Fourphase Pulse Compression Sequences Generated using Chaotic Maps

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ABSTRACT
Sequences with good correlation properties are in general required in various applications such as radar, sonar, spread spectrum and multiple access communication. The optimal sequences are the ones whose peak sidelobe of the aperiodic autocorrelation function is the minimum possible for a given sequence length. A chaotic sequence that provides auto and cross-correlation properties that are similar to those of random white noise is used for radar and spread spectrum systems. This paper presents a novel method of generating four phase codes applicable for radar pulse compression, which is derived from chaotic maps. It is also feasible to achieve simultaneously superior performances in detection range and range resolution using the proposed four phase coded sequences. The properties of these sequences like autocorrelation function, ambiguity function, Peak Sidelobe Ratio (PSR) and Integrated Sidelobe Ratio (ISR) have been studied. The properties of the codes generated by chaotic mapping method are very similar to those of the random four phase codes. The generation of these sequences is very simple, the available number of sequences is virtually infinite and number of sequences generated is not limited by the length of sequence.

Keywords
Pulse compression; peak sidelobe ratio; integrated sidelobe ratio; chaotic map;

1. INTRODUCTION
Pulse compression has been extensively used in operational radar systems since 1950s to solve the practical problem of extending the operating radar range while maintaining the required range accuracy and resolution. It finds its importance in various fields like radar, sonar, seismic exploration and spread spectrum communication to achieve the high transmit energy of a long pulse while preserving the range resolution of a short pulse. Pulse compression thus allows the use of low power transmitters with longer pulse lengths to maintain the energy content of a pulse. A matched filter, the front end of the receiver is used to maximize the signal to noise ratio of the received signal. Various pulse compressed waveforms have been used for this purpose, including the linear frequency modulated signal, the nonlinear frequency modulated signal, biphase modulated sequences and polyphase modulated sequences [1-3]. Thecode or sequence is used to describe the phase of the individual sub pulses of a phase coded waveform [4]. Each type of coded waveform has certain desirable properties and the choice often depends on the application.

Theoretically, in pulse compression, the code is used to modulate the carrier waveform during transmission, which is used as the reference signal at the receiver. At the receiver front end, the reference signal is used to combine with the received signal to achieve a high range resolution. Phase coded waveforms divide the long pulse into N time segments, referred to as chips, and apply a different phase to each. A binary phase coded waveform limits the chip phase to 0 or 180 degrees, while polyphase coded waveform support more levels [5]. The binary sequences can be easily generated, processed and stored in digital circuitry. But the limitation comes when longer length sequences with lower Peak Sidelobe Ratio (PSR) are needed. The Peak Sidelobe Ratio is defined from the autocorrelation pattern as the ratio of the peak sidelobe amplitude to the mainlobe peak amplitude and is expressed in decibels. It was proved that the binary Barker sequences exist only up to a length of 13 with a PSR of -22.3 dB [6]. But in some applications sequences with further reduction in PSR are required. To achieve this, Boehmer [7], Linder [8], Rao and Reddy [9] have obtained longer length binary codes violating the Barker conditions. But as the length of the sequence increases, the PSR did not reduce much. Therefore, it became much essential to switch over from biphase to fourphase sequences.

Though fourphase sequences do not have ideal energy efficiency like binary sequences, better sequences are available at larger lengths. The binary Barker sequences do not exist for length greater than 13, but four phase Barker sequences do exist at length 15. The generation of fourphase 15-bit generalized Barker sequence using acoustic surface waves is reported [10,11]. Richard J. Turyn proved that there are no even length Barker sequences of lengths 6, 8, 10, 12, 14 and also that there are no new fourphase Barker sequences of length ≤ 31 [12]. SAW devices are also used to generate fourphase sequences [13]. J. W. Taylor, Jr. generated good quadriphase or fourphase sequences using biphase to quadriphase transformation [14]. The advantages of these sequences are in the spectrum fall-off rate and the relative ease in which it lends itself to digital processing. Comparison with conventional pulse coding, in particular Barker's and Turyn'sbiphase sequences, the autocorrelation function is preserved when converted to fourphase sequences. The Doppler behavior of the quadriphase codes is proved by Taylor and Blinchikoff to be the same as the Doppler behavior of biphase codes. However, Levanon and Freedman has proved that the ambiguity diagram with a nonzero Doppler shift can be significantly different for a quadriphase code than that of the biphase code from which it was derived [15]. The ambiguity diagram of a quadriphase code derivedfrom a Barker code of length 13 has a diagonal ridge more like that of a linear-FM ambiguity function rather thanthethumbtack ambiguity function of the Barker code. But using these methods generation of fourphase sequences at large lengths has not been reported.

In this paper, a unique chaotic method of generating four phase radar signal having good correlation property is described. The use of chaotic sequences in spread spectrum communications is well documented [16]. A binary phase code generated using

chaotic logistic map to achieve a low PSR was reported earlier [17]. Similar concept has been extended to the generation of good ternary sequences [21]. In that the ternary sequences were proved to exhibit superior behavior than binary sequences at all lengths. In this paper the generation of fourphase sequences using chaotic maps is discussed. Results show that the fourphase sequences perform better than binary and ternary sequences. The method discussed here has an advantage over the previous methods already available for fourphase sequence generation. Here, it is possible to generate sequences at larger lengths easily as it is not an exhaustive search method.

2. CHAOTIC MAPS

Chaos theory is a field of study in mathematics, having applications in several disciplines including physics, engineering, economics, biology and philosophy. Chaos theory explores the strange behaviour found in nonlinear deterministic dynamical systems that are highly sensitive to initial conditions. This effect is popularly referred to as the butterfly effect. A very small disturbance in initial condition produces a drastically different final solution for the chaotic system and the long-term prediction is not possible. This happens even though these systems are deterministic, meaning that their future behavior is fully determined by their initial conditions, with no random elements involved in this process. This behavior is known as deterministic chaos, or simply chaos [18]. Chaotic Sequences can be generated using different types of chaotic maps. Some of the maps are Henon Map, Lorenz Map, Logistic Map, Improved logistic map, quadratic map, Tent map and Cubic map. Chaotic generated sequences have also been considered for the implementation of CDMA systems. The greater number of sequences and their inherent security improvement have been considered as the main advantages of chaotic generated sequences. In this paper an attempt is made to generate fourphase sequences using logistic map, improved logistic map, quadratic map and Cubic map and their properties like PSR and ISR for different lengths are studied [19-20].

2.1 The Logistic map

The logistic map is a second order polynomial mapping, often considered as a prototype model for how complex, chaotic behaviour can arise from very simple non-linear dynamical equations.

The equation for logistic map is written as

$$x_{n+1} = f(x_n) = \mu x_n (1 - x_n), \quad x_n \in (0,1) \quad (1)$$

Where, the constant μ is called the bifurcation parameter with $\mu \in (0, 4)$. The logistic equation is the basic example to illustrate the behaviour that is stable, periodic, and chaotic which depends on the value of μ in the equation. The series tends to a single value when μ is selected as $0 < \mu < 1$, independent of the initial value. With μ value between 1 and 3, the series will approach the value $(\mu - 1)/\mu$, independent of the initial value. The rate of convergence islinear, except for $\mu=3$. With μ value between 3 and 3.45 it can be observed that the system oscillates between 2 states for almost all initial conditions. With μ value between 3.5 and 3.54 the system oscillates between 4 states. When the system jumps between these states and it does not pass through any intermediary values. As the value of μ increases beyond 3.54 the number of states also increases to 8, 16 etc. This behavior is an example of a period-doubling cascade. The system undergoes increasingly frequent period doubling until it enters the chaotic regime at about 3.57. The system is in chaotic state when the value of μ is between 3.57 and 4. Beyond $\mu=4$, the value of x eventually leave the interval (0, 1) and diverge for almost all initial values.

Figure 1 shows the bifurcation diagram for the logistic map for all values of μ between 2.4 and 4. The horizontal axis shows the values of the parameter μ while the vertical axis shows the possible long term values of x and it can be seen that the value of x is within the interval (0,1) as long as the initial value is from the interval.
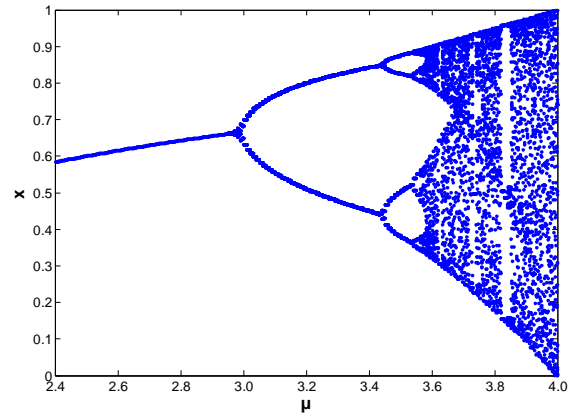

Fig1: Bifurcation diagram for the logistic map

The sequences generated using logistic map is inherently deterministic and a tiny disturbance in the initial condition x_o gives rise to a tremendous difference in the outcome. This sensitive dependence can be illustrated by giving two very close initial points to the iterative map. After a few iterations, the two resulting sequences will look completely uncorrelated. Figure2 shows two such sequences for $\mu=4$ with $x_o=0.1$ and 0.10001 respectively. This small difference in the initial value leads to two completely different waveforms after a few iterations.
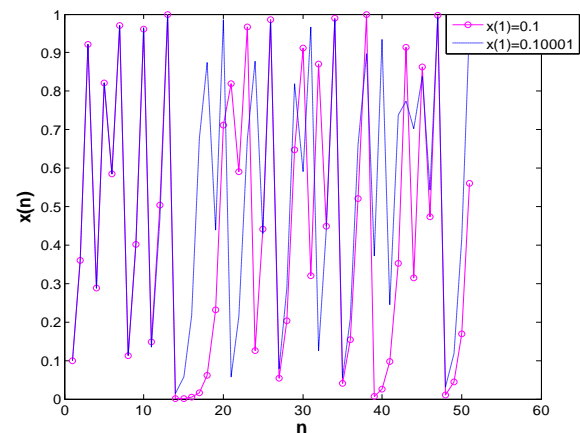

Fig 2:Chaotic behaviour of logistic map for $x_0 = 0.1, 0.10001$

2.2 The Improved Logistic Map

The improved logistic map is governed by the equation

$$x_{n+1} = f(x_n) = 1 - 2(x_n)^2, \quad x_n \in (-1,1) \quad (2)$$

The chaotic sequence generated by improved logistic map is neither periodic nor converging and it is sensitively dependent on initial value. This equation shows chaotic behaviour for a initial value of x_0 varying from -1 to 1. When the initial value x_0 is beyond this limit, x_n tends to infinity as n tends to infinity.

2.3 The Cubic Map

This map is described by the following equation

$$x_{n+1} = f(x_n) = 4(x_n)^3 - 3x_n, \quad x_n \in (-1,1) \quad (3)$$

This equation also shows chaotic behaviour for an initial value of x_0 varying from -1 to 1 like the improved logistic map. When $x_0 > 1$ as n tends to infinity, x_n also tends to infinity.

2.4 The Quadratic Map

The quadratic map is defined by the equation

$$x_{n+1} = f(x_n) = 0.5 - 4(x_n)^2, \quad x_n \in (-1,1) \quad (4)$$

Like improved logistic map this equation shows chaotic behaviour for an initial value of x_0 varying from -1 to 1. When $x_0 > 1$ as n tends to infinity, x_n also tends to infinity.

3. GENERATION OF FOURPHASE SEQUENCES

Even though there exists many methods for the generation of binary or fourphasesequences, they are either exhaustive search methods or they are techniques used to find only a finite number of sequences at a particular length. So, there is the chance of good sequences being neglected. Here, the method proposed for generation using chaotic maps has the advantage of generating virtually infinite number of sequences at any length. Thus this method has the advantage of finding almost all good sequences at a particular length. A Fourphase sequence usesfour alphabets for representation of data. A fourphase code is one whose subpulses are phased in one of four states: 0, 90, 180, 270 degrees; or equivalently +1, +j, -1, or –j. The algorithm or method used for generating the fourphase sequence of length N using a chaotic map is given below

Step 1. Ensure that initially the chaotic map is in the chaotic region. For example, in logistic map the bifurcation parameter µ is selected to be 4 so that the map is in chaotic region.

Step 2. Select an initial value for x_n.

Step3. Using the chaotic map equations generate a raw sequence $x_n$of length N.

Step.4. The sequence is then quantized into four defined levels based on the threshold levels a, b and cas given in equation (5) to obtain the fourphase sequence y_n. The threshold levels were chosen randomly between minimum and maximum values of x.For example, the x value varies between 0 and 1 for the logistic map. Therefore the threshold levels are chosen considering approximately the mean of the raw sequence obtained which is 0.5 in the case of logistic map. The level 'a' is chosen half of the mean of the raw sequence and level 'c' is chosen above the ¾ of the maximum value of the raw sequence, whereas the level 'b' is chosen as the mean value itself.

$$
\begin{aligned}
y_n &= -1 \quad if\ x_n < a \qquad\qquad (5)\\
&= -j \quad if\ a \le x_n < b\\
&= +j \quad if\ b \le x_n < c\\
&= +1 \quad if\ x_n \ge c
\end{aligned}
$$

The various maps are extremely sensitive to the initial condition. By varying the initial condition x_o, a totally uncorrelated fourphasesequence can be obtained. Therefore, the number of available sequences at a given length is virtually infinite.

Using the chaotic maps good fourphasesequences were generated for varied lengths using the above mentioned procedure. For logistic map the threshold levels a, b and c is selected as 0.25, 0.5 and 0.75 respectively. The values of a, b and c are chosen as -0.6, 0 and 0.6 for improved logistic map, -0.5, 0 and 0.4 for cubic map and -0.25, 0, 0.25 for quadratic map respectively. These levels are chosen after exhaustive searches of good sequences were done with different threshold levels. Thus the generation of sequences is simple, fast and reproducible. Using the above mentioned procedure fourphase sequences were generated for lengths from 10 to 5000 using the four chaotic maps. At every length the good sequences having a low value of PSR and ISR as defined by equation (7) and equation (8) are obtained. Several optimization techniques can be used to optimize the threshold levels, so that good sequences with good correlation properties can be obtained.

4. PROPERTIES OF FOURPHASE SEQUENCES

4.1 Correlaton function

The aperiodic autocorrelation r(k) of a sequence of length Nwhich is nothing but the output of the matched filter.Therefore the matched filter compressed output is givenby

$$r(k) = \sum_{n=0}^{N-1-k} x_n x^*_{n+k} \quad ---- \quad (6)$$

Where k =-(N-1)….+(N-1)

Theoretically, the autocorrelation characteristic of an infinitely long chaotic coded signal is nearly a Dirac Delta function. However, all the finite length sequences cannot be utilized due to rapid deterioration of correlation performance.

Thus the best sequences are selected based on the performance factor PSR. By varying the initial conditions a search for sequences with good autocorrelation pattern was done at every length for all the maps discussed above. The autocorrelation pattern of the good sequence generated using logistic map at length 5000 is shown in Fig.3. The figure has the shape of an impulse which is actually the desired response.

4.2 Peak Sidelobe Ratio (PSR)

One of the most commonly used performance measures is the Peak Sidelobe Ratio. The peak sidelobe is the largest sidelobe in the correlation of a sequence. The Peak Sidelobe Ratio is defined from the autocorrelation pattern $r(k)$ as the ratio of the maximum peak sidelobe amplitude to the mainlobe peak amplitude and is expressed in decibels. It is given by

$$PSR = 20\ log \frac{Max\ |r(k)|}{r(0)} \quad where\ k \ne 0 ---- \quad (7)$$

A sequence is defined as a good sequence if the PSR of the sequence is low. The reciprocal of PSR is known as the Discrimination Factor (D.F), which is expected to have a maximum value for better range detection.
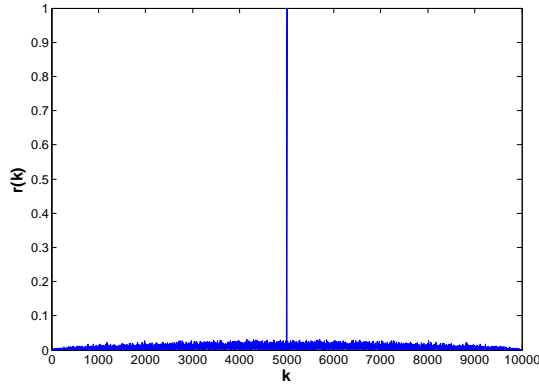
Fig 3:Normalized Autocorrelation pattern of logistic map of sequence length 5000

4.3 Integrated Sidelobe ratio

The Integrated Sidelobe Ratiois defined as the ratio of the total energy in the sidelobesto the energy in the main peak of the autocorrelation pattern and is expressed in decibels.

$$ISR = 10\,log\,\frac{2\sum_{k=1}^{N-1}r^2(k)}{r^2(0)} \quad where\ k \neq 0 ----- \quad (8)$$

A sequence is defined as a good sequence if the ISR evaluated for that sequence is as minimum as possible.

Although the fourphase sequence generation process is quite general and easy by chaotic mapping methods, all sequences generated are not good or adequate for the pulse compression application in radar. So a complete search for good sequences has to be done by varying the initial conditions in each mapping method and from all the generated sequences, the best sequences are selected based on the PSR and ISR criterion defined by the equations (7) and (8) respectively. The Peak Sidelobe Ratio and Integrated Sidelobe Ratio obtained for good four phase sequences at different lengths for all the maps are tabulated in Table.1 and Table.2. As observed from the Tables 1 and 2, the PSR decreases with the length of the sequence for logistic, improved logistic and quadratic maps. But, the PSR obtained with cubic map was found to decrease at lower lengths and increase at larger lengths. The variation of PSR as a function of sequence length for all the four chaotic maps can be clearly observed in the plot shown in Figure 4. From this plot it is clear that the three chaotic maps namely, the logistic map, the improved logistic map and the quadratic map work equally well at all lengths in the generation of good four phase sequences than the cubic map.
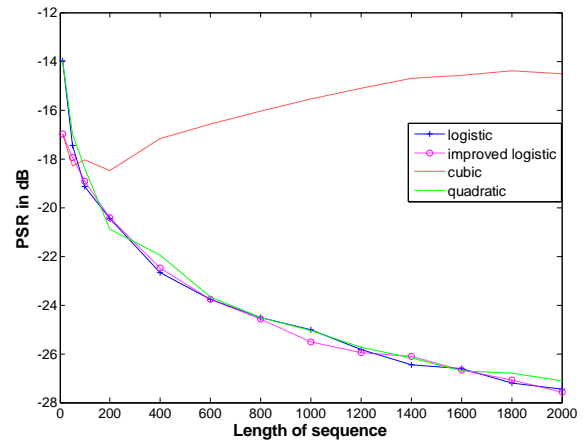

Fig4: PSR of fourphase sequences generated using Chaotic maps

Table 1.PSR and ISR variations of the fourphase sequences using logistic and improved logistic maps

Length of sequence	Logistic map		Improved logisticmap	
	PSR in dB	ISR in dB	PSR in dB	ISR in dB
10	-13.9794	-4.6852	-16.9897	-5.8503
20	-16.4782	-3.9794	-16.4782	-4.5593
30	-17.2379	-3.6318	-16.5321	-3.2078
40	-17.4172	-2.9243	-17.4172	-2.7573
50	-17.4473	-2.1098	-17.9588	-3.1516
60	-18.4030	-2.4243	-18.3203	-2.3326
70	-18.3286	-2.0532	-18.7728	-2.8914
80	-18.7676	-2.3331	-18.1941	-2.4270
90	-18.4403	-1.8437	-18.5541	-1.9500
100	-19.1364	-1.7289	-18.9279	-1.2587
200	-20.4455	-1.1390	-20.4335	-1.4746
300	-21.6891	-0.5392	-21.6891	-0.9004
400	-22.6660	-0.9594	-22.4940	-0.3774
500	-23.3348	-0.6607	-23.1158	-0.5364
600	-23.7618	-0.5632	-23.7646	-0.2865
700	-24.2302	-0.6677	-23.8035	-0.5109
800	-24.5265	-0.3980	-24.5710	-0.4438
900	-24.9151	-0.5260	-24.8655	-0.4722
1000	-25.0018	-0.5608	-25.5145	-0.4312
1500	-26.4760	-0.4061	-26.5252	-0.3337
2000	-27.4461	-0.4450	-27.5845	-0.3434
2500	-28.1180	-0.2294	-28.1135	-0.0976
3000	-28.7633	-0.1679	-28.7851	-0.2955
3500	-29.2245	-0.2568	-29.5716	-0.1854
4000	-29.7191	-0.1427	-29.6285	-0.1583
4500	-30.1026	-0.1958	-30.1717	-0.1641
5000	-30.5327	-0.1988	-30.1717	-0.1960

Table 2.PSR and ISR variations of the fourphase sequences using cubic and quadratic maps

Length of sequence	Cubic map		Quadratic map	
	PSR in dB	ISR in dB	PSR in dB	ISR in dB
10	-16.9897	-5.8504	-13.9794	-4.6852
20	-16.4782	-4.2022	-16.4782	-3.9794
30	-17.2379	-3.3724	-17.2379	-3.6318
40	-16.9897	-3.5902	-17.4172	-2.9243
50	-18.2974	-2.7935	-16.9897	-1.6800
60	-17.4339	-1.2752	-17.9287	-2.2185
70	-17.8711	-2.5723	-17.7638	-1.7671
80	-18.0186	-1.0272	-18.1495	-1.7393
90	-17.7495	-1.4491	-18.5541	-1.7337
100	-18.0410	-1.7941	-18.3863	-1.3525
200	-18.4695	-1.0282	-20.9018	-1.3847
300	-18.0287	0.4400	-21.6891	-0.6461
400	-17.1769	0.5622	-21.9680	-1.0392
500	-17.2170	0.7514	-23.3498	-0.9909
600	-16.5703	0.1131	-23.6709	-0.4783
700	-15.8445	0.1802	-23.9507	-0.0455
800	-16.0337	-0.1368	-24.5035	-0.5426
900	-15.9115	0.1320	-24.5747	-0.3385
1000	-15.5619	0.3254	-25.0515	-0.5088
1500	-14.5442	0.7812	-26.2261	-0.3840
2000	-14.5242	0.7143	-27.1036	-0.2863
2500	-14.2693	0.5901	-27.9186	-0.1295
3000	-13.9031	0.7415	-28.4958	-0.2024
3500	-13.7739	0.8742	-29.0296	-0.3253
4000	-13.5616	0.6852	-29.5605	-0.0111
4500	-13.4657	0.8932	-30.1744	-0.0307
5000	-13.5458	0.9144	-30.4412	-0.1379

The plot of variation of ISR for the good sequences at lengths up to 2000 is shown in Figure 5. The ISR of the good sequences is not found to be satisfactory especially at higher lengths. From the plot it is understood that the ISR of the sequences generated using logistic map, improved logistic map and quadratic map is better than that of cubic map.

Now, the performance of four phase sequences is compared with that of binary and ternary sequences generated using the logistic mapping method [17, 21]. The comparison plot of PSR of the binary, ternary and fourphase sequences are shown in Figure 6. The PSR of the four phase sequence decreases as the length of the sequence increases. It is found that at lower lengths up to 50, the PSR of ternary sequences is better than the binary and four phase sequences. But beyond length 50, the four phase sequences perform well compared to the binary and ternary sequences. As the chaotic sequence generation at higher lengths is easier, the

four phase codes performance is superior compared to other codes at higher lengths.
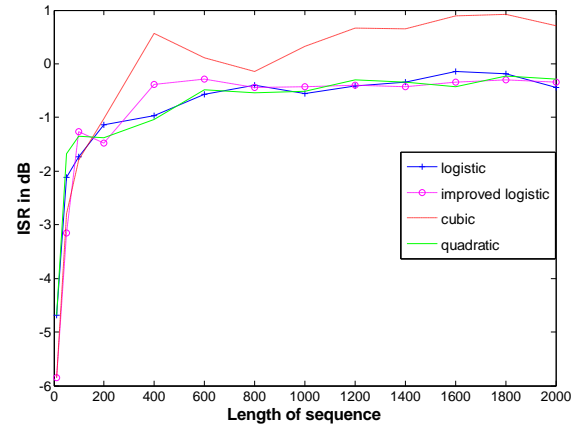

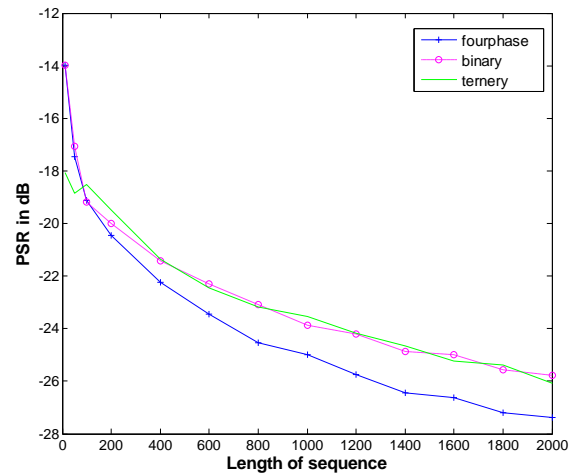**Fig 5: ISR of fourphase sequences generated using chaotic maps**

Fig6: Plot of PSR of binary, ternary and fourphase sequencesgenerated using logistic map

5. AMBIGUITY FUNCTION

The ambiguity function describes the complex envelope of the signal at the output of the radar receiver as a function of radar target range and radial velocity. The ambiguity function describes the output of a matched filter when the input signal is delayed by τ and Doppler shifted by ν relativeto nominal values for which the matched filter was designed. The ambiguity function is defined as

$$\chi(\tau, v) = \left| \int_{-\infty}^{\infty} u(t) u^*(t + \tau) \exp(j2\pi vt)\, dt \right| \qquad (9)$$

u(t) represents the envelop of the fourphase coded waveform. The time delay is indicated by τ and ν is the radial velocity depends on Doppler frequency. The ambiguity diagram of a 100 length fourphase sequence generated using logistic chaotic map is shown in figure 7. The ambiguity diagram approaches a thumbtack shape. The resolution of the waveform depends on the width of the spike centered at |x(0,0)|. As the width of the spike is very less, two closely spaced targets can be resolved easily both in range and velocity.
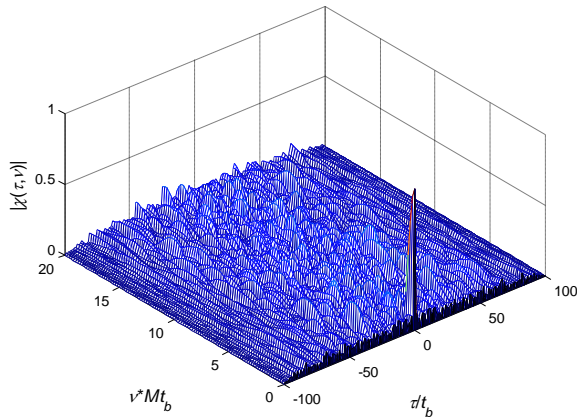
Fig7: Ambiguity diagram of 100 length fourphase sequence

6. CONCLUSION

Good four phase sequences are generated using logistic, improved logistic, quadratic and chaotic mapping method. The optimal codes are selected at various lengths based on the minimum Peak Sidelobe Ratio according to the performance requirements of the radar system. At larger lengths these codes offered very low PSR which is advantageous for the radar applications and spread spectrum communications. For lengths equal to and greater than 3000, PSR less than -28dB was obtained. These codes are able to achieve desired detection range and range resolution. Further improvement in performance can be achieved by applying optimization techniques for the three parameters namely the initial value x_0 of this mapping, the incremental value and the quantization levels and better sequences can be obtained.

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