On Prime Labeling of some Classes of Graphs

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ABSTRACT
Graph labeling is an important area of research in Graph theory. There are many kinds of graph labeling such as Graceful labeling, Magic labeling, Prime labeling, and other different labeling techniques.In this paper the Prime labeling of certain classes of graphs are discussed.It is of interest to note that H-graph which is a 3 –regular graph satisfy Prime labeling. A Gear graph is a graph obtained from Wheel graph, with a vertex added between each pair of adjacent vertices of an outer cycle. It is proved in general this graph is Prime. Yet another class of graphs is the Sun flower graph and corona of Cycle graph C_n and $K_{1,3}$ .A stepwise algorithm is given to prove that both these classes of graphs satisfy Prime labeling.

Keywords
Prime Labeling, H-Graph, Gear Graph, Sunflower Graph, Corona of C_n and $K_{1,3}$

1. INTRODUCTION
Labeling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions [2,3]. A dynamic survey on graph labeling is regularly updated by Gallian [1] and it is published in electronic journal of combinatorics. The different kinds of the graphs are studied by us are found in [5, 7, 8]. A Graph G = G(V,E) with V vertices is said to admit **prime labeling** if its vertices can be labeled with distinct positive integers not to exceeding V such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a **prime graph [6].**

2. PRELIMINARIES AND NOTATIONS
In this section, we give the basic definitions relevant to this paper. We always denote Let G=G(V,E) to be a finite, simple and undirected graph with 'V' vertices and "E "edges.

Definition 1:
Let G = G (V,E) be a graph. A bijection f: V→ {1,2,.......|v|} is called prime labeling if for each e = {u,v} belongs to E, we have GCD (f(u), f(v))= 1. A graph that admits a prime labeling is called prime graph.

Definition 2:
An H graph H(r) [4],[10] is a 3-regular graph with vertex set $\{(i,j) : 1 \le i \le 3;\ 1 \le j \le 2r\}$ and edge set $\{(i,j), (i,j+1) i = 1, 3, 1 \le j \le 2r-1\}$ $u\{((2,j),(2,j+1)),\ j \text{ odd},\ 1 \le j \le 2r-1\}$ $u\{((1,1),(1,n),(3,1),(3,n))\} u\{(i,j),(i+1,j+1) i, I = 1, 2, 1 \le j \le 2r$.

An H graph H(r) has 6r vertices and 9r edges .

Definition 3:
Gear graph G_r,[4] also known as a bipartite wheel graph is a wheel graph with a vertex added between each pair of adjacent vertices of the outer cycle. Gear graph G_r has 2r+1 vertices and 3r edges.

Definition 4:
The Helm H_n[9], is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

Definition 5:
A Flower is the graph obtained from a Helmgraph by joining each pendant vertex to the central vertex of the helm graph.

Definition 6:
The Sun flower graph V [n,s,t] is the resultant graph obtained from the flower graph of wheels W_n by adding n-1 pendant edges to the central vertex.

We construct V[n,s,t] as follows,

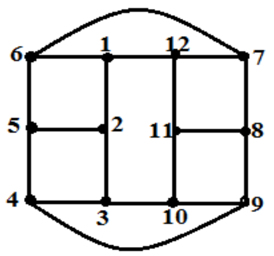
Consider the wheel graph W_n with 'n' vertices and 2(n-1) edges. It is the graph on 'n' vertices constructed by connecting a single vertex to every vertex in an (n-1) cycle. Then by attaching a pendant edge at each vertex of the n-cycle we get an helm graph with 2n-1 vertices and 3(n-1) edges. Now by joining each pendant vertex to the central vertex of the helm we get a flower graph with the same 2n-1 vertices and 4(n-1) edges. Finally, by adding n-1 pendant edges to the central vertex of the wheel we obtain the sunflower graph V[n,s,t] with s=(3n-2) vertices and t=5(n-1) edges..

Definition 7:
The graph corona of C_n and $k_{1,3}$ is obtained from a cycle C_n by introducing '3' new pendant edges at each vertex of cycle.

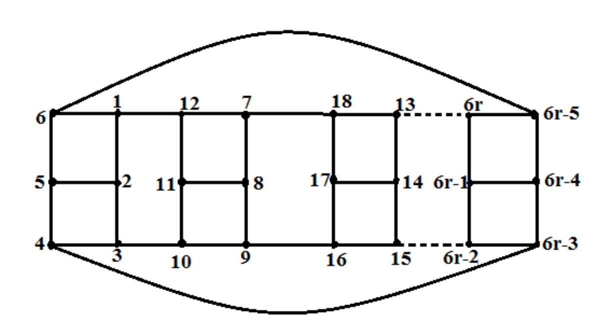
3. MAIN RESULTS
Theorem 1:
An H-graph H(r) is a 3-regular graph has 6r vertices and 9r edges. We prove H(r) is prime. We give below an algorithm of how to label H(r).So that it become prime.

Proof:
Step 1:
The prime labeling of H(1) is as follows;
The right arm of the H(1) can be labeled as in clockwise direction 1,2,3and left arm of the H(1) can be labeled as 4,5,6

Step 2:
We labeled H(1) as given in step 1 and H(2) is labeled as follows. The right arm of the H(2) can be labeled as in clockwise direction 7,8,9 and left arm of H(2) is 10,11,12 as shown in the following figure :



Step 3:
W have to be labeled H(1),H(2),….. as given in step 1 and step 2 and finally H(r) is labeled as in clockwise direction 6r-5, 6r-2, 6r-1,6r as shown in the following figure.



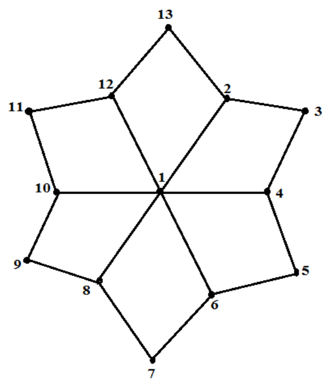
As seen by the algorithm the following pairs(6r,6r-5), (6r-1,6r-4), (6r-2,6r-3), (6r,6r-1), (6r-1,6r-2), (6r-5,6r-4), (6r-4,6r-3) ,(6r-5,6),(6r-3,4) are always prime, when 1≤ r ≤9.

Theorem 2:

A Gear graph G_r ,r≥3, there exists a (2r+1) vertices and 3r edges we prove G_r is prime.

Proof:
Step 1: The central vertex has to be labeled as 1.
Step 2: Rest of the vertices in the outer cycle has to be labeled as 2,3,…..n-1 where 'n' be the number of vertices

Example Gear graphG_6



Theorem 3:

If (n-1) pendant vertices are attached at the central vertex of a flower graph of wheel W_n for n=5,7,9… then we get sunflower graph having s vertices and t edges then the resulting graph V[n,s,t,] is prime.

Proof:
Step 1: The central vertex has to be labeled as 1.

Step2: Number of vertices on the wheel is equal to the number of pendant vertices (n-1) pendant vertices are attached with the central vertex. It has to be labeled as 2,3,….n.

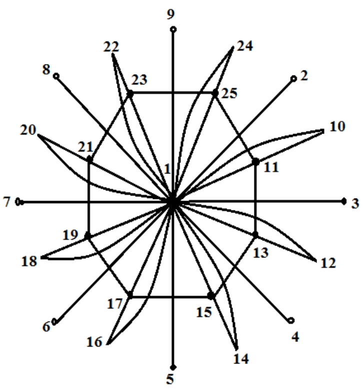
Step3: The prime labeling of W_n is given by

n+2,n+4,……n+x.

When n=5,7,9…..x will be 8,12,16… respectively

Step4: The rest of the vertices of the flower graph will be labeled as n+1,n+3,n+5….n+y when n=5,7,9….y will be 7,11,15 respectively.

Example: Sunflowergraph V[8,s,t]



Theorem 4:

We prove corona of C_n (n≥3) and $k_{,1,3}$ is prime.

Proof:
Prime labeling of C_n, will be classified in to following sub cases:

Case i)
When n=3, and n=4 these cases are discussed separately.

In C_n When n=3, has to be labeled as 1,2,3. In $k_{1,3}$ the labeling of pendant vertices joined with even number '2' will be labeled as odd numbers 5,7,9 satisfying the condition of prime labeling. Another labeling of pendant vertices joined with odd number 3 will be labeled as even numbers 4,8,10. Rest of the numbers 6,11,12 has to be labeled in the pendant vertices that should be joined with 1.

In C_n when n=4, has to be labeled as 1,2,3,4.In $k_{1,3}$ the labeling of pendant vertices joined with even number 2 will be labeled as odd number 5,7,9 satisfying the prime labeling condition. Another labeling of pendant vertices joined with odd number 3 will be labeled as 8,10,14. The labeling of pendant vertices joined with even number 4 will be labeled as odd number 11,13,15. Rest of the numbers 6,12,16 has to be labeled in the pendant vertices that should be joined with 1.

Case ii)
(i) In C_n when n=5, has to be labeled as 1,2,3,4,5.

(ii) In C_n (when n=6,7,8,9,10,11) has to be labeled as 1,2….n excluding 6.

(iii) In C_n [when n=12,13,14,15,16,17] has to be labeled from 1,2…n excluding 6,12.

(iv) Generally in C_n ($n\geq5$) the labeling of vertices with multiples of 6 are prohibited other integers are occur in order

Case iii)
In $k_{1,3}$ the labeling of pendant vertices joined with even number will be labeled as odd number satisfying the condition of prime labeling .

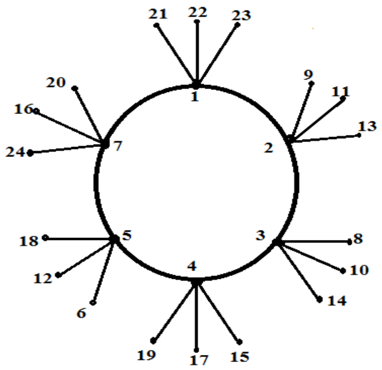
Case iv)
The labeling of pendant vertices joined with odd number will be labeled as even number satisfying the prime labeling conditions.

Case v)
The $k_{1,3}$ branch vertices which lies on the cycle C_n prime labeled vertex 5, has to be labeled as 6,12,18.

Case vi)
Similarly the $k_{1,3}$ branch vertices which lies on the cycle C_n with prime labeled vertex 7 has to be labeled as 24,30,36 provided the total number of vertices exceed 36, otherwise we follow case iv) similarly we continue this process for the vertices 11,13,17,…..

Case vii)
Rest of the numbers has to be labeled in the pendant vertices that should be joined with 1.

Example: Corona of $C_6$and $k_{1,3}$



4. CONCLUSION

We have presented the prime labeling of certain classes of graphs like H-graph,Gear graph,Sunflower graph and corona of C_n and $k_{1,3}$.It is very difficult to generalize these labeling due to the nature the prime numbers.It is of interest to look in to certain kind of graphs where in total prime labeling is possible up to a large prime and verify this by programming concepts.

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