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On Rainbow Coloring of Some Classes of Graphs

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ABSTRACT
A path in an edge colored graph is said to be a rain bow path if no two edges on the path have the same color. An edge colored graph is rainbow connected if there exists a rainbow path between every pair of vertices. The rainbow connectivity of a graph G, denoted by rc(G) is the smallest number of colors required to edge color the graph such that the graph is rainbow connected.

In this paper a rainbow coloring of the corona of P_n o$K_2$the corona of P_n∘C_4,flower graphs and fan graph are considered and rc(G) of these graphs are decided.

Keywords
Rainbow coloring, Flower graph, Fan graph.

1. INTRODUCTION
All graphs in this paper are finite, undirected and simple. We follow the notation and terminology of [1]. We have considered different kinds of labeling such as Bi magic labeling, Edge magic labeling, and Prime labeling of certain classes of graphs [2, 3, 4, 5].The Rainbow coloring is studied by[6,7,8,9].These people for graphs which exhibits some parameters in terms of the vertices and the minimum degree. Also the upper bound on Rainbow connection number is also a subject of investigation in these papers. The following interesting connectivity measure of a graph has recently attracted the attention of several researchers. An edge colored graph G is rainbow edge- connected if any two vertices are connected by a path whose edges have distinct colors. Clearly, if a graph is rainbow edge connected then it is also connected. Conversely, any connected graph has a trivial edge coloring that makes it rainbow edge- connected just color each edge with a distinct color. Thus the following natural graph parameter was defined by Chartrand et al in [6]. Let the rainbow connection of a connection graph G denoted by rc(G), be the smallest number of colors, that are needed in order to make rainbow edge-connected.

Let G be a nontrivial connected graph on which an edge coloring C : E(G) → {1, 2, … n}, n belongs to N, is defined where adjacent edges may be colored the same. A path is a rainbow, if no two edges of it are colored the same. An edge-coloring graph G is rainbow connected if any two vertices are connected by a rainbow path. An edge- coloring under which G is rainbow connected is called rainbow coloring. Clearly if a graph is rainbow connected, it must be connected. Conversely, any connected graph has a trivial edge –coloring that makes it rainbow connected; just color each edge with a distinct color. Thus, we define the rain bow connection number of a connected graph G, denoted rc(G), as the smallest number of colors needed in order to make G rainbow connected [8].

A rainbow coloring using rc(G) colors is called a minimum rain bow coloring.. By definition, if H is a connected spanning subgraph of G, then rc(G)≤rc(H) .It is easy to see that diam(G)≤rc(G) for any connected graph G, where diam(G) is the diameter of G.

Chakrarborty et al.[10] showed that computing the rainbow connection number of a graph is NP-Hard. To rainbow color a graph it is enough to ensure that every edge of some spanning tree in the graph gets a distinct color. Hence order of the graph minus one is an upper bound for rainbow connection number. Many authors view rainbow connectivity as one “quantifiable” way of strengthening the connectivity property of a graph [10, 11]. Hence tighter upper bounds on rainbow connection number for graphs with higher connectivity have been a subject of investigation.

Then, we consider the rainbow coloring of the following graphs. 1) corona of P_n°K_2 2) corona of P_n°C_4, 3) Flower graphs, 4) Fan graph

1) Corona of P_n°K_2

The corona graph G_1°G_2 [12] is obtained by taking one copy of G_1 of order P_1 and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.Let G_1 =P_n be a path graph with n vertices and G_2= K_2 be a complete graph with two vertices. The corona G=P_n°K_2 is obtained by taking one copy of P_n of order n and n copies of K_2 and then joining the i^{th} vertex of P_n to every vertex on the i^{th} copy of K_2.It is proved that the rainbow coloring rc(G) of P_n°K_2 is 2n-1.

2) Corona of P_n°C_4

Let G_1= P_n be a path graph with n vertices and G_2=C_4 be a cycle graph with 4 vertices. The corona G=P_n°C_4 is obtained by taking one copy of P_n with n vertices and n copies of C_4 and then joining the i^{th} vertex of P_n to every vertex in the i^{th} copy of C_4 .It is shown that the rainbow coloring of corona of P_n°C_4 is n+1.

3) Flower graph

A flower is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm. The Helm H_n is the graph obtained from a Wheel by attaching a pendant edge at each vertex of the n-cycle. The Prime labeling of sunflower graph is given in[13]. The Rainbow coloring of corona of flower graph is proved to be 3.

4) Fan graph

A Fan graph F_n can be constructed by joined n copies of the cycle graph C_3 with a common vertex. F_n is a planar undirected graph with 2n+1 vertices and 3n edges. Rainbow coloring of Fan graph is shown to be 3.

2. MAIN RESULTS

Theorem 1: If (n-1) petal edges are attached at the central vertex of wheel W_n for $n \geq 4$,then the rainbow coloring of the resulting graph G is rc(G)=3.The proof is given as follows;

Proof: From Wheel W_n ,$n \geq 4$,helm is obtained by adding pendant edges to the central vertex. Flower graph is drawn from the helm graph by joining each pendant vertex to the central vertex of the helm. The flower graph has 2n+1 vertices, 4n-4 edges[14].

We classify the edges of the flower graph as follows; i)spoke edges, ii)petal edges iii)cycle edges. We color these different edges as given below;

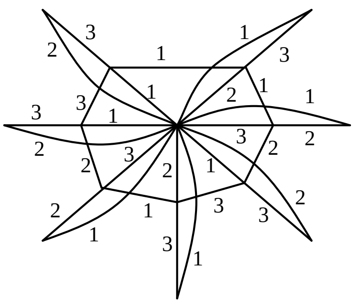
i) Spoke edges can be colored as 1,2,3 cyclically

ii) Among 1,2,3 the number that has not found in the Spoke ,the other two numbers has to be given in the two edges of the petal.

iii) The cycle edges has to be colored as,

Identify the spoke edge with the starting label 1.This edge is joined with a cycle edge. The next edge is to be labeled as 1,2,3 cyclically.

Example: Flower graph of W_8



Thus the Rainbow coloring of the Flower graph is 3.

Theorem 2: The corona graph G $P_n{}^oK_2$ admits a rainbow coloring. Where n is the number of vertices in the path. rc(G)=2n-1.
Proof: In $G=P_n \circ K_2$, construction of vertex set and edge set as follows. Let $V(G) = V(P_n) \cup V(nK_2)$, Where $V(P_n) = \{u_1,u_2,\ldots\ldots u_{n-1},u_n\}$ and $V(nK_2) = \{v_1,v_2,\ldots\ldots v_{2n-1}, v_{2n}\}$ $E(G) = \{u_iu_{i+1}:1 \leq i \leq n-1\} \cup \{v_{2i-1}v_{2i}:1 \leq i \leq n\} \cup \{u_iv_i:1 \leq i \leq n\} \cup \{u_iv_{2i}:1 \leq i \leq n\}$ be the vertex set and edge set of G respectively.

The Rainbow coloring of corona of $P_n \circ K_2$ as follows. We will classify the edges of corona of $P_n \circ K_2$ in to three cases:

i) Path edges
ii) K_2 edges
iii) Edges joining from K_2 with path.

The function f: $E(G) \rightarrow \{1,2,3,\ldots.2n-1\}$ is defined as below.

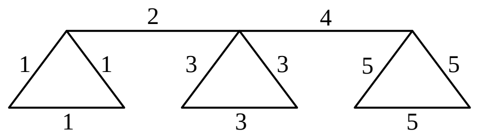
i) $f(u_i,u_{i+1})=2i;1 \leq i \leq n$.

ii) $f(v_{i,}v_{i+1})=1+2j$, when i+1,2,3….j=0,1,2… respectively.

iii) $f(u_i,u_{i+1})=1+2j$, when i=1,2,3,…. j=0,1,2,…respectively.

(iv) $f(u_i,v_{2i-1})=2i-1:1 \leq i \leq n$

Example: $P_3{}^oK_2$



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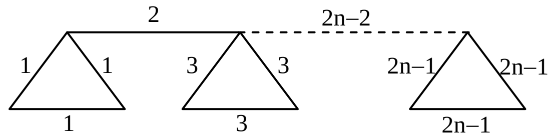


Thus the rain bow coloring of $P_n{}^oK_2$ is 2n-1.

Theorem 3: The corona graph $P_n{}^oC_4$ admits a rainbow coloring, where n is the number of vertices in the path rc(G)=n+1

In a graph $G=P_n \circ C_4$, construction of V(G) and E(G) as follows,

Let $V(G) = V(P_n) \cup V(C_4^1) \cup V(C_4^2) \cup \ldots. \cup V(C_4^n)$. where $V(P_n)=\{u_1,u_2,u_3,\ldots.u_{n-1},u_n\}$and $V(P_n)=\{u_1,u_2,\ldots..u_{n-1},u_n\}$ and $V(C_4^i) =\{p_i,q_i,r_i,s_i:1 \leq i \leq n\}andC_4^i$ is the i^{th} copy of C_4,be the vertex set and edge set of G respectively. The corona of $P_2{}^oC_4$ is given below. |V(G)|=5n and |E(G)|=9n-1. The function $f:E(G) \rightarrow \{1,2,3\ldots.2n-1\}$

i) $f(u_i,u_{i+1)=}2i;\ 1 \leq i \leq n$

ii) $f(p_i,q_i)=1;1 \leq i \leq n$

iii) $f(r_i,s_i)=1;1 \leq i \leq n$

iv) $f(p_i,s_i) =n+1;1 \leq i \leq n$

v) $f(q_i,r_i)=n+1;1 \leq i \leq n$,

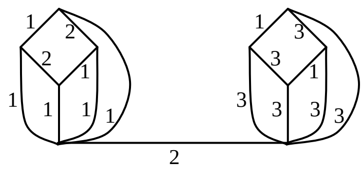
vi) $f(u_i,p_i)=2i-1;1 \leq i \leq n$

vii) $f(u_i,s_i)=2i-1;1 \leq i \leq n$

viii) $f(u_i,r_i)=2i-1;1 \leq i \leq n$

ix) $f(u_i,q_i)=2i-1;\ 1 \leq i \leq n$

Example: $P_2{}^oC_4$



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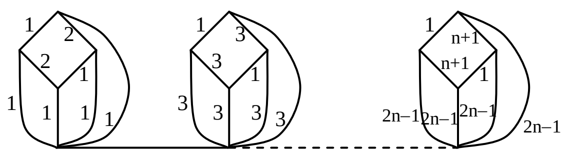


Thus the rainbow coloring of $P_n{}^oC_4$ is n+1.

Theorem 4: The Fan graph G has (2n+1) vertices and 3n edges admits a rainbow coloring. rc(G)=3.

Proof: The Fan graph denoted by F_n can be constructed by joining n copies of cycle graph C_3 with a common vertex. F_n is a planar undirected graph with 2n+1 vertices and 3n edges.

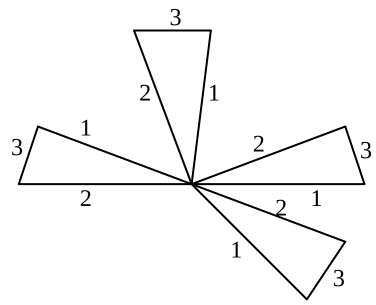
Construction of rain bow coloring as follows;

$f(u_i,u_{i+1})=3$:i=1,3,5,….2n-1

$f(u_i,v)=2$;i=1,3,5,….2n-1.

$f(u_i,v)=1$;i=2,4,6,….2n.where v is the central vertex.

Example: F_4



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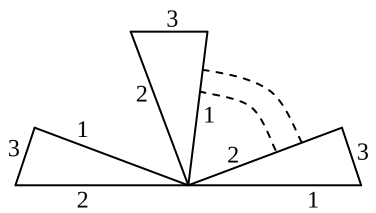

Thus the rainbow coloring of F_n is 3.

3. CONCLUSION

Rainbow connection number rc(G) of a connected graph G is the minimum number of colors needed to color its edges so that every pair of vertices is connected by atleast one path in which no two edges are colored the same.

In this paper the Rainbow coloring of corona of$C_4{}^oP_n$,and $K_2{}^o$ P_n, Flower graph, and Fan graph were discussed. It is observed that the rc(G) of these graphs is given only in terms of the number of vertices irrespective of the minimum degree of these graphs. It is known that computing the Rainbow connection number of a graph is NP-Hard[6]. Hence it is interesting to compute rc(G) of certain classes of graphs. Rainbow connection has an interesting application for secure transfer of classified information between agencies. Hence one can find prohibitive in traders to crack the password in sequrity information transfer. It is of interest to find the rc(G) of different classes of other graphs as given in [8,10].

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