-index fibers: Beyond the Gaussian approximation," *J. Lightwave Tech.* 18 (1), 54-59 (2001).

[8] A. K. Ghatak, R. Srivastava, I. F. Faria, K. Tyagarajan and R. Tewari, "Accurate method for characterizing single mode fibers: Theory and Experiment," *Elect.Lett.* 19(3), 97-99 (1983).

[9] J.C. Lagarias, J.A Reeds, M.H. Wright, and P.E Wright, "Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions", *SIAM J. Optim.* vol. 9, (1998), pp. 112–147.

[10] J.M Parkinson. and, D Hutchinson, "An investigation into the efficiency of variants on the simplex method", in *Numerical Methods for Nonlinear Optimization*, F.A. Lootsma (Ed.), Academic Press, New York, 1972, pp. 115–135,.

[11] M.H. Wright,., “Direct Search Methods: Once Scorned, Now Respectable”, in *Numerical Analysis 1995, Proceedings of the 1995 Dundee Biennial Conference in Numerical Analysis*, D.F. Griffiths and G.A. Watson (Eds.), Addison Wesley Longman, Harlow, UK, (1996), pp. 191–208,.

[12] J.A. Nelder. And R. Mead, “A simplex method for function minimization”, *Comput. J.* vol. 7, (1965), pp. 308–313,

[13] T.H Rowan, “*Functional Stability Analysis of Numerical Algorithms*”, Ph.D. thesis, University of Texas, Austin, 1990.

[14] Sanja Singer, and Saša Singer, “Complexity Analysis of Nelder-Mead Search Iterations”, in *Proceedings of the 1. Conference on Applied Mathematics and Computation, Dubrovnik, Croatia, 1999*, M. Rogina, V. Hari, N. Limić, and Z. Tutek (Eds.), PMF–Matematički odjel, Zagreb, (2001), pp. 185–196.

[15] Saša Singer, and Sanja Singer, , “Efficient Implementation of the Nelder-Mead Search Algorithm”, *Appl. Numer. Anal. Comput. Math.* vol. 1, No. 3,(2004), pp. 524–534.

[16] I. S. Gradshteyn and I. M Ryzhik, Table of Integrals, series and products, (Academic, 1980).

[17] M.Abramowitz and I. A Stegun, Handbook of Mathematical Functions, (Dover, 1981).

[18] K. Kawano, T. Kitoh: “Introduction to Optical Waveguide Analysis”, (John Willy and Sons, 2001)

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