Sensitivity Analysis in Radiological Risk Assessment using Double Monte Carlo Method

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ABSTRACT
Sensitivity analysis is a study of how changes in the inputs to a model influence the results of the model. Many techniques are available when the model is probabilistic. In this paper we consider a related problem of sensitivity analysis when the model includes uncertain variable that can involve both aleatory and epistemic uncertainty and the method of calculation is Probability bounds analysis. In this study, an advanced probabilistic technique called the Double Monte Carlo method is applied to estimate the radiological risk due to SR-90 through ingestion of food items. The variables of the risk model along with the parameters of these variables are described in terms of probability distribution (precise and imprecise).

Keywords: Uncertainty, Risk Assessment, Double Monte Carlo, Sensitivity Analysis.

1. INTRODUCTION
The Greek in the 4th century BC were the first recorded civilization to have considered uncertainty. Uncertainty plays a critical role in the analysis for a wide and diverse set in various fields. Ideals and concepts of uncertainty have long been associated with gambling and games. There are two kinds of uncertainty [7]. The first kind called aleatory uncertainty arises due to randomness and the other called epistemic uncertainty arises due to lack of data or insufficient information.

These two kinds of Uncertainty can propagate through various mathematical expressions with different calculation method. Probabilistic risk assessment (PRA) is related to one of these methods. Probabilistic risk assessment (PRA) applies the probability distribution for the input variables of the risk assessment model in order to quantitatively characterize their variabilities and uncertainties. Two interpretations are generally proposed [4] for the distribution of the input variable. First, Uncertainty regarding variability may be viewed in terms of probability regarding frequencies. Secondly variability is described by frequency distributions, and that uncertainty in general, including sampling error, measurement error, and estimates based upon judgment, is described by probability distribution. The most widely used method in PRA is Monte Carlo analysis (MCA), which is a means of quantifying uncertainty or variability in a probability framework using computer simulation. When inputs are tainted with both kinds of uncertainty, then an advanced modeling approach called Two-dimensional Monte Carlo analysis (2D MCA) can be used.

2. PROBABILITY BOUNDS AS A SENSITIVITY ANALYSIS

Sensitivity analysis [1] is the general term for quantitative study of how the inputs to a model influence the results of the model. Risk model involves uncertain inputs. The uncertainty of the inputs gets propagated to the output. In a decision making process it is desirable to have minimum of uncertainty in the conclusion. For this it becomes necessary to reduce the uncertainty of the inputs. However it is not always feasible to treat each input separately and reduce its uncertainty. In such situation sensitivity analysis can be done to identify the input which is most sensitive to the model. After identifying the most sensitive input, further investigation can be done to improve upon the estimate of the input and thereby reduce its uncertainty. If small changes in an input parameter result in relatively large changes in a model's output, the model is said to be sensitive to that parameter. Sensitive analysis has many manifestations in probabilistic risk analysis and there are many disparate approaches based on various measures of influence and response.

Monte Carlo analysis can be viewed as a kind of sensitivity analysis itself ([2], [3], [6]) in that it yields a distribution describing the variability about a point estimate. In sensitivity studies, many Monte Carlo simulations explore the possible impact on the assessment results by varying the inputs. The process of varying an input by replacing it with an input having reduced uncertainty is called *pinching*. The following strategies are usually followed for pinching in PBA assessments:

(i) replace an input with a point value,

(ii) replace an input with a precise distribution function,

(iii) replace an input with a zero-variance interval.

(iv) replace an input with an uncertain number with smaller uncertainty.

The fundamental purposes of sensitivity studies require estimating the value of additional empirical information. The value of information not yet observed cannot be measured, but it can perhaps be predicted. This might be done by comparing the uncertainty before and after pinching an input. The estimate of the value of information for a parameter will depend on how much uncertainty is present in the parameter, and how it affects the uncertainty in the final results. The sensitivity could be computed with an expression like [1]

$$100\left(1-\frac{\mathrm{unc(T)}}{\mathrm{unc(B)}}\right)\% \quad .....(1)$$

Where B is the base value of the risk expression, T is the value of the risk expression computed with an input pinched, and unc(.) is a measure of the uncertainty of a p-box. The result is an estimate of the value of additional empirical information about the input in terms of the percent reduction in uncertainty that might be achieved in the expression when the input parameter is replaced by a better estimate obtained from future empirical study. There are many possible ways to define unc(.) to measure uncertainty. In the context of PBA ([5], [8]), one obvious measure is the area between the upper and lower bounds of the p-box. An analyst may also define unc(.) as variance or some other measure of dispersion, or perhaps the heaviness of the tails of the p-box. The pinching can be applied to each input quantity in turn and the results used to rank the inputs in terms of their sensitivities. In this paper we have pinched an input by reducing its confidence interval. Further we have used average width as measure of the unc(.).

3. A CASE STUDY ON RADIOLOGICAL RISK OF THE RADIONUCLIDE SR-90

We have considered a case of Radiological Risk due to the radionuclide Strontium-90 (Sr-90) through the pathways of ingestion. The uncertain parameters of the risk model are food intake and food activity. For this case study, we considered some hypothetical data which are available in terms of Minimum, Most likely and Maximum values as shown in Table 1 below. Using this data, we calculated the radiological risk for the radionuclide Sr-90 by using 2DMCS and evaluate the sensitivity of the food items.

Risk due to ingestion of contaminated food is calculated using the following model:

Risk(/Yr) = Activity on food items(Bq/Kg) × Intake food(Kg/Yr) × Risk factor(/Bq)

3.1. Risk calculation using Double Monte Carlo method:

Data including intake of food items, activity of food items, risk factor (for the radionuclide Sr-90) are given in table 1. To incorporate the uncertainty involved in the determination of the most likely value we have considered an interval around it. For our problem we have considered 95% and 99% confidence intervals.

In this approach the inputs of the risk equation along with the parameters of these variables are described in terms of probability density functions (PDFs). A variable described in this way is called a “Second order random variable.”

For this method, we represent each uncertain input as a triangular distribution with support [min, max] where the mode is considered as uniform distribution over the 95% and 99% confidence interval of the most likely value. The confidence interval is obtained from the TFN by using α(alpha)-cut method. Table 2 gives (below) the 95% and 99% confidence interval of mode of different activities. Here, we use the Matlab Software to do the calculation.

Table 1: Intake and activity of food items for the radionuclide Sr-90

Intake of food items (Kg/Yr)				
Food items	**Value**			**Representation**
	Min	**Most likely Value(MLV)**	**Max**	
Wheat	105	110	115	2DMCS
Rice	22	25	27	2DMCS
Maize	1.5	2.5	6.5	2DMCS
Pulses	20	25	30	2DMCS
Vegetables	60	85	100	2DMCS
Milk	65	95	105	2DMCS
Mutton	10	13	20	2DMCS
Fish	11	15	19	2DMCS

Eggs	3	5	8	2DMCS
Activity of radionuclide Sr-90 in food items (Bq/Kg)				
Wheat	0.148	0.148	0.148	Fixed
Rice	0.148	0.148	0.148	Fixed
Maize	0.148	0.148	0.148	Fixed
Pulses	0.148	0.148	0.148	Fixed
vegetables	0.049	0.049	0.049	Fixed
Milk	0.0359	0.0359	0.0359	Fixed
Mutton	0.0418	0.0418	0.0418	Fixed
Fish	0.0847	0.0847	0.0847	Fixed
Eggs	0.0927	0.0927	0.0927	Fixed
Risk factor (/Bq) for Sr-90 in food				
1.62e-09				

Table 2: 95% and 99% confidence interval of MLV of intake of food using α(alpha)-cut method.

Food items	**Intake of food items (Kg/Yr)**				
	Most likely value(MLV)	95% confidence interval ([a, b]) of MLV		99% confidence interval ([c, d]) of MLV	
		A	b	c	d
Wheat	110	105.25	114.75	105.05	114.95
Rice	25	22.15	26.9	22.03	26.98
Maize	2.5	1.55	6.3	1.51	6.46
Pulses	25	20.25	29.75	20.05	29.95
vegetables	85	61.25	99.25	60.25	99.85
Milk	95	66.5	104.5	65.30	104.9
Mutton	13	10.15	19.65	10.03	19.93
Fish	15	11.2	18.8	11.04	18.96
Eggs	5	3.1	7.85	3.02	7.97

Table 3: Average width of risk at different confidence interval of the uncertain inputs.

Case	Confidence interval	Average width of Risk	
Base case	All input taken at 99% confidence interval	4.291085676875423e-009	
	Pinching of input at 95% confidence interval		Percentage reduction in uncertainty
Case 1	Wheat	4.133802772177716e-009	3.665340581
Case 2	Rice	4.202448664851546e-009	2.06560807
Case 3	Maize	4.253080954328651e-009	0.8856668314
Case 4	Pulses	4.133949105222079e-009	3.661930418
Case 5	Vegetable	4.043016564177955e-009	5.781033784
Case 6	Milk	4.084139227865366e-009	4.822706061
Case 7	Mutton	4.264526376625967e-009	0.6189412715
Case 8	Fish	4.221227475148873e-009	1.627984314
Case 9	Eggs	4.252254227988390e-009	0.9049329659

4. RESULT AND DISCUSSION

Sensitivity analysis is universally recognized as crucial in planning strategies to manage risk of adverse events, as well as in designing further empirical study to improve risk estimates.

We have discussed the sensitivity of the parameters of the risk model given in section 3. We have developed a computer code for triangular distribution. In the probabilistic calculation, 1000 iteration of probability distributions were used for the Monte Carlo random sampling.

The output of the model is first calculated with 99% confidence interval of the inputs (fig 1) which is considered as the base case. The uncertainty (average width) of the output parameter is 4.291085676875423e-009. The input parameters viz., intake of different food items are then pinched by considering at 95% confidence interval and the average width of each case is shown in table 3. Then the percentage reduction of uncertainty is calculated for each case. From the table we see that the percentage reduction is high in case 5 (i.e., the intake of vegetable) and least in case 7(i.e., intake of mutton). Hence we can conclude that the input vegetable intake is most sensitive among the input parameters. In other words this input is contributing maximum to the uncertainty of the output. To reduce the uncertainty of the output it is prudent to collect more information regarding that input so as to have any realistic conclusion regarding the risk involved.

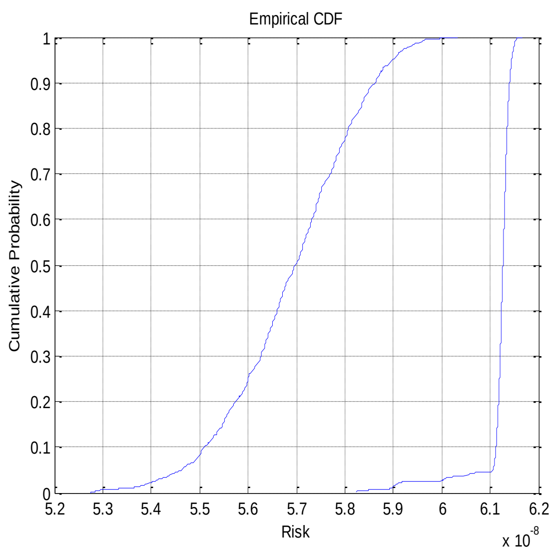

Figure1. Risk of food items when all items are in 99% confidence interval (Base case)

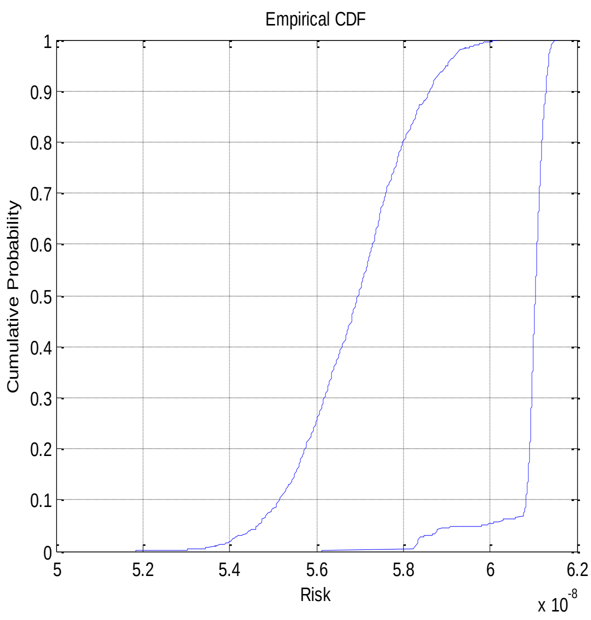

Figure2. Risk of food items when all items except vegetable (95%) are in 99% confidence interval (Case 5)

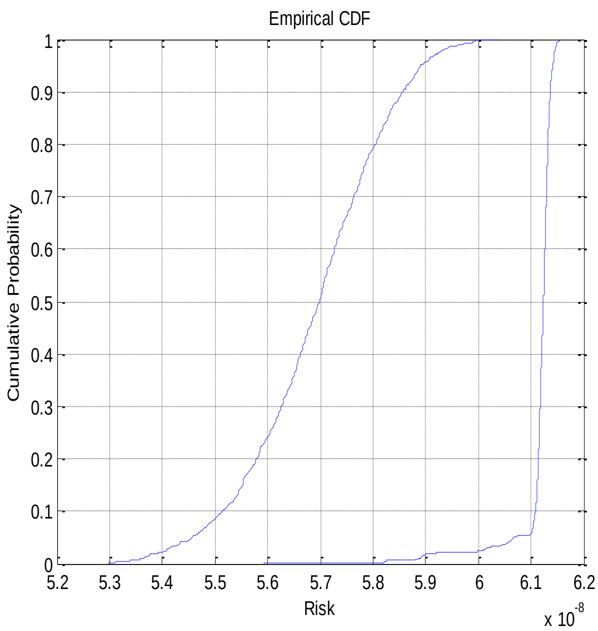

Figure3. Risk of food items when all items except mutton (95%) are in 99% confidence interval (case 7)

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