Flow Past an Exponentially Accelerated Infinite Vertical Plate and Temperature with Variable Mass Diffusion

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ABSTRACT
An exact solution of flow past an exponentially accelerated infinite vertical plate and temperature has been presented in the presence of variable mass diffusion. The dimensionless governing equations are solved using Laplace transform technique. The velocity, temperature, and concentration profiles are studied for different physical parameters like radiation parameter R, accelerating parameter a, thermal Grashof number Gr, mass Grashof number Gc, chemical reaction parameter K, Prandtl number Pr, Schmidt number Sc and time t. It is observed that the velocity increases with increase in Gc, Gr, R, a, Pr, Sc and t. It is also observed that temperature rise with increasing t, a, R while concentration increases with increasing t.

General Terms
Asogwa et. al.

Keywords
Accelerated, vertical plate, radiation, chemical reaction heat transfer, variable mass diffusion

1. INTRODUCTION

This study examines flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion. The dimensionless governing equations are solved using Laplace transform technique. The solutions are obtained in terms of exponential and complementary error functions.

2. FORMULATION OF THE PROBLEM
Here the flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion has been considered. The $x'$ -axis is taken along the plate in the vertically upward direction and also the $y'$ -axis is taken normal to the plate. At $t' > 0$, the plate is accelerated with a velocity $u = u_0 \exp(at)$ in its own plane and the temperature of the plane is raise at a uniform rate to velocity,
and the level of concentration near the plate is raised linearly with time. Then under the usual Boussinesq’s approximation the unsteady flow equations are momentum equation, energy equation, and mass equation respectively.

$$\frac{\partial u}{\partial t} = g \beta (T - T_{\infty}) + g \beta \theta' (C' - C_{\infty}') + \nu \frac{\partial^2 u}{\partial y^2}$$  \hspace{1cm} (1)$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \rho \frac{\partial q_r}{\partial y}$$  \hspace{1cm} (2)$$

$$\frac{\partial C'}{\partial t} = D \frac{\partial^2 C'}{\partial y^2} - K^* C'$$  \hspace{1cm} (3)$$

where $u$ is the velocity of the fluid, $T$ is the fluid temperature, $C'$ is the concentration, $g$ is gravitational constant, $\beta$ and $\beta'$ are the thermal expansions of fluid and concentration, $t'$ is the time, $\rho$ is the fluid density, $C_p$ is the specific heat capacity, $\nu$ is the viscosity of the fluid, $k$ is the thermal conductivity, $D$ is the diffusion term, $K^*$ is chemical reaction parameter and $q_r$ is the radiative heat flux.

In this research work the mathematical formulation has chemical reaction, rate of radiation heat flux which are not included in the work of Muthucumaraswamy et al. [15] and difference in the boundary conditions for velocity, concentration and temperature. By Rosseland approximation, we assume that the temperature differences within the flow are such that $T^4$ may be expressed as a linear function of the temperature $T$. This is accomplished by expanding Taylor series about $T_d^*$ neglecting higher order terms.

The initial and boundary conditions are:

$$t < 0$$

$$t' > 0: \begin{cases} u = u_0 e^{\frac{y}{\nu}}, T = T_{\infty}, \theta = C = C_{\infty}, \beta' = \beta' \theta' \gamma = 0 \\ u' \to 0, T' \to 0, C' \to 0 \text{ at } y \to \infty \end{cases}$$  \hspace{1cm} (4)$$

Where $B = \left( \frac{u_0^2}{\nu} \right)^{\frac{1}{3}}$, $B$ is a constant

The thermal radiation heat flux gradient may be expressed as follows

$$- \frac{\partial q_r}{\partial y} = 4 \sigma \sigma^* (T_{\infty}^4 - T^4)$$  \hspace{1cm} (5)$$

Where $q_r$ is the radiative heat flux, $\sigma$ is the absorption coefficient of the fluid and $\sigma^*$ is the Stefan-Boltzmann constant. We assumed that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. Expanding $T^4$ about $T_{\infty}$ in a Taylor’s series and neglecting higher order terms, we have

$$T^4 \approx 4T_{\infty}^3T^* - 3T_{\infty}^4$$  \hspace{1cm} (6)$$

By using equation (5) and (6), equation (2) reduces to

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16 \sigma \sigma^* T_{\infty}^3 \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (7)$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{(\nu u_0)^{\frac{1}{3}}} t' = t \left( \frac{u_0^2}{\nu} \right)^{\frac{1}{3}} Y = y \left( \frac{u_0}{\nu} \right)^{\frac{1}{3}}$$

$$\theta = \frac{T - T_{\infty}}{T_{\infty} - T_{\infty}}, C = C' - C_{\infty}', C_{\infty}' - C_{\infty}$$

$$Gr = g \beta (T_{\infty} - T_{\infty}), Gc = g \beta \sigma (C_{\infty}' - C_{\infty})$$

$$Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, R = \frac{4 \sigma \sigma^* T_{\infty}^3}{k a_{\infty}}$$  \hspace{1cm} (8)$$

Substituting the non-dimensional quantities of equation (8) into (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial y^2}$$  \hspace{1cm} (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\lambda} \frac{\partial^2 \theta}{\partial y^2}$$  \hspace{1cm} (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - KC$$  \hspace{1cm} (11)$$

Where $\lambda = \frac{3 Pr}{3 + 4R}$, $Gr$ is the thermal Grashof number, $Gc$ is the mass Grashof number, $Sc$ is the Schmidt number, $Pr$ is the Prandtl number and $R$ is the radiation parameter.

The initial and boundary conditions reduce:

$$U = 0, \theta = 0, C = 0, \text{ for all } y, t \leq 0$$

$$t > 0: \begin{cases} U = e^{\nu}, \theta = e^{\nu}, C = t \text{ at } y = 0 \\ U \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty \end{cases}$$  \hspace{1cm} (12)$$

3. SOLUTION TO THE PROBLEM

To solve equations (6) – (8), subjected to the boundary conditions of (9), the solutions are obtained for concentration, temperature and velocity flow in terms of exponential and complementary error function using the Laplace-transform technique as follows;
The Sherwood number for concentration field is obtain when

\[ C(y) = \left[ \frac{1}{2} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) + e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) \right] - \frac{\eta \sqrt{\rho}}{2\sqrt{K}} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) - e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) \right] \right] \]

\[ \frac{\eta \sqrt{\rho}}{2\sqrt{K}} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) - e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) \right] \]

(13)

\[ \theta(y,t) = e^{\eta \sqrt{\beta} \sqrt{K}} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) + e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) \right] \]

(14)

\[ U(y,t) = \frac{e^{\eta \sqrt{\beta} \sqrt{K}}}{2} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta + \sqrt{Kt} \right) + e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta - \sqrt{Kt} \right) \right] + \frac{Gc}{2d^2 (Sc-1)} \left[ 2 \text{erfc} \left( \eta \right) - 2 \eta \text{erfc} \left( \eta \right) - \frac{2\eta e^{\eta \sqrt{\beta} \sqrt{K}}}{\sqrt{\pi}} 
\]

\[ - e^{\beta} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta + \sqrt{Kt} \right) + e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta - \sqrt{Kt} \right) \right] \]

\[ - \frac{\eta \sqrt{\rho}}{2\sqrt{K}} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) - e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) \right] \]

\[ - e^{\beta} \left[ e^{2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} + \sqrt{Kt} \right) + e^{-2\eta \sqrt{\beta}} \text{erfc} \left( \eta \sqrt{K} - \sqrt{Kt} \right) \right] \]

\[ \frac{Gc}{2d^2 (Sc-1)} \left[ 2 \frac{1}{\sqrt{\pi}} - 2 \frac{1}{\sqrt{\pi}} e^{\eta \sqrt{\beta} \sqrt{K}} + \text{erfc} \left( \eta \sqrt{\beta} \sqrt{K} \right) - \frac{\eta e^{\beta \sqrt{K}}}{{\sqrt{\beta} \sqrt{K}} \sqrt{\pi}} \right] \]

(15)

where \( \eta = \frac{y}{\sqrt{2t}} \), \( d = \frac{kSc}{1-Sc} \) and \( b = d + K \)

The nusselt number for temperature field is obtain when \( y=0 \) as

\[ \theta|_{y=0} = \frac{e^{\sqrt{\beta} \sqrt{K}}}{2} \left[ \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} \right] \]

(16)

The Sherwood number for concentration field is obtain when \( y=0 \) as

\[ C|_{y=0} = \frac{e^{\sqrt{\beta} \sqrt{K}}}{2} \left[ \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} \right] \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

(17)

The skin friction gives

\[ U'_{y=0} = \frac{e^{\sqrt{\beta} \sqrt{K}}}{2} \left[ \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{\sqrt{\beta} e^{-\beta \sqrt{K}}}{\sqrt{\pi}} \right] \]

\[ + \frac{Gc}{2d^2 (Sc-1)} \left[ 2 \frac{1}{\sqrt{\pi}} - 2 \frac{1}{\sqrt{\pi}} e^{\eta \sqrt{\beta} \sqrt{K}} + \text{erfc} \left( \eta \sqrt{\beta} \sqrt{K} \right) - \frac{\eta e^{\beta \sqrt{K}}}{{\sqrt{\beta} \sqrt{K}} \sqrt{\pi}} \right] \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

\[ - \sqrt{\text{erfc} \left( \sqrt{\beta} \sqrt{K} \right)} + \frac{Sc}{\pi} e^{-\beta \sqrt{K}} - \sqrt{\text{erfc} \left( -\sqrt{\beta} \sqrt{K} \right)} - \frac{Sc}{\pi} e^{-\beta \sqrt{K}} \]

(18)

4. RESULTS AND DISCUSSION

The problem of flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion has been formulated, analysed and solved analytically. In order to point out the effects of physical parameters namely; Accelerating parameter \( a \), thermal Grashof number \( Gr \), mass Grashof number \( Gc \), Prandtl number \( Pr \), Schmidt number \( Sc \), time \( t \), radiation parameter \( R \), and chemical reaction parameter \( K \). On the flow patterns, the computation of the flow fields are carried out. The value of the Prandtl number \( Pr \) is chosen to represent air (\( Pr = 0.71 \)). The value of Schmidt number is chosen to represent water vapour (\( Sc = 0.6 \)). The values of velocity, temperature and concentration are obtained for the physical parameters as mention.

The velocity profiles has been studied and presented in figure 1 to 8. The effect of velocity for different values of Schmidt
number (Sc = 0.16, 0.22, 0.3, 0.6) is presented in figure 1. The trend shows that the velocity increases with increasing Schmidt number. The effect of velocity for different values of time (t = 0.2, 0.4, 0.6, 0.8) is also presented in figure 2. It is then observed that the velocity increases with increasing values of time. The effect of velocity profiles again have been studied for different values of thermal Grashof number (Gr = 2, 5, 10) and mass Grashof number (Gc = 3, 5, 10) is studied and then presented in figure 3 and 4 respectively. The results are here observed that the increase in the values of velocity increases with increasing values of Gr and Gc respectively.

The velocity profiles for different values of thermal Grashof number (Gr = 2, 5, 10) is seen in Figure 3. It is observed that velocity increases with increasing Gr. The velocity profiles for different values of mass Grashof number (Gc =3, 5, 10) is presented in Figure 4. It is observed that velocity increases with increasing Gc.

The velocity profiles for different values of accelerating parameter (a = 0.2, 2, 5, 10) is seen in Figure 5. It is observed that velocity increases with increasing a. The velocity profiles for different values of radiation parameter (R =2, 5, 10, 20) is presented in Figure 6. It is observed that velocity increases with increasing R.

The velocity profiles for different values of Prandtl number (Pr = 0.71, 0.85, 1, 7) is seen in Figure 7. It is observed that velocity increases with decreasing Pr. The velocity profiles for different values of chemical reaction parameter (K= 0.2, 0.4, 0.6, 0.8) is presented in Figure 8. It is observed that velocity increases with decreasing K.
The temperature profiles have been studied and presented in figure 9 to 12. The effect of temperature for different values of time ($t = 0.2, 0.6, 1, 2$) is presented in Figure 9. It shows that temperature rises with increasing $t$. The temperature profiles for different values of Prandtl number ($Pr = 0.71, 0.85, 1, 7$) is presented in Figure 10. It is observed that decreases the temperature.

The concentration profiles have been studied and presented in figure 13 to 15. The effect of concentration for different values of time ($t = 0.2, 0.4, 0.6, 0.8$) is presented in Figure 13. It is observed that concentration increases with increasing $t$. The concentration profiles for different values of Schmidt number ($Sc = 0.16, 0.22, 0.3, 0.6$) is presented in Figure 14. It is observed that the concentration increase with increasing $Sc$. The effect of concentration for different values of chemical reaction parameter ($K=0.2, 0.5, 2, 5$) is presented in Figure 15. It is observed that the concentration increases with a decrease in the values of $K$. 
Table 1 Skin friction $\tau$

<table>
<thead>
<tr>
<th>$Gr$</th>
<th>$Gc$</th>
<th>$t$</th>
<th>$R$</th>
<th>$a$</th>
<th>$Sc$</th>
<th>$Pr$</th>
<th>$K$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>0.71</td>
<td>0.5</td>
<td>6.5154</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>0.71</td>
<td>0.5</td>
<td>13.0467</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>0.71</td>
<td>0.5</td>
<td>13.3310</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
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<td>2</td>
<td>2</td>
<td>0.6</td>
<td>0.85</td>
<td>0.5</td>
<td>13.2660</td>
</tr>
<tr>
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<td>5</td>
<td>0.4</td>
<td>4</td>
<td>0.4</td>
<td>0.3</td>
<td>7</td>
<td>0.5</td>
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</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.4</td>
<td>4</td>
<td>0.4</td>
<td>0.3</td>
<td>7</td>
<td>0.5</td>
<td>298.3727</td>
</tr>
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Table 2 Nusselt number $Nu$

<table>
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<tr>
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<th>$Pr$</th>
<th>$Nu$</th>
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<tr>
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<td>2</td>
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<td>0.71</td>
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<td>0.85</td>
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<td>5</td>
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<td>0.85</td>
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Table 3 Sherwood number $Sh$

<table>
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<th>$K$</th>
<th>$t$</th>
<th>$Sh$</th>
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<td>0.0670</td>
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<tr>
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<td>0.5</td>
<td>0.2</td>
<td>0.0997</td>
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<td>0.5</td>
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<td>1</td>
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<td>0.5272</td>
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</table>

Tables 1 to 3 gives the Skin friction, Nusselt number and Sherwood number respectively.

Table 1 shows the effect of physical parameters $Gr$, $Gc$, $R$, $a$, $t$, $Sc$ and $Pr$ on the skin friction, it's observed that the shear stress increases when $Gr$, $Gc$, $R$, $a$, $t$, $Sc$ and $Pr$ increases.

Table 2 represents the variation of physical parameters $Pr$, $a$, $t$, $R$ on the nusselt number which determines the rate of heat transfer, it's observed that rate of heat transfer increases when $R$, $a$, $t$, $Pr$ increases.

Table 3 gives the effect of physical parameters $Sc$, $K$ and $t$ on the Sherwood number, it's observed that mass transfer increases when $K$, $t$, $Sc$ increases.
5. SUMMARY AND CONCLUSION
Flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion has been studied. The dimensional governing equations are solved by Laplace transform technique. The effect of different parameters like accelerating parameter, radiation parameter, chemical reaction parameter, Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time are presented graphically. It is observed that velocity profile increases with increasing parameter namely R, t, Gc, Gr and a while Sc, Pr and K decreases with increasing velocity. It also observed that temperature rise with increasing t, a, R while Pr decreases with increasing Gr and a while Sc, Pr and K decreases with increasing temperature. The concentration profiles increases with increasing t while Sc and K decreases with increase in concentration.

6. REFERENCES