Relative Superior Julia Sets for Complex Carotid-Kundalini Function

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ABSTRACT
Carotid Kundalini function broadly known as C-K function was introduced by Gordon R.J.Cooper. It is given by the function $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ where z,c andN are complex constants. Cooper presented interesting Julia sets by taking c=(0,0).Rani and Negi introduced a new process for generation of the C-K function and obtained interesting variants of Julia set generated by Cooper an some exciting figures for parameter $N > 2$, for values of c other than (0, 0). In this paper we apply a different iteration process for generation of the Julia set for C-K function and will call them relative superiorC-K Julia sets. Further, different properties like trajectories and fixed point arealso discussed in the paper. We also obtain some exciting figures for the C-K function for values of c other than (0, 0).

General Terms
Your general terms must be any term which can be used for general classification of the submitted material such as Pattern Recognition, Security, Algorithms et. al.

Keywords
Carotid Kundalini function, Ishikawa iteration, Relative superior C-K Julia set.

1. INTRODUCTION
In mathematics the Mandelbrot set, named after Benoit Mandelbrot, is a set of points in the complex plane, the boundary of which forms a fractal. Mathematically, the Mandelbrot set can be defined as the set of complex values of c for which the orbit of 0 under iteration of the complex quadratic polynomial $z_{n+1} = z_n^2 + c$ remains bounded see[1-16]. Julia sets [13] provide a most striking illustration of how an apparently simple process can lead to highly intricate sets. Function on the complex plain c as simple as $z_{n+1} = z_n^2 + c$ give rise of fractals of exotic appearance. Using other functions in place of this function yield Julia sets of different kinds.

One such function is Carotid-Kundalini function given by $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ where N and c are parameters and $\cos(z) = \left(\frac{e^{iz} + e^{-iz}}{2}\right)$. This function falls under the category of transcendental functions, the study of which was initiated by Fatou[10]. Points with unbounded orbits are not in Fatou sets but they must lie in Julia sets. Attractive points of a function have a basin of attraction, which may be disconnected. A point z in Julia for cosine function has an orbit that satisfies $|\mathrm{Im}(z)| \geq 50$.

The iteration of complex analytic function F decomposes the complex plane into two complementary sets:

Julia set, which consists of values with the property such that an arbitrarily small perturbation can cause drastic changes in the sequence of iterated function values. The iteration on the Julia set exhibits ‘chaotic’ behaviour, &

Fatou set, which consists of values with the property that all nearby values behave similarly under repeated iteration of the function. The iteration on Fatou set exhibits a “regular” behaviour.

Günter Rottenfußer, Dierk Schleicher[14] showed that the escaping points which converge to ∞ under iteration of cosine functions are organized in the form of dynamic rays and, as in the exponential family, every escaping point is either on one of these rays or the landing point of a unique ray. It is well known that the set of escaping points is an open neighbourhood of ∞, which can be parameterized by dynamic rays. For the entire transcendental functions, the point ∞ is an essential singularity (rather than super attracting point). Eremenko[9] studied that for entire transcendental functions, the set of escaping points is always non-empty. R.L. Devaney, see [3-7], provided answers to his queries, for the special case of exponential function, where every escaping point can be connected to ∞, along with unique curve running entirely through the escaping points.

A dynamic ray is connected component of the escaping set, removing the landing points. It turns out to be union of all uncountable many dynamic rays, having Hausdorff dimension equal to one. However, by a result of McMullen[15], the set of escaping points of a cosine family has an infinite planar Lesbegue measure. Therefore, the entire measure of escaping points sits in the landing points of those rays which land at the escaping points.

Recently, many cosine functions of the following forms have been considered by various researchers, see[20,22-25]:

$$\cos(z^n + c), where\ n \geq 2$$

$$f(z) = (e^{iz} + e^{-iz})/2$$

In this paper we have usedthe cosine function of the type $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ where N and c are parameters and $\cos(z) = \left(\frac{e^{iz} + e^{-iz}}{2}\right)$. This function is also known as Carotid- Kundalini(C-K) function [1, 2, 24]. When the C-K equation which is given by:

$$z_{n+1} = K_{n,c}(z_n) \quad (1)$$

Where n= number of iterations, is iterated, many fascinating properties are revealed as given by Cooper[1-2].By using the above equation, Cooper produced a range of fractal Julia sets in the complex plane as parameters c and N were varied [2]. Also periodic behaviour was observed in some trajectories that remained bounded. Recently, Rani and Negi [17] studied the C-K equation by using superior iterates given by:

$$z_{n+1} = s(K_{n,c}(z_n)) + (1-s)z_n \quad (2)$$

where $0 < s \le 1$.

Following Rani and Kumar [11], the orbit generated by equation (2) was called the superior iterations. In this paper we have applied a new iteration process i.e. Ishikawa iteration to the C-K function given by:

$$y_n = s'_n(K_{n,c}(z_n)) + (1-s'_n)x_n$$
$$x_{n+1} = s_n(K_{n,c}(y_n)) + (1-s_n)x_n \quad (3)$$

where $0 < s'_n \le 1, 0 < s_n \le 1$ and s_n, s'_n are convergent to non zero number.

Following the study of Rana et. al. [19], the orbit generated by the equation (3) will be called relative superior orbit. Julia sets may be computed for any complex map. In this paper, we focus our study on Julia sets for Carotid-Kundalini(C-K) functions using Ishikawa iterates. We obtain new C-K Julia sets using relative superior Iterates. Here we have used different valuesof the new parameter s' and the values for the parameter N, c and s will be taken as taken by Rani and Negi.[17]

2. PRELIMINARIES

The process of generating a fractal image from $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ consists of iterating the function up to n time's see [23].

Definition2.1: Ishikawa Iteration [16]: Let X be a subset of real or complex numbers and $f : X \to X$ .For $x_0 \in X$, we have the sequences $\{x_n\}$ *and* $\{y_n\}$ in X in the following manner:

$$y_n = s'_n f(x_n) + (1-s'_n)x_n$$
$$x_{n+1} = s_n f(y_n) + (1-s_n)x_n$$

where $0 \le s'_n \le 1,\ 0 \le s_n \le 1,\ \{s'_n\}$ & $\{s_n\}$ are convergent to non zero number

Definition 2.2[18, 19]: The sequence of $\{x_n\}$ and $\{y_n\}$ described above is called the Ishikawa sequence of iterations or Relative Superior sequences of iterates. We denote it by $RSO(x_o, s_n, s'_n, t)$. Notice that $RSO(x_o, s_n, s'_n, t)$ with $s'_n = 1$ is $SO(x_o, s_n, t)$ i.e. Mann orbit and if we place $s_n = s'_n = 1$ then $RSO(x_o, s_n, s'_n, t)$ reduces to $O(x_o, t)$. The set of points whose orbits are bounded under relative superior iteration of the function $f(z)$ is called Relative Superior Julia sets.

Definition 2.3[18, 19] Relative superior C-K sets for the function $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ where n=1,2,3,… is defined as the collection of $c \in C$ for which the orbit of 0 is bounded.

The collection of points that are bounded, i.e. there exists M, such that $|Q^n(z)| \le M$, for all n, is called as a prisoner set while the collection of points that are in the stable set of infinity is called the escape set. Hence, the boundary of the prisoner set is simultaneously the boundary of escape set and that is Julia set for Q.

Definition 2.4 [18, 19]: The set of points whose orbits are bounded under relative superior iteration of the function Q (z) is called Relative Superior Julia sets.

3. GEOMETRY OF RELATIVE SUPERIOR JULIA SETS FOR C-K FUNCTION

The fractals generated from the equation $z_{n+1} = \cos(Nz_n \cos^{-1}(z_n)) + c$ shows symmetry along the real axis for both Real and Imaginary values:

- **a)** When the powers of eq2 and function3 is kept constant and the power of first function is varied then the following properties of composite fractal us as found as follows:
  - a. At p=2,q=2,r=2 , we observe that the composite fractal exhibits symmetry across the real axes i.e. x-axis and there are 5 petals each of which contains 4 mandelbrot sets of order 2 . The fifth petal contains the fractal similar to mandelbar. This fractal contains a tail.
  - b. All the petals contains one of infinite Mandelbrot sets contained within all the petals of the composite fractal.
- **b)** For all the imaginary values we observe that the relative superior C-K Julia set consists of disjoint curves (hairs) tending to infinity in all the three arms.

4. GENERATION OF RELATIVE SUPERIOR CAROTID-KUNDALINI SETS

Fig 4.1: Relative Superior C-K set for N=(0.02,0), s=0.5 and s'=0.5

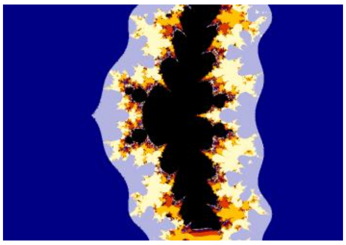


Fig4.2: Relative Superior C-K set for N=(1,0), s=0.5 and s'=0.5

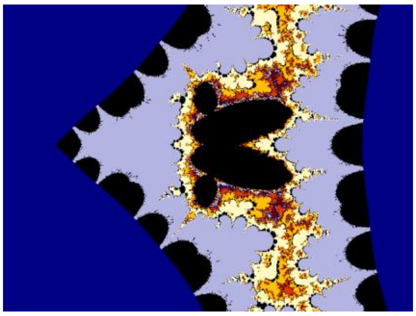


Fig. 4.3: Relative Superior C-K set for N=(1,0), s=0.1 and s'=0.1

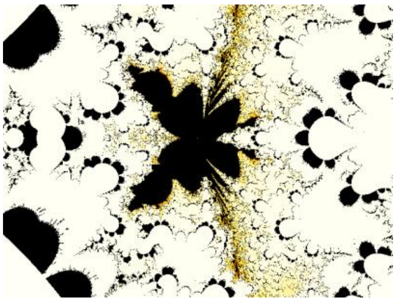


Fig 4.4: Relative Superior C-K set for N=(0,0.7), s=0.0 and s'=0.1

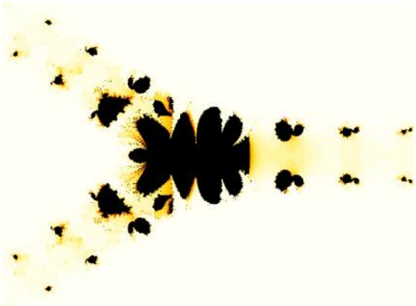


Fig. 4.5: Relative Superior CK set for N=(3,0), s=0.5 and s'=0.5

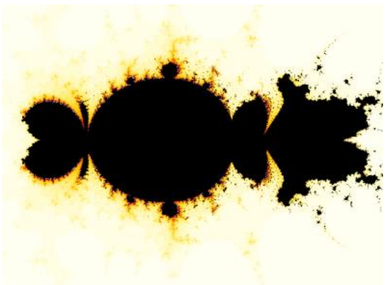


Fig. 4.6: Relative Superior CK set for N=(5,0), s=0.1and s'=0.1

Fig. 4.7: Relative Superior CK set for N=(0,3), s=0.1 and s'=0.04

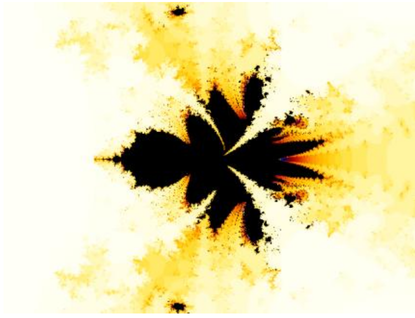
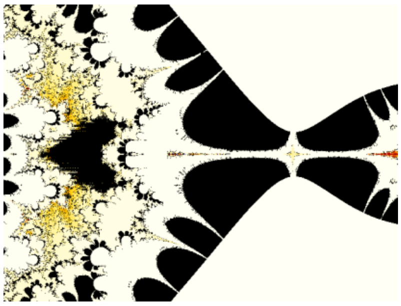


Fig.4.8: Relative Superior CK set for N=(0,14.5), s=0.01 and s'=0.01



5. GENERATION OF RELATIVE SUPERIOR CAROTID-KUNDALINI JULIA SETS

Fig 5.1: Relative Superior C-K Julia set for c=(0,0) at N=(0.02,0), s=0.5 and s'=0.5

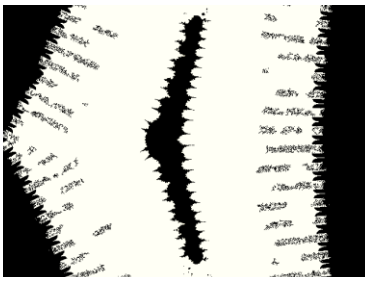


Fig 5.2: Relative Superior C-K Julia set for c = (-0.03588, 0.01962) at N=(1,0) ,s=0.5,s'=0.5

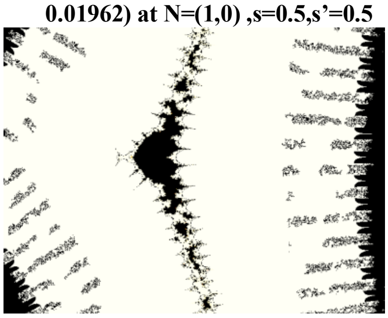


Fig 5.3: Relative Superior C-K Julia set for c=(-0.03588, 0.01962) at N=(1,0) ,s=0.1,s'=0.5

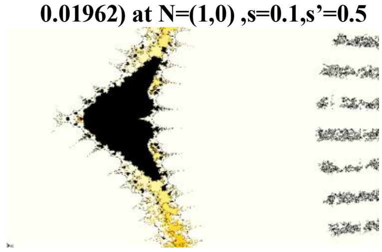


Fig 5.4: Relative Superior C-K Julia set for c = (-0.03588, 0.01962) at N=(1,0) ,s=0.1,s’=0.

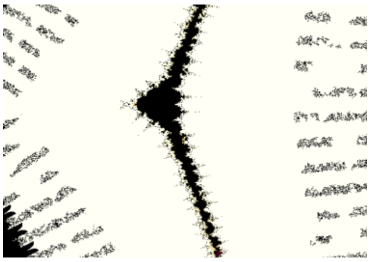


Fig 5.5: Relative Superior C-K Julia set for c= (-0.325,-0.01875) at N=(3,0), s=0.5 and s’=0.5

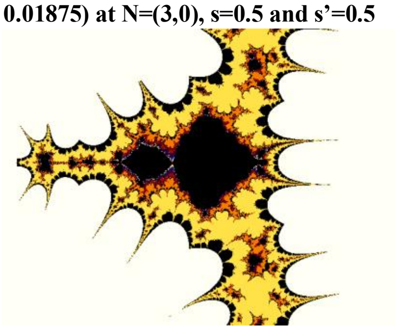


Fig 5.6 : Relative Superior C-K Julia set for c =(0.3518,-0.0803) at N=(5,0), s=0.1,s’=0.1

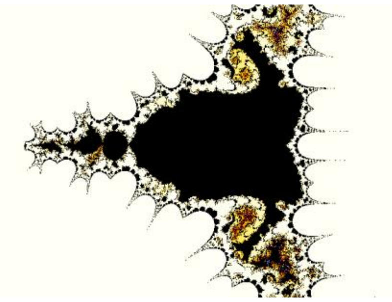


Fig.5.7. Relative Superior C-K Julia set for c=(1.0678,-0.0381) n=(0,0.7),s=0.9,s’=0.1

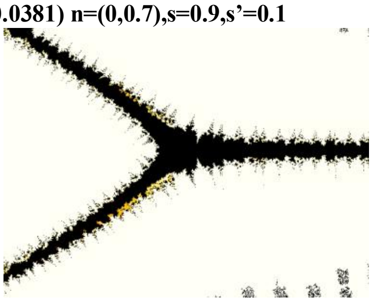


Fig 5.8: Relative Superior C-K Julia set for c=(-0.3549,-0.0015) at N=(0,3), s=0.1 and s’=0.04

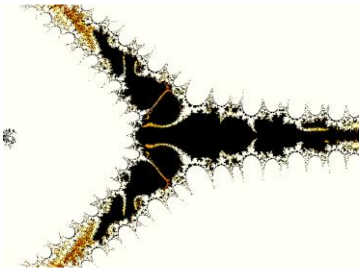


Fig.5.9: Relative Superior C-K Julia set for c =(0,0) at N=(0,14.5),s=0.01 and s’=0.01

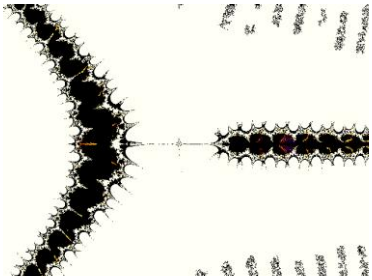


Fig.7.1: Filled relative superior C-K Julia set for c= (0,0) at N=(0.02,0),s=0.5,s’=0.5,

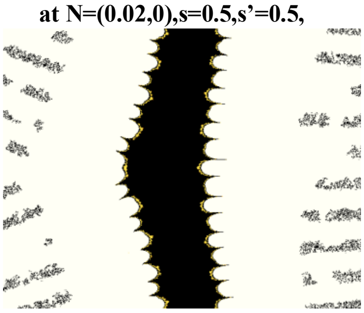


Fig. 7.2: Relative Superior C-K Julia set for c=(-0.325,-0.01875) at n= (3,0),s=0.5,s’=0.5

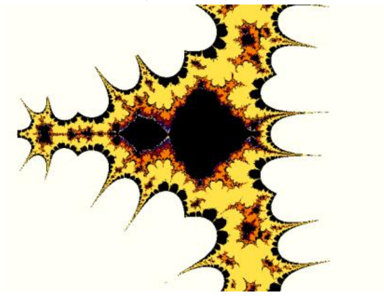


Fig. 7.3: Relative Superior C-K Julia set for c=(-1.3607, 0.0089) at N=(4.5,0),s=0.01, s’=0.5

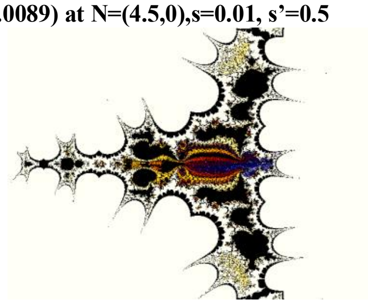


Fig. 7.4 : Relative Superior C-K Julia set for c=(-0.8407,-0.051) at N=(4.5,0), s=0.01, s’=0.5

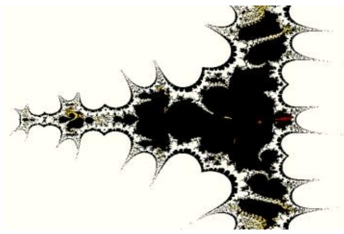


Fig 7.5: Relative Superior C-K Julia set for N=(5.5,0),s=0.05,s'=0.04,c=(0.3492,0.0875)

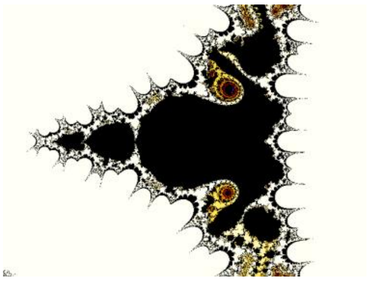


Fig.7.6: Relative Superior C-K Julia for N= (9.5, 0), s = 0.09, s'=0.1,c = (0, 0)

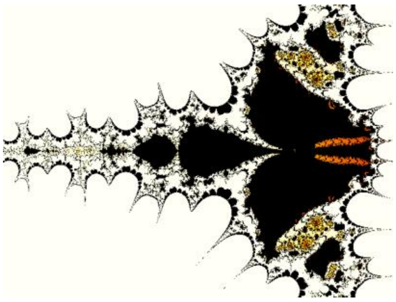


Fig.7.7: Relative Superior C-K Julia set for c=(-0.09,0.1) at n= (9.5,0) s=0.01,s'=0.5

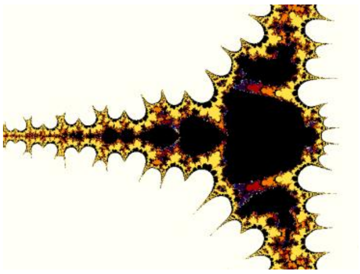


Fig.7.8: Zoom of portion of Superior Julia set for N=(0,0.2), s=0.1,s'=0.01 C=(0,0)

Fig.7.9: Relative Superior C-K Julia set for c =(-0.7374,0.0009) at N=(0,4.5), s=0.0,1s'=0.5

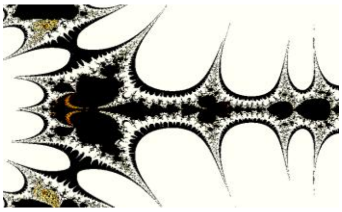


Fig. 7.10: Relative Superior C-K Julia set for c = (0, 0) at N= (0,9.5), s = 0.01, s'=0.1

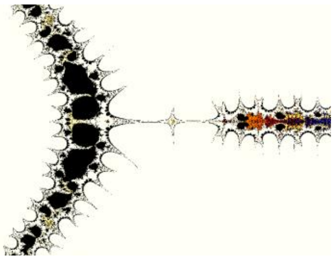


Fig. 7.11: Relative Superior C-K Julia set for c=(1.0678,-0.0381) at N =(0,0.7), s=0.9, s'=0.1

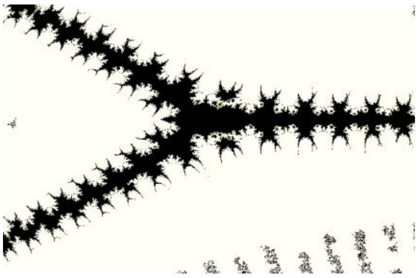


Fig. 7.12: Part of Relative Superior C-K Julia set for N= (1, 0), s = 0.6, s'=0.5,c = (0, 0)

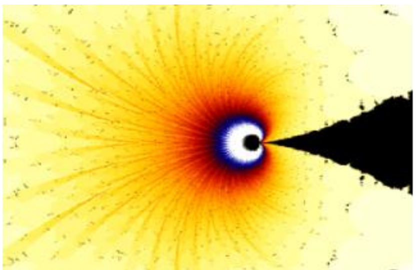
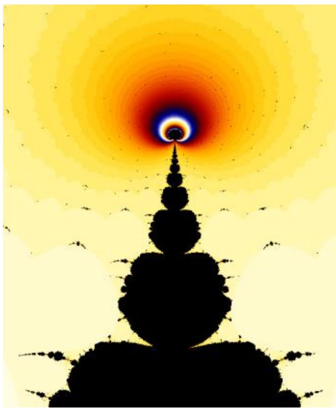


Fig. 7.13: Part of Relative Superior C-K Julia set for *N*= (1, 0), *s* = 1, s'=0.5,c = (0, 0) rotated 90 degrees.



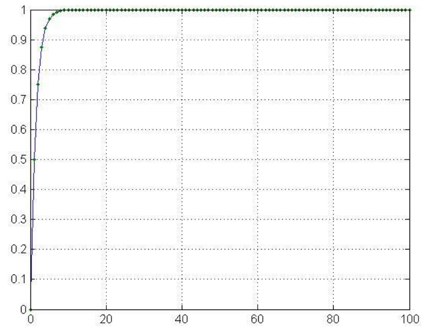
6. FIXED POINTS

6.1 Fixed points for Real

Table 6.1.1: Orbit of F(z) for c=(0,0) at N=(0.02,0), s=0.5 and s'=0.5

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	2.8528
2.	1.4614	17.	2.8528
3.	2.1883	18.	2.8528
4.	2.5381	19.	2.8528
5.	2.7046	20.	2.8528
6.	2.7836	21.	2.8528
7.	2.8208	22.	2.8528
8.	2.8382	23.	2.8528
9.	2.8462	24.	2.8528
10.	2.8499	25.	2.8528
11.	2.8515	26.	2.8528
12.	2.8523	27.	2.8528
13.	2.8526	28.	2.8528
14.	2.8527	29.	2.8528
15.	2.8528	30.	2.8528

Fig. 6.1.1: Orbit of F(z) for c=(0,0) at real(N)=0.02 and



s=0.5 and s'=0.5

Table 6.1.2: Orbit of F(z) for c=(-0.0359,0.01962) at N=(1,0) and s=0.5, s'=0.5

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	0.7935
2.	0.4020	17.	0.7936
3.	0.5894	18.	0.7936
4.	0.6826	19.	0.7936
5.	0.7316	20.	0.7936
6.	0.7584	21.	0.7936
7.	0.7735	22.	0.7936
8.	0.7822	23.	0.7936
9.	0.7872	24.	0.7936
10.	0.7900	25.	0.7936
11.	0.7916	26.	0.7936
12.	0.7925	27.	0.7936
13.	0.7930	28.	0.7936
14.	0.7933	29.	0.7936
15.		30.	
16.	0.7935	31.	0.7936

Fig. 6.1.2: Orbit of F(z) for c=(-0.0359,0.01962) at N=(1,0) and s=0.5,s'=0.5

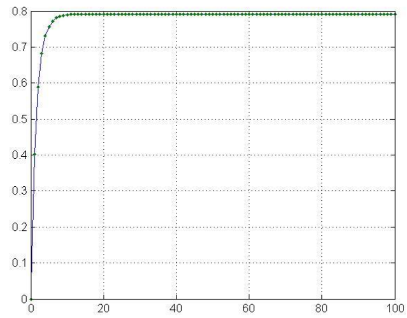


Table 6.1.3: Orbit of F(z) for c= =(-0.325,-0.01875) at (N=3,0) and s=0.5, s'=0.5
Here the value converges after 16 iterations

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	0.0669
2.	0.0475	17.	0.0668
3.	0.0563	18.	0.0668
4.	0.0609	19.	0.0668
5.	0.0635	20.	0.0668
6.	0.0650	21.	0.0668
7.	0.0659	22.	0.0668
8.	0.0665	23.	0.0668
9.	0.0668	24.	0.0668
10.	0.0669	25.	0.0668
11.	0.0670	26.	0.0668
12.	0.0670	27.	0.0668
13.	0.0669	28.	0.0668
14.	0.0669	29.	0.0668
15.	0.0669	30.	0.0668

Fig. 6.1.3 : Orbit of F(z) for c= =(-0.325,-0.01875) at (N=3,0) and s=0.5,s'=0.5

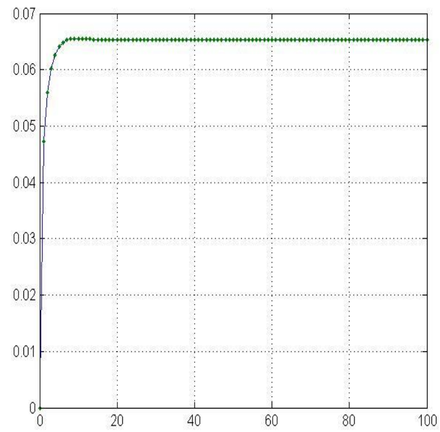
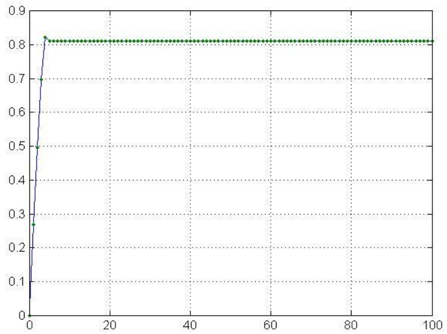


Table 6.1.4 : Orbit of F(z) for c= =(0.3518,-0.0803) at N=(5,0) and s=0.1,s'=0.1

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	0.8425
2.	0.2756	17.	0.8425
3.	0.5144	18.	0.8425
4.	0.7264	19.	0.8425
5.	0.8563	20.	0.8425
6.	0.8405	21.	0.8425
7.	0.8425	22.	0.8425
8.	0.8427	23.	0.8425
9.	0.8424	24.	0.8425
10.	0.8426	25.	0.8425
11.	0.8425	26.	0.8425
12.	0.8425	27.	0.8425
13.	0.8425	28.	0.8425
14.	0.8425	29.	0.8425
15.	0.8425	30.	0.8425

Here the value converges after 10 iterations

Fig. 6.1.4 : Orbit of F(z) for c= =(0.3518,-0.0803) at N=(5,0) and s=0.1,s'=0.1



6.2 Fixed points when real N = 0

Table.6.2.1: Orbit of F(z) for , c= (1.067813364,-0.038102687) at N=(0, 0.7), s=0.9, s'=0.1

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	1.5670
2.	1.8726	17.	0.3831
3.	0.3564	18.	1.5670
4.	1.5981	19.	0.3831
5.	0.3737	20.	1.5670
6.	1.5777	21.	0.3831
7.	0.3797	22.	1.5670
8.	1.5708	23.	0.3831
9.	0.3819	24.	1.5670
10.	1.5684	25.	0.3831
11.	0.3827	26.	1.5670
12.	1.5675	27.	0.3831
13.	0.3830	28.	1.5670
14.	1.5672	29.	0.3831
15.	0.3831	30.	1.5670

Here we get two fixed points after 15 iterations

Fig. 6.2.1: Orbit of F(z) for , c= (1.067813364,-0.038102687) at N=(0, 0.7), s=0.9, s'=0.1

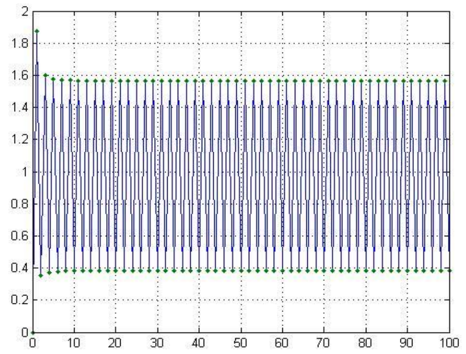


Table.6.2.2Orbit of F(z) for c=(-0.3549,-0.0015) at N=(0,3), s=0.1, s'=0.04

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
1.	0.0000	16.	0.0908
2.	0.0481	17.	0.0996
3.	0.0138	18.	0.0908
4.	0.0602	19.	0.0996
5.	0.0349	20.	0.0908
6.	0.0742	21.	0.0996
7.	0.0625	22.	0.0908
8.	0.0858	23.	0.0996
9.	0.0879	24.	0.0908
10.	0.0904	25.	0.0996
11.	0.0987	26.	0.0908
12.	0.0909	27.	0.0996
13.	0.0997	28.	0.0908
14.	0.0908	29.	0.0996
15.	0.0996	30.	0.0908

Here we get two fixed points after 14 iterations

Fig 6.2.2 Orbit of F(z) for c=(-0.3549,-0.0015) at N=(0,3), s=0.1,s'=0.04

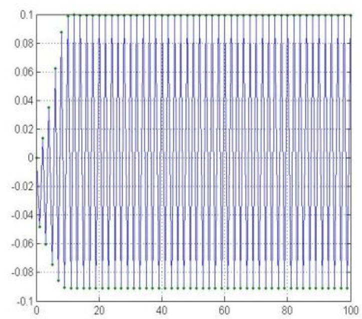
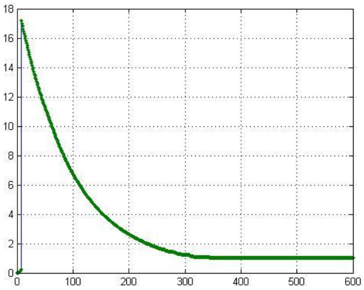


Table 6.2.3: Orbit of F(z) for N=(0,14.5), s=0.01,s'=0.01, c=(0,0)

No. of Iteration	|F(z)|	No. of Iteration	|F(z)|
340.	1.0718	355.	1.0195
341.	1.0698	356.	1.0203
342.	1.0673	357.	1.0196
343.	1.0639	358.	1.0201
344.	1.0589	359.	1.0197
345.	1.0508	360.	1.0200
346.	1.0366	361.	1.0198
347.	1.0164	362.	1.0200
348.	1.0223	363.	1.0198
349.	1.0180	364.	1.0200
350.	1.0214	365.	1.0199
351.	1.0188	366.	1.0199
352.	1.0208	367.	1.0199
353.	1.0192	368.	1.0199
354.	1.0205	369.	1.0199

Here we get two fixed points after 364 iterations

Fig. 6.2.3: Orbit of F(z) for N=(0,14.5),s=0.01,s'=0.01, c=(0,0)



7. CONCLUSIONS AND REMARKS

In this paper we studied the Carotid Kundalini function which is one of the members of transcendental family. We generated different Julia set for C-K function using the Relative Superior iterates and studied their characteristics. The fractals and the Julia sets generated shows symmetry along the real axis for both Real and Imaginary values. The relative superior Carotid Kundalini set and its Julia set takes up the shape of < and appears to be an invariant Cantor set in the form of curves or "dendrites "that extends to ∞ . The orbit of any point on the dendrites tends to infinity under iteration. This geometry of the dendrites is quite similar to that of exponential family showing the property of transcendental function.

Finally we concluded that for both the real and imaginary values , the repeated iteration of C-K function produced values which exhibited three different types of behaviour, i.e the values either converged to a fixed point, displayed periodic motion or increased indefinitely at each iteration.
We get some beautiful images that resemble the Yoga Kundlaini from Hindu Mythology; hence we can say that the name Carotid-Kundalini is justified.

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