The Study of Results Simulation of Collective Motion

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ABSTRACT
The collective behavior/motion has always been one of the most fascinating phenomena since men started to observe nature which remains a real natural phenomenon, were it is typical in our social environment. The study of collective behavior on a large scale also enables us to better understand different approaches to study in the small scale. In this study, we discuss the principal effect of the control parameters: The binder cumulant, density and the size of system with three zones repulsion, orientation and attraction on the collective motion in the 2D. Furthermore a simple model with a novel type of dynamics is introduced in order to investigate the emergence of self-ordered motion in system of particles. In our simulation, the particles equivalent to agents interact with their neighbors by choosing at each time step a velocity depending on their direction. The aim of this article is to extend the model proposed earlier by Viscek et al. Numerical simulations showed that depending on the control parameters both disordered and long-range ordered phases can be observed and the corresponding phase space domains are separated by singular critical lines.

Keywords
Collective motion, Noise, Density, size of system, Binder cumulant and Kinetic phase transition.

1. INTRODUCTION
Recently there has been an increasing interest in the studies of far-from-equilibrium systems typical in our natural and social environment. Concepts originated from the physics of phase transitions in equilibrium system such as collective behavior, scale invariance and renormalization have been shown to be useful in the understanding of various non-equilibrium system as well. Simple algorithmic models in section 2 have been helpful in the extraction of the basic properties of various far-from-equilibrium phenomena. There is the several models where have been suggested and simulated to evaluate the main features of the collective motion such as flocks of birds, schools of fish and group of bacteria,…. The primary study of the collective motion was given by Vicsek and his collaborators [1] that studied the problem with computer simulation they treated identical point-wise particles, the present contribution is a continuation of our study of collective behavior of interacting agents Najem & al. [3] which we tacked two zones, in this article we consider the effect of the three zones: repulsion, orientation and attraction of the flocking model. In our model, the agent corresponding to particles are driven with a constant absolute velocity locally interact with his neighbors and at each time step the velocity depending on the direction of the motion [2-5]. This direction is being subject to error i.e noise. This movement describes the behavior of each individual depending on the state of the position and the velocity [6-7]. Our intention is to present a detailed numerical study on the effect of the density and size of system on the kinetic phase transition, with three zones of the flocking model, and find the nature of this transition using the Binder cumulant where measure to distinguish between first and second order phase transitions. The numerical simulation of individuals on the $D=2$ undergo a kinetic phase transition [8-11] from an ordered phase where all the particles move in the same direction, to a disordered phase where the particles move in random directions i.e from no transport: zero average velocity , to finite net transport [12-17].Our system depends on many different variables and parameters, such as: the noise, density and the size of system that plays a very important role and its variations can affect significantly the collective behavior motion. The effect of noise causes the change the value of density, size of system and velocity. These crucial parameters influence the motion of flocks which become much more coherent.

2. PRESENTATION AND DESCRIPTION OF THE MODEL
To understand the complex behavior of non-equilibrium multi-agent system, we will extend the two-dimensional Viscek model [1] by considering three zones that are defined by three concentric circles around every agent as shown in Fig.1. The model that we think here studies the effect of each zone on the movement of whole flock, where each zone or radius around each agent attempts to maintain a minimum distance between him and others at all time. Each agent has a specified range of awareness where it recognizes other mates or obstacles.

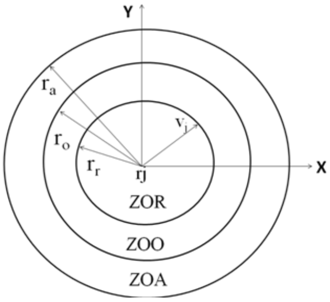


Fig.1. Schematic representation of the three zones for individual *i.* ZOR is the Zone Of Repulsion (r_r radius of repulsion); ZOO is the Zone Of Orientation (r_o radius of orientation) while ZOA is the Zone Of Attraction (r_a radius of attraction).

In our implementation as in model of Vicsek we consider an $L \times L$ square shaped surface with periodic boundary conditions. We define N individuals present within the system $i = 1,...., N$ each with a unique position vector $r_i(t)$ and a unit direction vector $v_i(t)$ at time *t*, where it's partitioned into discrete time steps Δt. At each time, every individual assesses the other individuals' positions and velocities within a local neighborhood, to find their preferred travel direction $V_i^d(t)$.

The zone of repulsion is modeled by a circular surface with radius r_r the set of individuals within this zone is of size N_r. The individual i has N_r neighbors determined by the condition $0 < r_j(t) - r_i(t) \le r_r$, where $r_j(t)$ is the position of the *j-th* neighboring individual $(j = 1,..., N_r, j \ne i)$. We define

$$V_i^{repulse}(t+1) = -\frac{\sum_{j \ne i}^{N_r} r_{i,j}(t)}{\left|\sum_{j \ne i}^{N_r} r_{i,j}(t)\right|} \quad \textbf{(1)}$$

The vector $r_{i,j}(t)$ is the unit vector pointing from individual i in the direction of neighbor j. so if neighbors are present in an individual's zone of repulsion i.e. $N_r > 0$ then the preferred direction of travel for the next time step is

$$V_i^d(t+1) = V_i^{repulse}(t+1)$$

If there are no neighbors within the zone of repulsion, then individual i respond to neighbors within the zone of orientation and zone of attraction; radius r_o and r_a respectively. There are N_o detectable neighbors present in the zone of orientation, determined by the condition $r_r < r_j(t) - r_i(t) \le r_o$. Similarly, there are N_a detectable neighbors in the zone of attraction, such $r_o < r_j(t) - r_i(t) \le r_a$ is satisfied. The preferred travel direction resulting from the zone of orientation is the average of the neighbor's velocities

$$V_i^{orient}(t+1) = \frac{\sum_{j \ne i}^{N_o} V_j(t)}{\left|\sum_{j \ne i}^{N_o} V_j(t)\right|} \quad \textbf{(2)}$$

Where $(j = 1,..., N_o, j \ne i)$, if neighbors are found in zone of orientation then

$$V_i^d(t+1) = V_i^{orient}(t+1)$$

The preferred direction due to the zone of attraction is

$$V_i^{attract}(t+1) = \frac{\sum_{j \ne i}^{N_a} r_{i,j}(t)}{\left|\sum_{j \ne i}^{N_a} r_{i,j}(t)\right|} \quad \textbf{(3)}$$

Where $(j = 1,..., N_a, j \ne i)$, if neighbors are present in zone of attraction then

$$V_i^d(t+1) = V_i^{attract}(t+1)$$

In the rare case when the social forces cancel one another out and give a zero vector, or if no neighbors are detected, then

$$V_i^d(t+1) = v_i(t)$$

In our simulations, the position $r_i(t+1)$ for each particle at the unit, is

$$r_i(t+1) = r_i(t) + v_i(t+1)\Delta t \quad \textbf{(4)}$$

Each group member orients towards their desired travel directions at a turning rate of γ. Otherwise, the individual rotates their current direction from the expression:

$$\gamma_i(t+1) = \langle \gamma(t) \rangle_i + \Delta\gamma \quad \textbf{(5)}$$

The quantity $\langle \gamma(t) \rangle_i$ describes the average direction of the *i-th* particles. A noise η has been introduced as a random variable $\Delta\gamma$ chosen with uniform distribution in interval [- $\eta/2$, $\eta/2$].

The natural order parameter suitable to describe the collective behavior of the individuals is the normalized average velocity which presents a sense of the ordered/disordered motion for the particles, given by

$$v_a = \frac{1}{N v_0} \left| \sum_{i=1}^{N} v_i \right| \quad \textbf{(6)}$$

We have studied in detail the nature of kinetic phase transition by determining the absolute value of the average normalized velocity. In the next, we will adopt the value $v_0 = 0.1$; this value used for particles that always interact with their actual neighbors and move fast enough to change the configuration after a few updates of the directions, we announce obviously the results are not changed by the parameter v_0 chosen from the wide range ($0.1 \le v_0 \le 0.5$) the $\Delta t = 1$ means the time interval between two updating of the directions/positions.

3. SIMULATION RESULTS AND DISCUSSIONS

For the statistical characterization of the configuration, a well-suited order parameter is the magnitude of the average momentum of the system: eq. (6), this measure of the net flow is non-zero in the ordered phase, and vanishes (for an infinite system) in the disordered phase. In the simulation random initial conditions and periodic boundary conditions were applied. In our presentation, the density ρ and the size of system L in our study play a very important role and they variation can affect significantly the collective behavior motion. In the simple model of Viscek presented only for one zone, it is shown that for small densities and noise, the particles move coherently in random directions. While for higher densities and noise the motion becomes correlated. The density it is defined by $\rho = \frac{N}{L^2}$ where N corresponds to the number of particles and L the system size. It measures the number of particles per unit of surface. In the limit of big systems, N and L tend to infinity while ρ remains finite. We declare that the numerical errors in our results are probable in the range of 2% due to the correlations and the number of iterations.

In the following, Fig.3.1 (a) investigates the behavior of the transport properties (velocity v_a) as a function of density ρ. We present our results by calculating the variation of velocity in range of noise parameters from *η=0* to *η=5*. For a fixed noise (η=0) the velocity increases with the density to a value of ρ after well becomes stable, and much greater the noise (η=5) the velocity decreases quantitatively as function of the density. These results indicate the strong role of density effects in our model.

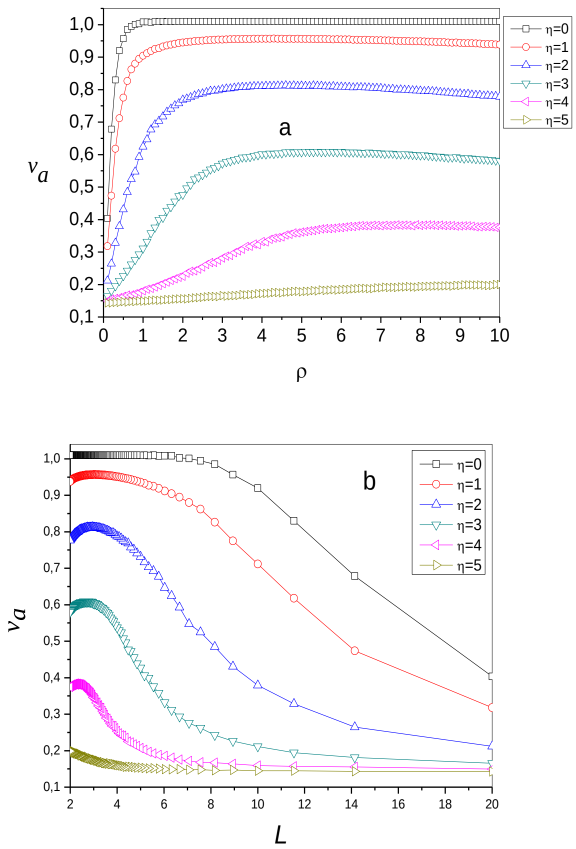


Figure 3.1: Variation of the average velocity v_a as a function of: (a) density ρ and (b) size of system L for different values of noise η for fixed value of the r_r=0.1,r_o=0.9 and r_a=2.0.

To explain better the motion of flocks, Fig.3.1 (b) shows the change shape of v_a caused by the effect size L of system, as can be seen from the figure, for some range of values of η , the velocity v_a decreases with size L from its maximal value ($v_a \sim 1$) until its minimum value ($v_a \sim 0$), indicating that the transition regime is not fast enough as like as in second-order transitions. The L is changed by varying the density of individuals between ρ = 0.1 and ρ = 10.0 with constant number of individuals N. Consequently, the quantitative effect of increasing the noise decreases gradually the velocity. These results are in good agreement with the results acquired in the framework of the similar model finding by Vicsek. These finding indicate the strong role and the effect of size system in our model. So, the system undergoes a kinetic phase transition, which occurs at some critical density ρ_c and at some critical size of system L_c from net transport phase to no transport phase.

Another useful observable in the study of equilibrium critical behavior is the susceptibility that according to fluctuation–dissipation can be taken by calculating the variance of the order parameter v_a. However, the fluctuations of the order parameter given by the Ising models

$$\chi = Var(v_a) N^D \qquad \textbf{(7)}$$

With $$Var(v_a) = [\langle v_a^2 \rangle - \langle v_a \rangle^2] \qquad \textbf{(8)}$$

The $Var(v_a)$ is the order parameter variance and $D=2$. Early numerical calculation and theoretical arguments strongly propose that the dependence of the critical density and the critical size of system corresponding to different noise of particles.

It turned out we analyses in Fig.3.2 (a) the variation of susceptibility χ as a function of density ρ for different values of the noise η: initially we note that there is an increased value of susceptibility as a function of density, this increase reaches a maximum value and drops to a minimum value equal to zero when the density tends to infinity $\rho \rightarrow \infty$. The dependence of the susceptibility on the density practically is no monotonic exhibiting a turnover. The data in this figure shows that for a fixed noise the critical density ρ_c and the L_c are determined from the maximum value of the curve depends strongly on the noise. More precisely, by decreasing the noise in Fig.3.2 (b), the peak position is shifted to the large size of system L. Therefore, our simulation shows that at what critical values of $\rho_c(\eta)$ and $L_c(\eta)$ the whole flock can stay together and also determines the ordered and disordered movement, this transition does not depend only on the density as found in the Viscek model but also the effect of system size.

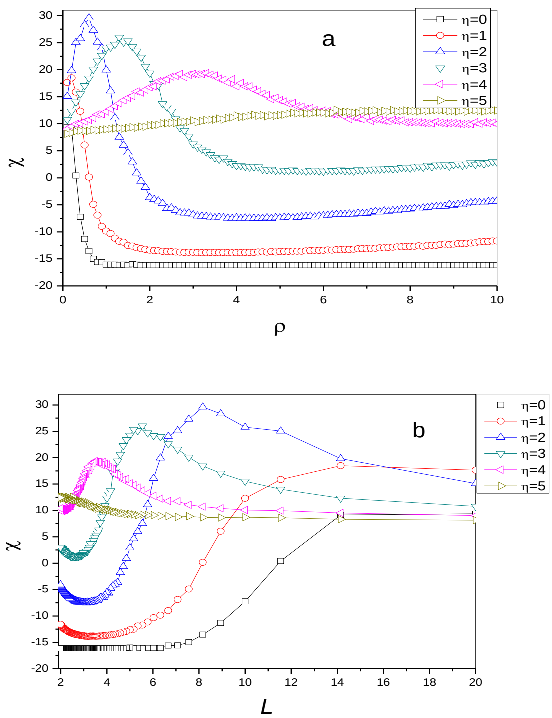


Figure 3.2: Variation of the susceptibility χ as a function of: (a) density ρ and (b) size of system L for different values of noise η, for fixed value of the r_r=0.1,r_o=0.9 and r_a=2.0.

To study the nature of the transition found in Fig.3.1, and in Ref [1, 3] (described the velocity as a function of noise) we use the fourth order cumulant of the order parameter which is the Binder cumulant [16-18], defined as:

$$B = 1 - \frac{\langle v_a^4 \rangle}{3 \times \langle v_a^2 \rangle^2} \qquad \textbf{(9)}$$

The Binder cumulant measures the fluctuations of the order parameter and is a good measure to distinguish between first and second order phase transitions. In case of a first order phase transition B has a definite minimum; on the other hand, the Binder cumulant exhibits a sharp drop toward negative values. This behavior in this case reflects the transition of first order. The Fig.3.3 described the variation of Binder cumulant B as a function of noise η for different values of L. The Binder cumulant has no minimum distinguishable in small sizes systems, we conclude that the finite size effects are very important and the minimum can be observed only for very large sizes. Note that in the case of small sizes of L the finite size effects can mask the pic of the first order transition. This minimum is due to the simultaneous contributions of the two phases coexisting. When $\eta \rightarrow 0$ the fluctuations of the $B \rightarrow 0$, it is easy to compute that $B = \frac{2}{3}$ in the ordered phase. The movement is completely random when $B = \frac{1}{3}$.

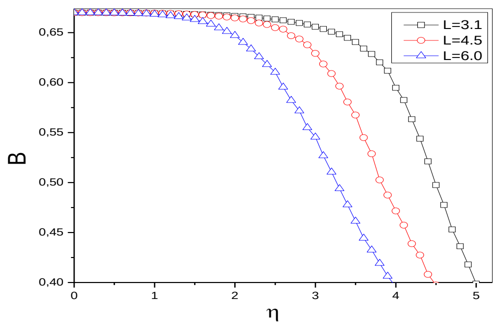


Figure 3.3. Variation of the Binder cumulant G as a function of the noise η, for different values of system sizes L and for fixed value of the density $\rho = 2$, r_r=0.1, r_o=0.9 and r_a=2.0.

In Fig.3.4 we present the variation of susceptibility χ as a function of noise η for different values of the radius of orientation r_o: initially we note that there is an increased value of susceptibility as a function of noise, this increase reaches a maximum value and drops to a minimum value equal to zero when the noise tends to infinity $\eta \rightarrow \infty$. The dependence of the susceptibility on the noise is no monotonic exhibiting a turnover. The data in this figure shows that for a fixed density and radius of repulsion r_r the critical noise η_c is determined from the maximum value of the curve depends strongly on the radius of orientation r_o. In fact, by increasing the radius of orientation, the peak position is shifted to the large noise the result is always the same if we increase the densityρ . And for much larger value of radius of repulsion r_r the peak position is oriented also towards large noises. Therefore, our simulation shows that at what critical values $\eta_c(\rho, r_r, r_o)$

the whole flock can stay together and also determines the ordered and disordered movement, this transition does not depend only on the density as found in the Viscek model but also the effect of radius of repulsion r_r and radius of orientation r_o.

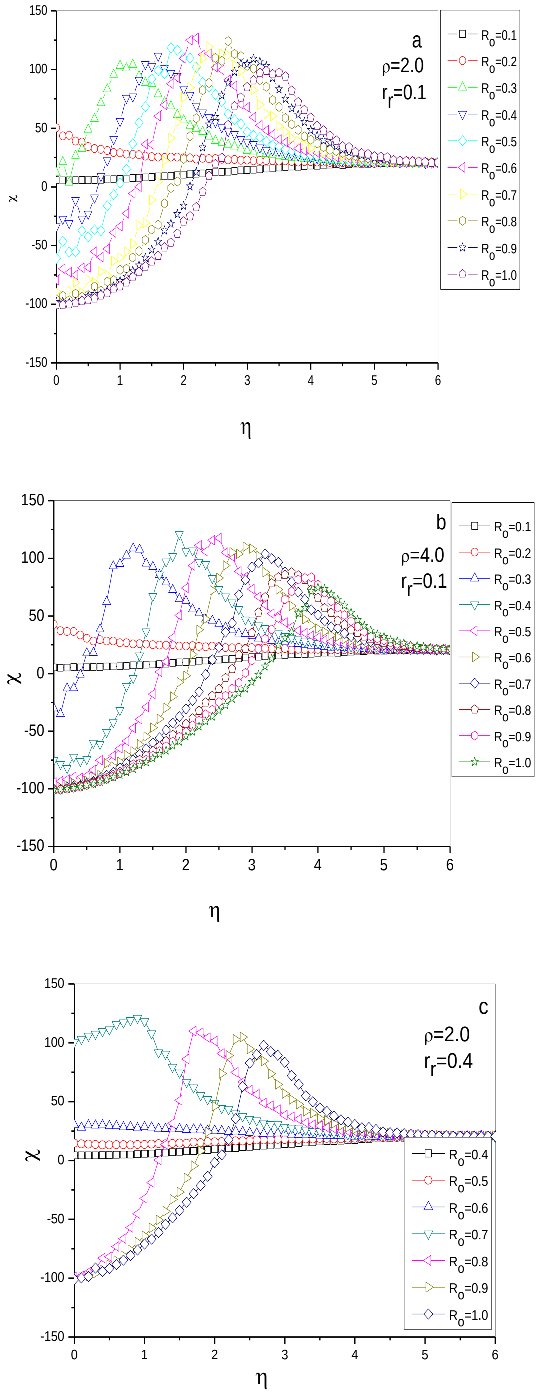


Figure 3.4. The susceptibility χ as a function of noise η for different values of the radius r_o and r_a=2.0.

However, in nature, groups are likely to move between the collective states if conditions change, therefore the initial orientation of individuals and the form of the group can influence the future of collective behavior if the parameters behavioral change.

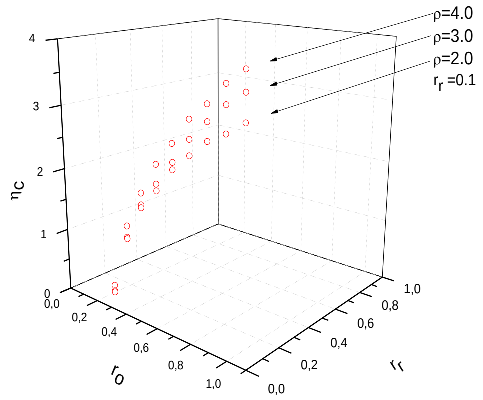
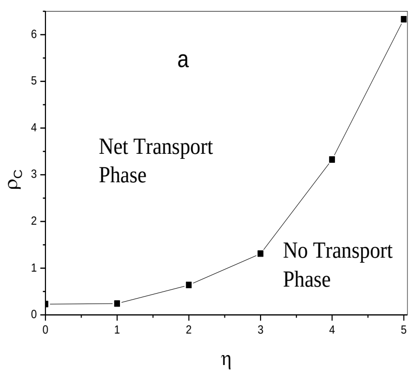


Figure 3.5. Kinetic phase diagram with the effect of two radiuses r_o and r_r

In this Figure 3.5 we will determine the evolution of critical noise η_c as a function of radius of orientation r_o and radius of repulsion r_r for different values of density ρ. This figure also describes the variation of the kinetic phase transition in flock model of three-dimensional space. From these results, we found that the critical noise η_c increases with increasing the density. However, the determination of the phase diagram and more precise with the effect of two radiuses r_o, r_r and density in our new model is outside of the scope of the study which concentrate on demonstrating the main features of a novel no equilibrium system.



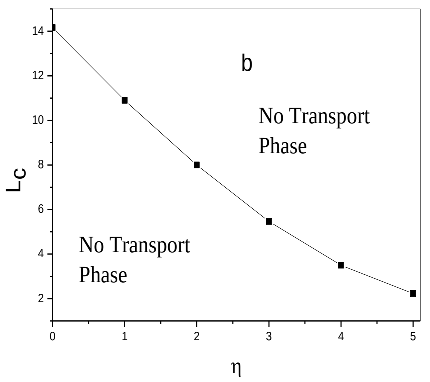


Figure 3.6: Variation of the: (a) critical density ρ_c as a function of noise η and (b) critical size of system L_c as a function of noise η

Finally, in Fig. 3.6 (a, b) we determine the kinetic phase diagram of the Vicsek model with three zones of the flock model in the plane of density versus the noise and the size of system versus the noise also. From these results it is evident the find the ρ_c increases with η and L_c decreases with η also. In fact and in all cases, the relation between ρ_c , L_c and η is trivial.

4. CONCLUSION

The aim of this work is an extension of the Viscek model of the collective displacement of self-propelled individuals, aimed to contribute and to understand the role: of density and the effect size of system in the onset of order motion. Using the computation of the susceptibility our simulation proves that the ρ and the L plays a very important role to determine the kinetic phase transition from no transport to finite net transport. Another important result is that the noise influences greatly the critical value of L_c and ρ_c. We observed a sharp decreasing of B with η indicating that the transition regime is fast enough like the first-order transitions. These finding are qualitatively in good agreement with the elegant results obtained in the framework of the similar model developed by Grégoire and collaborators, As a guide in this study, we have examined the effect of the third zone of attraction in the model of Viscek and we concluded that this zone has no influence on the results found in the movement of flock in our simulation. To maintain an order in a flock with higher density, it is essential to decrease the noise of the flock. In this article, we have not considered the effect of the movement in space of three dimensions, and the open boundaries which permitted us to have much more information about the collective behavior motion of the flock. Other studies are being conducted to explore further this subject.

5. REFERENCES

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