Cost-Benefit Analysis of a Parallel System with Arrival Time of the Server and Maximum Repair time

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ABSTRACT
An attempt has been made to develop a cost-benefit analysis for a system of two identical units with parallel redundancy. Each unit has direct complete failure from normal operative mode. The system is considered in up-state if at least one unit is operative. There is a single server who takes some time to arrive at the system for doing repair activities. If server unable to repair the failed unit in a pre specific time (called maximum repair time), the unit is replaced by new one with some replacement time. The failure time of the unit and maximum repair time taken by the server are exponentially distributed while the distributions of arrival time of the server, repair and replacement of the unit are taken as arbitrary with different probability density functions. The expressions for various parameters of vital significance have been derived using semi-Markov process and regenerative point technique. The graphical study of the results obtained for a particular case has also been made.

General Terms
Reliability Theory.

Keywords
Reliability, Parallel-Unit System, Replacement, Maximum Repair Time and Cost-Benefit Analysis.

1. INTRODUCTION
It is a common knowledge that redundancy can be used to improve the reliability of a system without changing the reliability of the individual unit that forms the system. Standby systems often find applications in various industrial and other areas. Therefore, systems with cold standby redundancy have widely been studied by many authors including Gopalan and Naidu[2] and Singh[3]. Whenever, the introduction of standby redundancy is not possible, it is desirable to introduce parallel redundancy in order to achieve better reliability. Nakagawa and Osaki[1] discussed a two-unit parallel redundant system with repair maintenance. In most of these studies, it is assumed that repair facility becomes available immediately as and when required. But, in practice, this assumption is not always true may because of the pre occupations of the repair facility and in such a situation service facility may take some time to arrive at the system. Chander [5] has suggested reliability models of a standby system with arrival time of the server.

Furthermore, reliability and performance of the system can be increased by making replacement of the failed components (or units) by new one in case repair of these is not possible in a pre-specific time. Singh and Agrafoitis [4] have analyzed a two-unit cold standby system subject to maximum operation and repair times. Recently, Kumar and Malik [6] have proposed a reliability model of a computer system with the concepts of maximum operation and repair times. It is also pointed out here that the work on parallel system considering arrival time of the server and maximum repair time has not been reported so far in the literature of reliability.

In view of the above circumstances and facts, this paper has been designed with an object to carry out cost-benefit analysis of a system of two identical units with parallel redundancy. Each unit has two modes- operative and complete failure. The system is considered in up-state if at least one unit is operative. There is a single server who takes some time to arrive at the system for doing repair activities. If server unable to repair the failed unit in a pre specific time (called maximum repair time), the unit is replaced by new one with some replacement time. The random variables are uncorrelated and statistically independent. The failure time of the unit and maximum repair time taken by the server are exponentially distributed while the distributions of arrival time of the server, repair and replacement of the unit are taken as arbitrary with different probability density functions. The expressions for various parameters of vital significance have been derived using semi-Markov process and regenerative point technique. The unit works as new after repair. The switch devices are perfect and switch over is instantaneous. Numerical results pertaining to the case when arrival, repair and replacement times are exponentially distributed have been obtained to depict the graphical behavior of MTSF, availability and profit with respect to the replacement rate. The applications of the present study can be visualized in h/w and s/w industries.

2. NOTATIONS

E	:	Set of regenerative states
O	:	Unit is operative
α_0	:	Maximum constant rate of repair time
λ	:	Constant failure rate of the unit
$f(t)/F(t)$	:	pdf / cdf of the replacement time of the unit
$g(t)/G(t)$	:	pdf / cdf of the repair time of the unit
$w(t)/W(t)$	:	pdf / cdf of the waiting time of the server for repairing of the unit

$FU_r/\ FU_R$	:	Unit is failed and under repair / under repair continuously from previous state
$FW_r\ /\ FW_R$	:	Unit is failed and waiting for repair / waiting for repair continuously from previous state
$FUR_p/\ FUR_P$	:	Unit is failed and under replacement / under replacement continuously from previous state
m_{ij}	:	Contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state if occurs before transition to $S_j \in E$. Mathematically, it can be written as $m_{ij} = \int_0^\infty t\, d\left(Q_{ij}(t)\right) = -q_{ij}^{*\prime}(0)$
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, where T denotes the time to system failure.
$\sim / *$	:	Symbol for Laplace Stieltjes transform / Laplace transform
$\boxed{S}$ /©	:	Symbols for Stieltjes convolution / Laplace convolution.
′ (desh)	:	Symbol for derivative of the function

The possible transitions between states along with transitions rates for the system model are shown in figure 1.The states S_0, S_1, S_2 and S_3 are regenerative while the other states are non-regenerative.

3. RELIABILITY INDICIES

3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\, dt$ as

$p_{01} = f^*(0)$, $\quad p_{12} = w^*(\lambda)$,

$p_{14} = 1 - w^*(\lambda)$,

$p_{20} = g^*(\lambda + \alpha_0)$, $\quad p_{23} = \frac{\alpha_0}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)]$,

$p_{27} = \frac{\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)]$, $\quad p_{30} = f^*(\lambda)$,

$p_{38} = 1 - f^*(\lambda)$, $\quad p_{45} = w^*(0)$,

$p_{52} = g^*(\alpha_0)$, $\quad p_{56} = 1 - g^*(\alpha_0)$,

$p_{62} = f^*(0)$,

$p_{12.45} = [1 - w^*(\lambda)]g^*(\alpha_0)$,

$p_{12.456} = [1 - w^*(\lambda)][1 - g^*(\alpha_0)]f^*(0)$,

$p_{22.7} = \frac{\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)]g^*(\alpha_0)$,

$p_{22.76} = \frac{\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)][1 - g^*(\alpha_0)]f^*(0)$,

$p_{32.8} = 1 - f^*(\lambda)$ ... (1)

It can easily be verified that

$p_{01} = p_{12} + p_{14} = p_{12} + p_{12.45} + p_{12.456} = p_{20} + p_{23} + p_{27} = p_{20} + p_{23} + p_{22.7} + p_{22.76} = p_{30} + p_{38} = p_{30} + p_{32.8} = p_{45} = p_{62} = 1$ ... (2)

The mean sojourn times μ_i in state S_i is given by

$\mu_0 = \int_0^\infty P(T > t)\, dt = m_{01} = \frac{1}{2\lambda}$,

$\mu_1 = m_{12} + m_{14} = \frac{1}{\lambda}(1 - w^*(\lambda))$,

$\mu_2 = m_{20} + m_{23} + m_{27} = \frac{[1 - g^*(\alpha_0 + \lambda)]}{\alpha_0 + \lambda}$ $\quad \mu_3 =$

$\mu_1' = m_{12} + m_{12.45} + m_{12.456} = (1 - w^*(\lambda))\left[\frac{1}{\lambda} - w^{*\prime}(0) + (1 - g^*(\alpha_0))\left(\frac{1}{\alpha_0} - f^{*\prime}(0)\right)\right]$,

$\mu_2' = m_{20} + m_{23} + m_{22.7} + m_{22.76} = \frac{[1-g^*(\alpha_0+\lambda)]}{\alpha_0+\lambda}\left[1 - \lambda(1 - g^*(\alpha_0))\left(\frac{1}{\alpha_0} - f^{*\prime}(0)\right)\right]$,

$\mu_3' = m_{30} + m_{32.8} = (1 - f^*(\lambda))\left(\frac{1}{\lambda} - f^{*\prime}(0)\right)$ ... (3)

3.2 Mean Time to System Failure (MTSF)

Let $\emptyset_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\emptyset_i(t)$:

$\emptyset_0(t) = Q_{01}(t)\ \boxed{S}\ \emptyset_1(t)$,

$\emptyset_1(t) = Q_{12}(t)\ \boxed{S}\ \emptyset_2(t) + Q_{14}(t)$,

$\emptyset_2(t) = Q_{20}(t)\ \boxed{S}\ \emptyset_0(t) + Q_{23}(t)\ \boxed{S}\ \emptyset_3(t) + Q_{27}(t)$

$\emptyset_3(t) = Q_{30}(t)\ \boxed{S}\ \emptyset_0(t) + Q_{38}(t)$, ... (4)

Taking $L.S.T$ of relations (4) and solving for $\widetilde{\emptyset}_0(s)$, we get

$MSTF(T_0) = \lim_{s\to 0} \frac{1-\widetilde{\emptyset}_0(s)}{s} = \frac{\mu_0+\mu_1+p_{12}\mu_2+p_{12}p_{23}\mu_3}{1-p_{12}p_{20}-p_{12}p_{23}p_{30}} = \frac{N_1}{D_1}$ (5)

where $N_1 = \frac{1}{2\lambda} + \frac{1}{\lambda}(1 - w^*(\lambda)) + \frac{[1-g^*(\alpha_0+\lambda)]}{\alpha_0+\lambda}(w^*(\lambda))(1 - \alpha_0 f^{*\prime}(0))$ and

$D_1 = \left(1 - w^*(\lambda)\left(g^*(\alpha_0 + \lambda) + \frac{\alpha_0}{\lambda+\alpha_0}(1 - g^*(\alpha_0 + \lambda))f^*(\lambda)\right)\right)$

3.3 Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t = 0$.The recursive relation for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t)© A_1(t)$
$A_1(t) = M_1(t) + \left(q_{12}(t) + q_{12.45}(t) + q_{12.456}(t)\right)© A_2(t)$
$A_2(t) = M_2(t) + q_{20}(t)© A_0(t) + \left(q_{22.7}(t) + q_{22.76}(t)\right)© A_2(t) + q_{23}(t)©A_3(t)$
$A_3(t) = M_3(t) + q_{30}(t)© A_0(t) + q_{32.8}(t)© A_2(t)$

where

$M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-\lambda t}\bar{W}(t)$, $M_2(t) = e^{-(\lambda+\alpha_0)t}$, $M_3(t) = e^{-\lambda t}\bar{F}(t)$ Taking $L.T.$ of relation (6) and solving for $A_0^*(s)$,

we get steady-state availability as

$$A_0 = \lim_{s\to 0} s\, A_0^*(s) = \frac{\left(M_0^*(0) + M_1^*(0)\right)(p_{20} + p_{23}p_{30}) + \left(M_2^*(0) + p_{23}M_3^*(0)\right)}{(\mu_0 + \mu_1')(p_{20} + p_{23}p_{30}) + (\mu_2' + p_{23}\mu_3')}$$

$$= \frac{N_2}{D_2} \qquad \ldots (7)$$

Where $N_2 = \left(\frac{1}{2\lambda} + \frac{1}{\lambda}\left(1 - w^*(\lambda)\right)\right)g^*(\alpha_0 + \lambda) + \frac{1}{\alpha_0+\lambda}\left(1 - g^*(\alpha_0+\lambda)\right)\left(1 + \frac{\alpha_0}{\lambda}\left(1 - f^*(\lambda)\right) + \alpha_0 f^*(\lambda)\left(\frac{1}{2\lambda} + \frac{1}{\lambda}\left(1 - w^*(\lambda)\right)\right)\right)$ and

$$D_2 = \frac{1-g^*(\alpha_0+\lambda)}{\alpha_0+\lambda}\left[1 + \lambda\left(1 - g^*(\alpha_0)\right)\left(\frac{1}{\alpha_0} - f^{*'}(0)\right) + \left(1 - f^*(\lambda)\right)\left(\frac{1}{\lambda} - f^{*'}(0)\right)\right]$$

$$+\alpha_0 f^*(\lambda)\left(\frac{1}{2\lambda} + \left(1 - w^*(\lambda)\right)\left(\frac{1}{\lambda} - w^{*'}(0) + \left(1 - g^*(\alpha_0)\right)\left(\frac{1}{\alpha_0} - f^{*'}(0)\right)\right)\right) +$$

$$\left[\left(\frac{1}{2\lambda} + \left(1 - w^*(\lambda)\right)\left(\frac{1}{\lambda} - w^{*'}(0) + \left(1 - g^*(\alpha_0)\right)\left(\frac{1}{\alpha_0} - f^{*'}(0)\right)\right)\right)g^*(\alpha_0 + \lambda)\right]$$

3.4 Busy Period Analysis Due to Repair

Let $B_i^1(t)$ be the probability that the server is busy in repairing the unit at an instant 't'given that the system entered regenerative state i at $t = 0$.The recursive relations for $B_i^1(t)$ are given as

$B_0^1(t) = q_{01}(t)© B_1^1(t)$,
$B_1^1(t) = \left(q_{12}(t) + q_{12.45}(t) + q_{12.456}(t)\right)© B_2^1(t)$

$B_2^1(t) = W_2(t) + q_{20}(t)©B_0^1(t) + \left(q_{22.7}(t) + q_{22.76}(t)\right)©B_2^1(t) + q_{23}(t)©B_3^1(t)$

$B_3^1(t) = q_{30}(t)©B_0^1(t) + q_{32.8}(t)© B_2^1(t)$

$$\ldots (8)$$

where

$$W_2(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + \left(\lambda e^{-\lambda t}©1©e^{-\alpha_0 t}\right)\bar{G}(t) \qquad \ldots (9)$$

Taking $L.T.$ of relations (9) and solving for $B_0^{1^*}(s)$, we get in the long run the time for which the system is under repair is given by

$$B_0^1 = \lim_{s\to 0} s\, B_0^{1^*}(s) = \frac{N_3}{D_2} \qquad \ldots (10)$$

Where $N_3 = \frac{1}{\alpha_0+\lambda}\left(1 - g^*(\alpha_0 + \lambda)\right)[1 + \lambda]$ and D_2 is already specified.

3.5 Busy Period Analysis Due to Replacement

Let $B_i^2(t)$ be the probability that the sever is busy in replacing the unit at an instant 't'given that the system entered regenerative state i at $t = 0$.The recursive relation for $B_i^2(t)$ are given by:

$B_0^2(t) = q_{01}(t)©B_1^2(t)$
$B_1^2(t) = \left(q_{12}(t) + q_{12.45}(t) + q_{12456}(t)\right)© B_2^2(t)$
$B_2^2(t) = q_{20}(t)©B_0^2(t) + \left(q_{22.7}(t) + q_{22.76}(t)\right)©B_2^2(t) + q_{23}(t)©B_3^2(t)$
$B_3^2(t) = W_3(t) + q_{30}(t)©B_0^2(t) + q_{32.8}(t)© B_2^2(t)$

where

$$W_3(t) = e^{-\lambda t}\bar{F}(t) + \left(\lambda e^{-\lambda t}©1\right)\bar{F}(t) \qquad \ldots (12)$$

Taking $L.T.$ of relations (11) and solving for $B_0^{2*}(s)$, we get the time for which the system is under replacement is given by

$$B_0^2 = \lim_{s\to 0} s\, B_0^{2*}(s) = \frac{N_4}{D_2} \qquad \ldots (13)$$

Where
$N_4 = \frac{1}{\alpha_0+\lambda}\left(1 - g^*(\alpha_0 + \lambda)\right)\left[\frac{\alpha_0}{\lambda} - \frac{\alpha_0}{\lambda}f^*(\lambda) + \alpha_0 - \alpha_0 f^*(\lambda)\right]$
and D_2 is already specified.

3.6 Expected Number of Visits By The Server

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $N_i(t)$ are given by

$$N_0(t) = Q_{01}(t)\boxed{S}\, N_1(t)$$
$$N_1(t) = \left(Q_{12}(t) + Q_{12.45}(t) + Q_{12.456}(t)\right)\boxed{S}\,[1 + N_2(t)]$$
$$N_2(t) = Q_{20}(t)\boxed{S}\, N_0(t) + \left(Q_{22.7}(t) + Q_{22.76}(t)\right)\boxed{S}\, N_2(t) + Q_{23}(t)\boxed{S}\, N_3(t)$$
$$N_3(t) = Q_{30}(t)\boxed{S}\, N_0(t) + Q_{32.8}(t)\boxed{S}\, N_2(t) \quad \ldots(14)$$

Taking $L.S.T.$ of relations (14) and solving for $\widetilde{N_0}(s)$, we get the expected number of visits per unit time as

$$N_0 = \lim_{s\to 0} s\,\widetilde{N_0}(s) = \frac{N_5}{D_2} \quad \ldots(15)$$

Where $N_5 = \frac{\alpha_0}{\alpha_0+\lambda}\left(1 - g^*(\alpha_0 + \lambda)\right) f^*(\lambda) + g^*(\alpha_0 + \lambda)$ and D_2 is already specified.

3.7 Expected Number of Replacements of the Unit

Let $R_i(t)$ be the expected number of replacements by the unit in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $R_i(t)$ are given by:

$$R_0(t) = Q_{01}(t)\boxed{S}\, R_1(t)$$
$$R_1(t) = \left(Q_{12}(t) + Q_{12.45}(t) + Q_{12.456}(t)\right)\boxed{S}\, R_2(t)$$
$$R_2(t) = Q_{20}(t)\boxed{S}\, R_0(t) + \left(Q_{22.7}(t) + Q_{22.76}(t)\right)\boxed{S}\, R_2(t) + Q_{23}(t)\boxed{S}\, R_3(t)$$
$$R_3(t) = Q_{30}(t)\boxed{S}\,[1 + R_0(t)] + Q_{32.8}(t)\boxed{S}\,[1 + R_2(t)] \quad \ldots(16)$$

Taking $L.S.T.$ of relations (16) and solving for $\widetilde{R_0}(s)$, we get the expected number of replacements of the unit per unit time as

$$R_0 = \lim_{s\to 0} s\,\widetilde{R_0}(s) = \frac{N_6}{D_2} \quad \ldots(17)$$

Where $N_6 = \frac{\alpha_0}{\alpha_0+\lambda}\left(1 - g^*(\alpha_0 + \lambda)\right)$ and D_2 is already specified.

3.8 Cost-Benefit Analysis

Profit incurred to the system model in steady state is given by:

$P = K_1 A_0 - K_2 B_0^1 - K_3 B_0^2 - K_4 R_0 - K_5 N_0$

Where

K_1 = Revenue per unit uptime of the system

K_2 = Cost per unit time for which server is busy due to repair

K_3 = Cost per unit time for which server is busy due to replacement

K_4 = Cost per unit time replacement of the unit

K_5 = Cost per unit visits by the server

3.9 Conclusion

The graphical behavior of mean time to system failure(MTSF),availability and profit with respect to replacement rate(β) has been shown in figures 2,3 and 4 respectively. The values of these performance measures go on increasing with the increase of replacement rate (β) , arrival rate (γ) of the server and repair rate(θ) of the unit for fixed values of other parameters including K2>K4. However, the effect of θ and γ on these measures is more than that of other parameters. And, if we take K2<K4, the system becomes less profitable. Further there is an increase in the values of MTSF with the increase of maximum repair time of the server while profit declines. Hence on the basis of the results obtained for a particular case , it is concluded that a parallel system with arrival time of the server and maximum repair time can be made more reliable and profitable to use by one of the following ways:

i. Increasing the repair rate of the unit in case server is taking much time to arrive at the system.
ii. Paying minimum cost for replacement of the unit by new one in case repair time taken by the server is too long.
iii. Increasing arrival rate of the server.
iv. Making immediate replacement of the unit after the completion of maximum repair time taken by the server.

4. REFERENCES

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State Transition Diagram

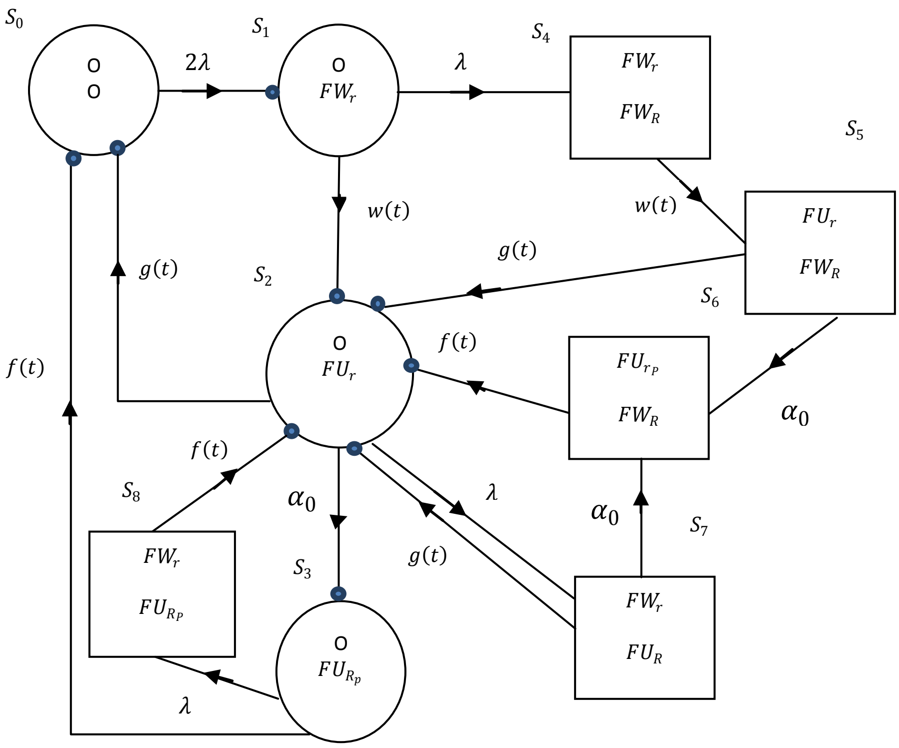
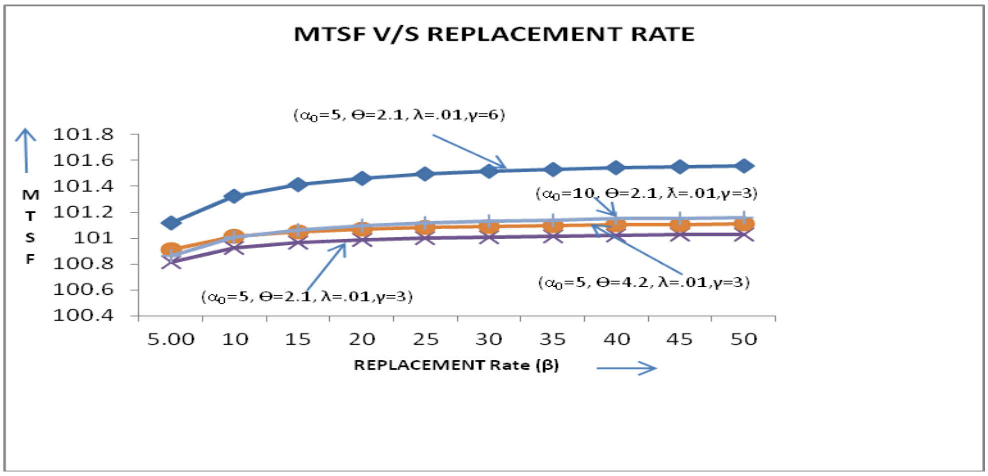
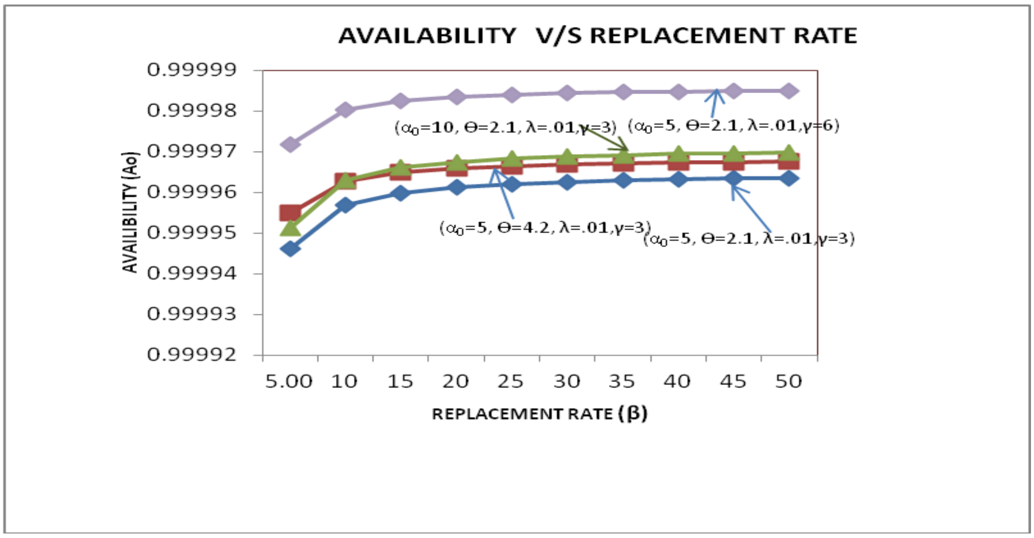
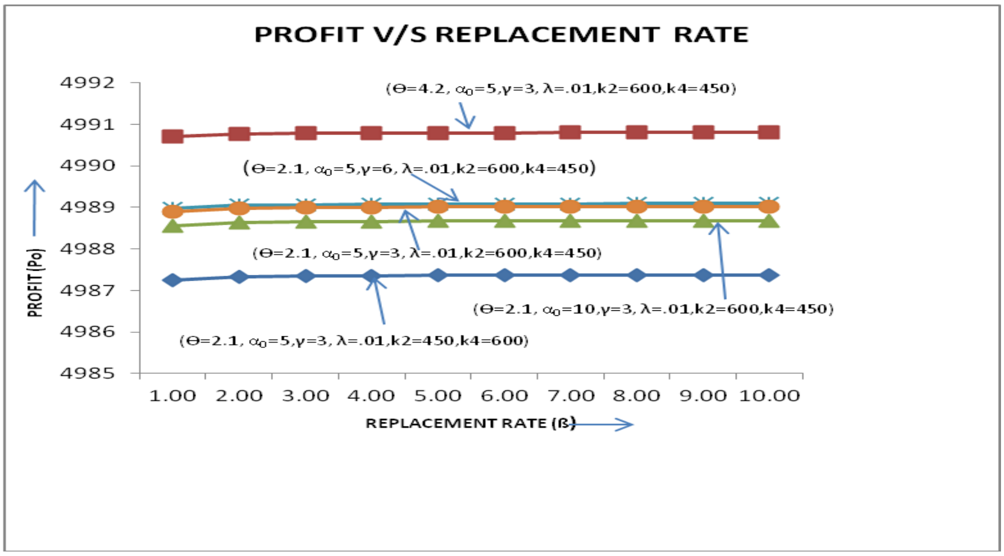


Figure1

● : Regenerative Point
O : Upstate
□ : Failed State


MTSF V/S REPLACEMENT RATE
(α0=5, Θ=2.1, λ=.01,γ=6)
(α0=10, Θ=2.1, λ=.01,γ=3)
(α0=5, Θ=4.2, λ=.01,γ=3)
(α0=5, Θ=2.1, λ=.01,γ=3)
MTSF
101.8
101.6
101.4
101.2
101
100.8
100.6
100.4
5.00 10 15 20 25 30 35 40 45 50
REPLACEMENT Rate (β)


AVAILABILITY V/S REPLACEMENT RATE
AVAILIBILITY (Ao)
0.99999
0.99998
0.99997
0.99996
0.99995
0.99994
0.99993
0.99992
(α0=10, Θ=2.1, λ=.01,γ=3)
(α0=5, Θ=2.1, λ=.01,γ=6)
(α0=5, Θ=4.2, λ=.01,γ=3)
(α0=5, Θ=2.1, λ=.01,γ=3)
5.00 10 15 20 25 30 35 40 45 50
REPLACEMENT RATE (β)


PROFIT V/S REPLACEMENT RATE
PROFIT (Po)
4992
4991
4990
4989
4988
4987
4986
4985
(Θ=4.2, α0=5,γ=3, λ=.01,k2=600,k4=450)
(Θ=2.1, α0=5,γ=6, λ=.01,k2=600,k4=450)
(Θ=2.1, α0=5,γ=3, λ=.01,k2=600,k4=450)
(Θ=2.1, α0=10,γ=3, λ=.01,k2=600,k4=450)
(Θ=2.1, α0=5,γ=3, λ=.01,k2=450,k4=600)
1.00 2.00 3.00 4.00 5.00 6.00 7.00 8.00 9.00 10.00
REPLACEMENT RATE (ß)