Stability of Functional Equations in Multi-Banach Spaces via Fixed Point Approach

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Dong Yun Shin [14] proved the Fuzzy stability of equation (1.1) and (1.2).
In the section 2, we adopt some usual terminology, notion and
convention of the theory of Multi-Banach spaces. In the last
section, we prove the stability problem in the sense
of Hyers-Ulam-Rassias for the functional equations (1.1)
and (1.2) on Multi-Banach spaces by using fixed point
approach. We also present some corollaries in reference
to our results.

2. PRELIMINARIES

The multi-Banach space was first investigated by Dales
and Polyakov [4]. Theory of multi-Banach spaces is
similar to the operator sequence space and has some
connections with operator spaces and Banach spaces.
In 2007, H. G. Dales and M. S. Moslehian [5] first proved the
stability of mappings on multi-normed spaces and also
gave some examples on multi-normed spaces.
The asymptotic aspects of the quadratic functional
functions in multi-normed spaces was investigated by M. S. Moslehian,
the stability of functional equations on multi-normed
was proved by many mathematicians ([7], [10],
[21]).

Now, we adopt some usual terminology, notion and
convention of the theory of multi-Banach spaces from [4]
and [5].

Let (E, ||·||) be a complex normed space, and let k ∈N. We denote by E the linear space E ⊕· · · ⊕E consisting of k-tuples (x1,..., xk), where x1,..., xk ∈E. The linear operations on E are defined coordinate-wise. The zero element of either E or E is denoted by 0. We denote by Nk the set {1, 2,..., k} and by Sk the group of permutations on k symbols.

Definition 2.1(Multi-norm) A multi-norm on {Ek : k ∈N} is a sequence {Nk : k ∈N} such that Nk is a norm on Ek for each k ∈N, ||x|| = ||x||k for each x ∈E, and the following axioms are satisfied for each x ∈E with k ≥ 2:

(N1) ||(xσ(1),..., xσ(k))||k = ||(x1,..., xk)||k,
for σ ∈ Sk, x1,..., xk ∈E;

(N2) ||(αx1,..., αxk)||k ≤ (maxi∈Nk |αi|) ||(x1,..., xk)||k,
for α1,..., αk ∈ C, x1,..., xk ∈E;

(N3) ||(x1,..., xk−1, 0)||k = ||(x1,..., xk−1)||k−1,
for x1,..., xk−1 ∈E;

(N4) ||(x1,..., xk−1, xk)||k = ||(x1,..., xk−1)||k−1,
for x1,..., xk−1 ∈E.

In this case, we say that {Ek : k ∈N} is a multi-

normed
space (see [4], [5]). Suppose that \((E_k, \| \| ; k \in \mathbb{N})\) is a multi-normed space, and take \(k \in \mathbb{N}\). We need the following two properties of multi-norms. They can be found in [4], \[\|(x, \ldots, x)k = \|x\|, \text{ for } x \in E.\]

\[
\max_{i \in \mathbb{N}} \|x_i\| \leq \|(x_1, \ldots, x_k)k \leq k
\]

It follows from (b) that if \((E, \| \|)\) is a Banach space, then \((E_k, \| \| ; k \in \mathbb{N})\) is a Banach space for each \(k \in \mathbb{N}\); in this case, \((E_k, \| \| ; k \in \mathbb{N})\) is a multi-Banach space.

**Lemma 2.2.** Suppose that \(k \in \mathbb{N}\) and \((x_1, \ldots, x_k) \in E_k\). For each \(j \in \{1, \ldots, k\}\), let \((x_i^j)_{i=1,2,\ldots}\) be a sequence in \(E\) such that \(\lim_{n \to \infty} x^j_n = x_j\). Then

\[
\lim_{n \to \infty} (x_1^j - y_1^j, \ldots, x_k^j - y_k^j) = (x_1 - y_1, \ldots, x_k - y_k)
\]

holds for all \((y_1, \ldots, y_k) \in E_k\) (see [4], [5]).

**Definition 2.3.** Let \((E_k, \| \| ; k \in \mathbb{N})\) be a multi-normed space. A sequence \([x_n]\) in \(E\) is a multi-null sequence if for each \(\varepsilon > 0\), there exists \(n_0 \in \mathbb{N}\) such that

\[
\sup_{k \in \mathbb{N}} \|x_n - x_n + k - 1\| \leq \varepsilon, \quad (n \geq n_0).
\]

**Theorem 2.4.** (Fixed Point Alternative) Let \((X, d)\) be a complete generalized metric space and \(J : X \to X\) be a strictly contractive mapping, that is,

\[
d(Jx, \psi y) \leq Ld(x, y), \quad \forall x, \psi y \in X,
\]

for some \(L \leq 1\). Then, for each fixed element \(x \in X\), either

\[
d(Jnx, Jn+1x) = \infty, \quad \forall n \geq 0,
\]

or

\[
d(Jnx, Jn+1x) < \infty, \quad \forall n \geq n_0.
\]

for some natural number \(n_0 \geq 0\). Moreover, if the second alternative holds, then the sequence \([Jnx]\) is convergent to a fixed point \(y^*\) of \(J\).

**Lemma 2.5.** If a mapping \(f : X \to Y\) satisfies the functional equation (1.1) and (1.2) then \(f\) is a Quadratic mapping.

### 3. MAIN RESULTS

In this section, we prove the Hyers – Ulam – Rassias stability of functional equations (1.1) and (1.2). Throughout this section, let \(E\) be a linear space and \((F_n, \| \| ; n \in \mathbb{N})\) be a multi-Banach space.

#### 3.1 STABILITY OF THE FUNCTIONAL EQUATION (1.1) BY FIXED POINT METHOD

First, we prove a lemma, which gives a useful strictly contracting mapping.

**Lemma 3.1.** Let \(E\) be a linear space and \((F_n, \| \| ; n \in \mathbb{N})\) be a Banach space for all \(n \in \mathbb{N}\). Let \(0 < \alpha < a^2\) and a mapping \(\psi : E^n \to \mathbb{R}\) such that

\[\psi(ax_1, ax_2, \ldots, ax_k) \leq \alpha \psi(x_1, x_2, \ldots, x_k)\]

for all \(x_1, \ldots, x_k \in E\). Let \(S = \{h : E \to F : h(0) = 0\}\), and the generalized metric \(d\) defined on \(S\) by

\[
d(g, h) = \inf \{\varepsilon \in (0, \infty) : \sup_{k \in \mathbb{N}} \|g(x_1) - h(x_1), \ldots, g(x_k) - h(x_k)\| \leq \varepsilon, \forall x_1, \ldots, x_k \in E\}
\]

Then, it is easy to show that \((S, d)\) is a complete generalized metric on \(S\) (see [8]). Define a mapping \(J_0 : S \to S\) by

\[
J_0 g(x) = g(a^n x) / a^{2n}
\]

for all \(g \in S\) is a strictly contractive mapping.

**Proof:** It is easy to show that \(d\) is a complete metric on \(X\). (see [8]). Given \(g, h \in S\), let \(\varepsilon \in (0, \infty)\) be an arbitrary constant with \(d(g, h) \leq \varepsilon\). Then from the definition of \(d\), it follows for each \(x_1, \ldots, x_k \in E\) that

\[
\sup_{k \in \mathbb{N}} \|g(x_1) - h(x_1), \ldots, g(x_k) - h(x_k)\| \leq \varepsilon \psi(x_1, \ldots, x_k), \quad k \in \mathbb{N}
\]

and so

\[
\sup_{k \in \mathbb{N}} \|J_0 g(x_1) - J_0 h(x_1), \ldots, J_0 g(x_k) - J_0 h(x_k)\| \leq \varepsilon \psi(x_1, \ldots, x_k), \quad k \in \mathbb{N}
\]

for all \(x_1, \ldots, x_k \in E\). Hence, it holds that

\[
d(J_0 g, J_0 h) \leq \alpha^n d(g, h)
\]

for all \(g, h \in S\). Hence \(J_0\) is a strictly contractive mapping with Lipschitz constant \(\alpha^n / a^{2n}\).

**Theorem 3.1.** Let \((E, \| \|)\) be a normed space and let \((F_n, \| \| ; n \in \mathbb{N})\) be a multi-Banach space. Let \(\varepsilon \geq 0\) and let \(f : E \to F\) be a mapping satisfying \(f(0) = 0\) such that

\[
\sup_{k \in \mathbb{N}} \|D f(x_1, y_1), \ldots, D f(x_k, y_k)\| \leq \varepsilon
\]

for all \(x_1, \ldots, x_k, y_1, \ldots, y_k \in E\). Then there exists a unique quadratic mapping \(Q : E \to F\) such that

\[
\|f(x_1) - Q(x_1), f(x_2) - Q(x_2), \ldots, f(x_k) - Q(x_k)\| \leq \varepsilon
\]

for all \(x_1, \ldots, x_k \in E\).
\[ \frac{a^{2n} \epsilon}{(a^{2n} - 1)(2a^2 - 2)} \text{ for all } x_1, \ldots, x_k \in E. \]

(3.2)

**Proof:** - Let \( y_1, \ldots, y_k = 0 \) in (3.1), we get

\[ \sup_{x \in \mathbb{K}} \left\{ \frac{f(ax_1)}{a^3} - f(x_1), \ldots, \frac{f(ax_k)}{a^3} - f(x_k) \right\} \leq \frac{\epsilon}{2a^3} \]

(3.3)

Again replacing \( x \) with \( ax \) and dividing by \( a^2 \), in (3.3), we get

\[ \sup_{x \in \mathbb{K}} \left\{ \frac{f(ax^2_1)}{a^4} - f(x_1), \ldots, \frac{f(ax^2_k)}{a^4} - f(x_k) \right\} \leq \frac{\epsilon}{2a^4} + \frac{\epsilon}{2a^3} \]

(3.4)

By using induction for a positive integer \( n \), we get

\[ \sup_{x \in \mathbb{K}} \left\{ \frac{f(ax^n_1)}{a^{2n}} - f(x_1), \ldots, \frac{f(ax^n_k)}{a^{2n}} - f(x_k) \right\} \leq \frac{\epsilon}{2^n} \sum_{i=1}^{n} \frac{1}{a^i} \]

for all \( x_1, \ldots, x_k \in E. \)

(3.5)

Now, let \( S = \{ h : h : E \to F, h(0) = 0 \} \), and introduce the generalized metric \( d \) on \( E \) defined by

\[ d(g, h) = \inf_{\|x\| = 1} \sup_{k \in \mathbb{N}} \left\{ \|D f(ax^n x) - h(ax^n x)\| \right\} \]

\[ \leq \nu; \quad x_1, \ldots, x_k \in E \]

(3.6)

Then, it is easy to show that \( d \) is a complete generalized metric on \( S \) [see [8]]. Let us define a function \( J_0 : S \to S \) by

\[ J_0(h(x)) = h(ax^n x) \]

We claim that \( J_0 \) is a strictly contractive mapping. Let \( g, h \in S \) and \( v \in (0, \infty) \) be an arbitrary constant with \( d(g, h) \leq v \). Now by using the definition of \( d \), we get

\[ \sup_{x \in \mathbb{K}} \left\{ \|g(x_1) - h(x_1), \ldots, g(x_k) - h(x_k)\| \right\} \leq \nu \]

for all \( x_1, \ldots, x_k \in E. \)

Therefore,

\[ \sup_{x \in \mathbb{K}} \left\{ \|J_0 g(x_1) - J_0 h(x_1), \ldots, J_0 g(x_k) - J_0 h(x_k)\| \right\} = \]

\[ \sup_{x \in \mathbb{K}} \left\{ \|g(ax^n x) - h(ax^n x), \ldots, g(ax^n x) - h(ax^n x)\| \right\} \]

\[ \leq \frac{\nu}{a^{2n}} \text{ for all } x_1, \ldots, x_k \in E. \]

Hence, we found that

\[ d(J_0 g, J_0 h) \leq \frac{\nu}{a^{2n}} \leq \frac{1}{a^{2n}} d(g, h) \]

(3.7)

for all \( g, h \in S. \) From (3.5), \( d(J_0 f, f) \leq \frac{\epsilon}{2(a^{2n} - 1)} \). Using

\[ \text{fixed point alternative, we show the existence of a fixed point of } J_0, \text{ that is, the existence of a mapping } Q : E \to F \text{ satisfying the following:} \]

(i) \( Q \) is a fixed point of \( J_0 \), that is \( Q(ax^n x) = ax^n Q(x) \) for all \( x \in E. \)

(ii) For any \( x \in E \), we have \( d(J_0^2 f, Q) \to 0 \), which implies

\[ Q(x) = \lim_{n \to \infty} f(ax^n x), \text{ for all } x \in E \]

(3.8)

(iii) Also, \( d(f, g) \leq \frac{1}{1-L} d(J_0 f, f) \) implies the inequality

\[ d(f, g) \leq \frac{1}{1-L} d(J_0 f, f) \leq \frac{a^{2n} \epsilon}{(a^{2n} - 1)(2a^2 - 2)} \]

(3.9)

Now, to prove that the mapping \( Q : E \to F \) is quadratic, set \( x_1 = x_k = a^n x, y_1 = y_l = a^n y \) in (3.1) and dividing both sides by \( a^n \), we have

\[ \frac{1}{a^n} \sup_{k \in \mathbb{N}} \|D f(ax^n x, a^n y), \ldots, D f(ax^n x, a^n y)\| \]

\[ \leq \lim_{n \to \infty} \left\{ \|D f(ax^n x, a^n y)\| \right\} \leq \lim_{n \to \infty} \frac{\epsilon}{a^{2n}} = 0 \]

for \( x, y \in E. \) Which shows that \( Q \) is a quadratic mapping satisfying (1.1).

Since, \( Q \) is a unique fixed point of \( J_0 \), then if \( Q' \) is another fixed point of \( J_0 \), thus \( Q = Q' \) which completes the proof of theorem.

**Theorem 3.2.** Let \( E \) be a linear space, and let \( (F^p, \|\cdot\|_p) : p \in \mathbb{N} \) be a multi-Banach space. Suppose \( \psi : E^2 \to [0, \infty) \) for some \( 0 < \alpha < a^2 \), \( k \in \mathbb{N} \).

\[ \psi(ax_1, ay_1, \ldots, ax_k, ay_k) \leq \alpha \psi(x_1, y_1, \ldots, x_k, y_k) \]

(3.10) for all \( x_1, \ldots, x_k, y_1, \ldots, y_k \in E. \) If \( f : E \to F \) is a mapping satisfying \( f(0) = 0 \) such that

\[ \|D f(x_1, y_1), \ldots, D f(x_k, y_k)\| \leq \psi(x_1, y_1, \ldots, x_k, y_k) \]

(3.11) for all \( x_1, \ldots, x_k, y_1, \ldots, y_k \in E. \) Then, there exists a unique quadratic mapping \( Q : E \to F \) such that

\[ \|f(x_1) - Q(x_1), \ldots, f(x_k) - Q(x_k)\|_k \]

\[ \leq \frac{a^{2n}}{\alpha(a^{2n} - \alpha^n)(a^2 - \alpha)} \psi(x_1, 0, \ldots, x_k, 0) \]

(3.12) for all \( x_1, \ldots, x_k \in E. \)
Proof - Let \( y_1, \ldots, y_k = 0 \) in (3.1), we get
\[
\|f(\alpha x) - \alpha^2 f(x)\|_k \\
\leq \frac{1}{2} \psi(x_1, 0, \ldots, x_k, 0) \tag{3.13}
\]
again replacing \( x \) with \( ax \), in (3.13) we obtain
\[
\|f(\alpha^2 x) - \alpha^2 f(x)\|_k \\
\leq \frac{a^2}{2} \psi(ax_1, 0, \ldots, ax_k, 0) + \frac{1}{2} \psi(x_1, 0, \ldots, x_k, 0).
\]

By using induction for a positive integer \( n \), we get
\[
\left\| \frac{f(a^n x)}{a^{2n}} - \frac{f(x)}{a^n} \right\|_k \\
\leq \frac{1}{2} \sum_{i=0}^{n-1} \frac{1}{a^{2(i+1)}} \psi(x_1, 0, \ldots, x_k, 0)
\]
for all \( x \in E \), which is a strictly contractive mapping (see Lemma 3.1). Given \( g, h \in S \), let \( v \in (0, \infty) \) be an arbitrary constant with \( d(g, h) \leq v \). From the definition of \( d \), it follows that
\[
\sup_{k \in \mathbb{N}} \left\| \frac{g(x_1) - h(x_1)}{a} \right\|_k \leq v \psi(x_1, \ldots, x_k).
\]

By using (3.14), we obtain
\[
d(J_0 f, f) \leq \frac{1}{2(a^2 - \alpha)} \psi(x_1, 0, \ldots, x_k, 0).
\]
Using fixed point alternative, we deduce the existence of a unique fixed point of \( J_0 \) that is, the existence of mapping \( Q : E \rightarrow F \) such that \( Q(a^n x) = a^{2n} Q(x) \) for all \( x \in E \).

Moreover, we have \( d(J_0^n f, Q) \rightarrow 0 \), which implies that
\[
Q(x) = \lim_{n \to \infty} \frac{f(a^n x)}{a^{2n}}, \text{ for all } x \in E.
\]

Hence,
\[
d(f, Q) \leq \frac{1}{1-L} d(J_0 f, f) \text{ implies the inequality}
\]
\[
d(f, Q) \leq \frac{a^{2n}}{2(a^{2n} - \alpha^3)(a^2 - \alpha)} \psi(x_1, 0, \ldots, x_k, 0)
\]

For \( \text{fix } x \in E \), let us replace \( x_1, x_2, \ldots, x_k \) by \( a^x \) and \( y_1, \ldots, y_k \) by \( a^y \) in (3.11) and dividing by \( a^2n \). Then, using property (a), we get
\[
\left\| \frac{f(a^n(ax + ay)) - f(a^n(ax - ay))}{a^{2n}} \frac{2a^{2}f(a^x)}{a^{2n}} \frac{2a^{2}f(a^y)}{a^{2n}} \right\| \\
\leq \frac{1}{a^{2n}} \psi(a^x, a^y, \ldots, a^x, a^y).
\]

as limit \( n \to \infty \), we obtain
\[
T(ax + ay) + T(ax - ay) = 2a^{2}T(ax) + 2a^{2}T(a y)
\]
for all \( x, y \in E \).

Thus, the uniqueness of \( Q \) follows from the fact that \( Q \) is the unique fixed point of \( J_0 \). This completes the proof of the theorem.

Corollary 3.1. Let \( E \) be a linear space, and let \( (\mathbb{F}^n, \|\|_n) : n \in \mathbb{N} \) be a multi-Banach space. Let \( 0 \leq f \in E \rightarrow F \) be a mapping satisfying \( f(0) = 0 \) such that
\[
\|D f(x_1, y_1), \ldots, D f(x_n, y_k)\| \leq \theta(||x_1||+||y_1|| + \ldots + ||x_n||+||y_k||)
\]
for all \( x_1, \ldots, x_n, y_1, \ldots, y_k \in \mathbb{E} \). Then, there exists a unique quadratic mapping \( Q : E \rightarrow F \) such that
\[
\|f(x_1) - Q(x_1), \ldots, f(x_n) - Q(x_n)\|_k \\
\leq 2(a^{2n} - 1)(a^{2n} - a) \psi(x_1, \ldots, x_n, y_1, \ldots, y_k)
\]
for all \( x_1, \ldots, x_n, y_1, \ldots, y_k \in \mathbb{E} \).

3.2 STABILITY OF THE FUNCTIONAL EQUATION (1.2) BY FIXED POINT METHOD

Theorem 3.3. Let \( E \) be a linear space and \((\mathbb{F}^n, \|\|_n) : n \in \mathbb{N} \) be a multi-Banach space and \( f : E \rightarrow F \) satisfies \( f(0) = 0 \) such that
\[
\sup_{k \in \mathbb{N}} \|D^n f(x_1, \ldots, x_n)\|_k \leq \varepsilon \tag{3.17}
\]
for all \( x_1, \ldots, x_n, y_1, \ldots, y_k \in \mathbb{E} \). Then, there exists a unique mapping \( C : E \rightarrow F \) such that
\[
\sup_{k \in \mathbb{N}} \|f(x) - C(x_1), \ldots, f(x_n) - C(x_n)\|_k \leq \frac{\varepsilon}{3} \tag{3.18}
\]
for all \( x_1, \ldots, x_n \in \mathbb{E} \).

Proof. Let \( y_1, \ldots, y_k = 0 \) and replacing \( x_1, \ldots, x_k \) with \( 2x_1, \ldots, 2x_n \) in (3.17), we get
\[
\sup_{k \in \mathbb{N}} \|f(2x_1) - 4 f(x_1), \ldots, f(2x_n) - 4 f(x_n)\|_k \leq \varepsilon \tag{3.19}
\]
for all $x_1, \ldots, x_k \in E$. Now, let $S = \{g : g : E \to F : g(0) = 0\}$, and introduce the generalized metric $d$ defined on $S$ by
\[
d(g, h) = \inf\{v \in (0, \infty) : \sup_{k \in \mathbb{N}} \|g(x_1) - h(x_1), \ldots, g(x_k) - h(x_k)\|_k \leq v\}
\]
(3.20)
Then, it is easy to show that $d$ is a complete generalized metric on $S$ (see [8]). Let us define a function $J_0 : S \to S$ by
\[
J_0 h(x) = \frac{1}{4} h(2x), \quad \forall x \in E.
\]
(3.21)
where $J_0$ is strictly contractive mapping with Lipschitz constant $1/4$. Given $g, h \in S$, let $v \in (0, \infty)$ be an arbitrary constant with $d(g, h) \leq v$. It follows from (d) that
\[
\sup_{k \in \mathbb{N}} \|g(x_1) - h(x_1), \ldots, g(x_k) - h(x_k)\|_k \leq v
\]
(3.22)
for all $x_1, \ldots, x_k \in E$. Therefore,
\[
\sup_{k \in \mathbb{N}} \|J_0 g(x_1) - J_0 h(x_1), \ldots, J_0 g(x_k) - J_0 h(x_k)\|_k = \sup_{k \in \mathbb{N}} \left\|\left(\frac{1}{4} g(2x_1) - \frac{1}{4} h(2x_1), \ldots, \frac{1}{4} g(2x_k) - \frac{1}{4} h(2x_k)\right)\right\|_k \leq \frac{v}{4}
\]
(3.23)
for all $x_1, \ldots, x_k \in E$. Hence, it shows that $d(J_0 g, J_0 h) \leq \frac{v}{4}$
that is
\[
d(J_0 g, J_0 h) \leq \frac{v}{4} d(g, h)
\]
for all $g, h \in S$. Now by using (3.19), it holds that
\[
d(J_0 f, f) \leq \frac{\varepsilon}{4}
\]
Using fixed point alternative, there exists a fixed point of $J_0$ that is, the mapping $C : E \to F$ satisfying the following:
\begin{itemize}
  \item[(i)] $C$ is a fixed point of $J_0$, that is
  \[
  C(2x) = 4C(x), \quad \forall x \in E.
  \]
  Moreover the mapping $C$ is unique fixed point of $J_0$ in the set $\Omega = \{h \in S : d(g, h) < \infty\}$.
  \item[(ii)] We have $d(J_0^n f, C) \to 0$, which implies that
  \[
  C(x) = \lim_{n \to \infty} (J_0^n f)(x) = \lim_{n \to \infty} \frac{f(2^n x)}{4^n}, \quad \forall x \in E.
  \]
\end{itemize}
(iii) Also, $d(f, C) \leq \frac{d(f, f)}{1 - \varepsilon}$ with $f \in \Omega$ implies that
\[
d(f, C) \leq \frac{1}{1 - \varepsilon} d(f, f) \leq \frac{\varepsilon}{3}
\]
for all $x \in E$. Which implies that the inequality (3.18) holds.

Now taking $x_1 = \ldots = x_k = 2^nx, y_1 = \ldots = y_k = 2^ny$ in (3.17), and dividing both sides by $4^n$, we get,
\[
\sup_{k \in \mathbb{N}} \left\|\left(\frac{Df(2^n x, 2^ny)}{4^n}, \ldots, \frac{Df(2^n x, 2^ny)}{4^n}\right)\right\| \leq \frac{\varepsilon}{4^n}
\]
taking limit as $n \to \infty$, we obtain.
\[
\sup_{k \in \mathbb{N}} \left\|Df(x,y)\right\| = 0
\]
for all $x, y \in E$. It shows that $C$ is a mapping satisfying the functional equation (1.2). The uniqueness of $C$ follows from the fact that $C$ is the unique fixed point of $J_0$. Hence completes the proof of the theorem.

**Corollary 3.2.** Let $E$ be a linear space, and let $((F^n, \|\|_n) : n \in \mathbb{N})$ be a multi-Banach space. $f : E \to F$ satisfies $f(0) = 0$ such that
\[
\sup_{k \in \mathbb{N}} \left\|Df(x_1, y_1), \ldots, Df(x_k, y_k)\right\| \leq \psi(x_1, y_1, \ldots, x_k, y_k)
\]
for all $x_1, \ldots, x_k, y_1, \ldots, y_k \in E$ and $\psi : E^2 \to [0, \infty)$. Then there exists a unique mapping $C : E \to F$ such that
\[
\sup_{k \in \mathbb{N}} \left\|(f(x_1) - C(x_1), \ldots, f(x_k) - C(x_k))\right\|_k \leq \frac{\varepsilon}{3}
\]
\[
\leq \frac{1}{3} \psi(x_1, 0, \ldots, x_k, 0) \forall x_1, \ldots, x_k \in E.
\]
**Proof** :- Proof is similar to that of Theorem 3.3 by using the general condition instead of $\varepsilon$.

4. CONCLUSION

Throughout the paper we concluded the following results:
(i) In section 3 using the ideas of multi normed spaces from H. G. Dales and M. E. Polyakov [4], we proved the Hyers-Ulam-Rassias stability of the Jensen Type Quadratic functional equations (1.1) and (1.2) in Multi Banach spaces.
(ii) We also present some corollaries related to our results by using the general conditions.
5. REFERENCES


