# Digital Image Interpolation via the Contour let Transform

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## ABSTRACT
A suggested digital image interpolation algorithm based on the contourlet transform is presented in this paper. The use of the contourlet transform improves the regularity of object boundaries in the generated high-resolution images. An edge-based image interpolation technique that uses wavelet transform with symmetric biorthogonal wavelets is used as the initial estimate for the contourlet interpolation algorithm. An iterative projection process is then used to drive our solution aimed towards an improved high-resolution image. Experimental results show that the proposed algorithm objectively and subjectively outperforms other commonly used algorithms such as the bilinear, and linear wavelet algorithms, and also the image interpolation with geometric representations.

## General Terms
Image processing.

## Keywords
Image interpolation, wavelet transform, and contourlet transform.

## 1. INTRODUCTION
In most electronic imaging applications, high resolutions are desired. As the pixel density within a high-resolution image is increased, such an image can provide supplementary details that may be critical in various applications. Medical images must have higher resolutions to be helpful for a doctor to make correct diagnosis. For satellite images, it may be easy to distinguish an object from similar ones using high-resolution image. Pattern recognition performance in computer vision can be improved if a high-resolution image is provided. Image interpolation is considered as a means to increase the image resolution.

Simple linear interpolation algorithms generally cannot produce high-quality images. The pixel replication yields blocking effects, and linear interpolation produces blurring effects to images. Higher-order interpolation schemes have been introduced for better-quality image. Also, some efficient interpolation algorithms are found in [1-3]. The Key's cubic convolution scheme (Bicubic) [4] based on a third-order formula has found some popularity in the interpolation field. Higher-order interpolation schemes however still blur images, especially image edges. In order to solve this problem, edge-based image interpolation algorithms began to emerge in the last decade. This gives a better visual effect, because clear edge contours appear in the interpolation results. Allebach and Wong [5] generated a high-resolution edge map from a low-resolution image using a sub-pixel edge estimation technique. They used the high-resolution edge map to guide the interpolation of the low-resolution image to a final high-resolution version. The two most famous approaches to edge-based image interpolation are Kimmel's and Li & Orchard's. In Kimmel's approach [6], they likelihood that each pixel belongs to an edge is estimated by calculating the directional derivatives at this pixel. Then, the interpolation is performed such that larger weights are given to the pixels lined up along the edge than the weights of the pixels across the edge. The Li & Orchard's algorithm [7, 8] is based on the assumption that the high-resolution covariance is the same as the low-resolution covariance.

Edge-based image interpolation often leads to an image with good quality because of the importance of sharp edges and smooth contours to the human vision. The wavelet-based image interpolation algorithm has good potential in producing interpolated images with high quality around edges [9]. Su and Ward [10] presented an edge-based image interpolation algorithm that uses the symmetric biorthogonal wavelet transform. They formed a list of ideal step edges, and studied the relationships between the wavelet approximation sub-image of each edge and its wavelet detail sub-images. Based on these relationships, an algorithm that predicts the edge information of high-resolution images was presented.

The appearance of new transforms such as curvelet [11], Ridgelet [12] and contourlet transforms, which are more efficient from the information representation perspective than the discrete wavelet transform, opens a new field of research in the image interpolation area. These new transforms are highly anisotropic and produce a much more efficient extraction of spatial details in different directions.

Recently, some algorithms have been proposed implementing interpolation with the contourlet transform [13]. The contourlet transform was presented by Do and Vetterli [14] as a directional multi-resolution transform. This transform can efficiently capture and represent smooth object boundaries in natural images. In this paper, we present a new image interpolation algorithm that combines the contourlet transform and the edge-based image interpolation algorithm to improve the regularity of object boundaries in the generated images. An iterative projection process introduced in [15] is then used to produce better-quality images. A sparsity constraint is enforced on the coefficients of the contourlet transform, which provides an efficient mechanism that helps to improve the regularity along edges in the resulting images. Also, to satisfy the observation constraint provided by the given low-resolution image, an iterative projection procedure is employed in the wavelet domain as in [16].

The rest of the paper is organized as follows. Section 2, discusses the contourlet transform. In Section 3, image interpolation with a multi-scale geometric representation algorithm is discussed. In section 4, the suggested interpolation algorithm is presented. The simulation results are given in Section 5, followed by the concluding remarks in Section 6.

## 2. CONTOURLET TRANSFORM

This contourlet transform has a more efficient performance in representing the image salient features such as edges, lines, curves and contours than wavelet transform because of its anisotropy and directionality property. It is therefore well-suited for multi-scale image enhancement. The contourlet transform consists of two steps; the sub-band decomposition and the directional transform. The sub-band decomposition is performed with a Laplacian pyramid, which is first used to capture point discontinuities. After that, the directional transform is employed with directional filter banks to link point discontinuities into linear structures. After these two steps, the overall result is an image expansion using basic elements like contour segments. Figure 1 shows the framework of the contourlet transform [17]. The multi-scale contourlet filter bank structure is show in Figure 2 [17].

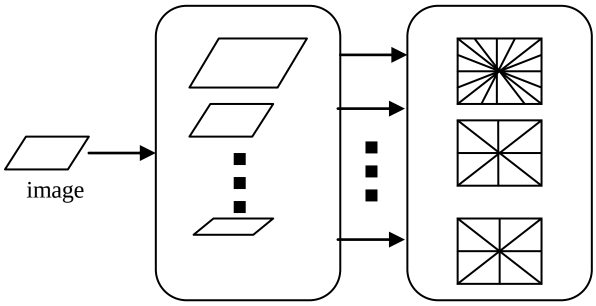



**Fig 1 Contourlet transform framework [17].**

In this paper, we employ a new version of the contourlet transform, recently proposed by Lu and Do. [13]. As an improvement to their previous work [17], the basis images from the new transform are sharply localized in the frequency domain and they exhibit smoothness along their main ridges in the spatial domain.

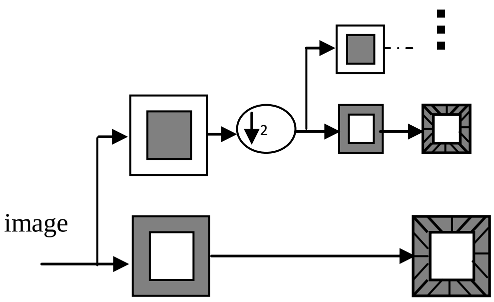



**Fig 2 Contourlet Filter Bank.**

Experimentally, this improved regularity of basis images, produced in [13], making this new contourlet transform a favorable choice for various image processing applications because of the artifacts reduction in the processed images.

## 3. IMAGE INTERPOLATION WITH A SYMETRIC REPRESENTATION

Mueller et al. [13] presented an interpolation algorithm that upsamples a given low-resolution image by a factor of $2^n$ with n = 1, 2, . . ., and improves the quality of interpolation around image edges. Like several papers that perform interpolation in a transform domain, they assume that the low-resolution image is the output of the lowpass output of an n-level wavelet transform applied to the ground truth high-resolution image (to which we do not have access). The main idea of this algorithm is enforcing two constraints [16]. The first constraint considers the given low-resolution image as the down-sampled output of the lowpass anti-aliasing filter in a wavelet transform. This observation constraint can be employed by requiring the high-resolution image to have the given low-resolution image as its lowpass output (approximation sub-band) of the wavelet transform. The second constraint is the sparsity constraint. The contourlet transform described in Section 2 allows a sparse image representation to be well-suited to preserve contours and edges, that the unknown high-resolution image should be sparse in the contourlet transform domain. This constraint is employed through hard-thresholding of the contourlet coefficients.

### 3.1 Observation Constraint

The observation constraint can be employed by requiring the high-resolution image to have the given low-resolution image as its lowpass output of the wavelet transform (approximation part). Note that the set of all images, whose lowpass wavelet sub-bands are equal to the given low-resolution image $\mathbf{X_L}$ forms a linear variety. Consequently, for any input image **y**, we can calculate the best approximation (in $L^2$ norm) to **y**, by the application of the observation constraint through the orthogonal projection. Denote $\mathbf{W}$ and $\mathbf{W}^{-1}$ as the forward and inverse wavelet transforms, respectively. Also denote **P** as the diagonal projection matrix of 1's and 0's that keeps the lowpass wavelet coefficients and zeros out the high-frequency sub-band coefficients, and let $\mathbf{P'} = \mathbf{I} - \mathbf{P}$ perform the reverse action. If we use orthonormal wavelet transforms, then Equation 1 presents the projection of any image **y** onto the constraint set:-

$$\hat{\mathbf{y}} = \mathbf{W}^{-1}(\mathbf{P'}\,\mathbf{Wy} + \mathbf{PW}\,\hat{\mathbf{x}}_{\mathbf{0}}), \tag{1}$$

where $\hat{\mathbf{x}}_{\mathbf{0}}$ is the initial zero details high-resolution image.

### 3.2 Sparisty Constraint

Because of the multi-directionality and sparsity nature of the contourlet transform, there are many more insignificant coefficients than significant ones, and these coefficients well-represent the edge regions of the image. These insignificant coefficients will be removed using a direct hard- thresholding scheme to enforce the sparseness constraint. Denote **C** and $\mathbf{C}^{-1}$ as the forward and inverse contourlet transforms, respectively; $\mathbf{D_T}$ as the diagonal matrix that, given some threshold value T, zeros out insignificant coefficients in the coefficient vector, whose absolute values are smaller than T. Let $\hat{\mathbf{x}}$ the noisy high-resolution image; and $\tilde{\mathbf{x}}$ the denoised high-resolution image. The sparseness constraint by hard thresholding can be written as

$$\tilde{\mathbf{x}} = \mathbf{C}^{-1}\mathbf{D_T}\mathbf{C}\hat{\mathbf{x}} \tag{2}$$

# 4. THE PROPOSED INTERPOLATION ALGORITHM

Mueller et al. [15] start their algorithm with an initial high-resolution image with zero details ( high frequency sub-image images) images, $\hat{\mathbf{x}}_{\mathbf{0}}$. After that, they iteratively enforce the observation and sparsity constraints to obtain the high-resolution image. We will start our algorithm with a high-resolution image, whose detail sub-images are detected by an edge-based image interpolation algorithm using a symmetric biorthogonal wavelet transform [10]. The following section summarizes this algorithm.

## 4.1 Edge-based Image Interpolation Using Symmetric Biorthogonal Wavelet Transform

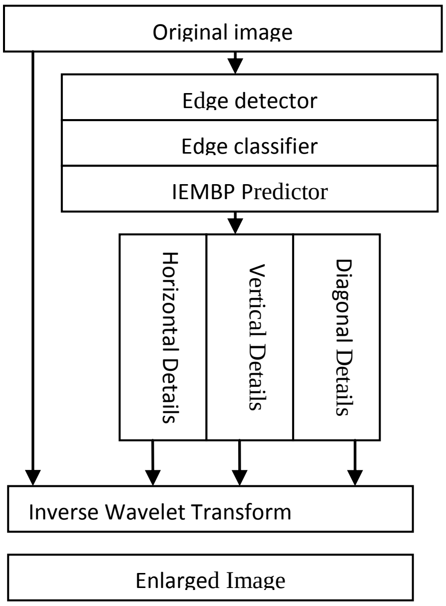



**Fig 3 Edge-based image interpolation using a symmetric biorthogonal wavelet transform algorithm [10].**

Figure 3 present the algorithm in [10] with the following steps:

1- Each ideal step edge is decomposed into a low-resolution image and the three wavelet detail images.

2- The gradients $G_x$ and $G_y$ of the original image are calculated using the following equation:

$$G_x = f(x, y) - f(x-1, y)$$
$$G_y = f(x, y) - f(x, y-1) \tag{3}$$

3- The extrema of the gradients are found using the thresholding constraints. The value of the threshold depends on the content of the image and the wavelet transform used. The threshold value also influences the computational time.

4- Both $G_x$ and $G_y$ are scanned from top to bottom and from left to right. If there are extrema at a point either in the $G_x$ block or in the $G_y$ block, we determine whether or not it belongs to an edge with a certain orientation as in [10].

5- If there is an edge at the current point, we predict the extrema of the wavelet horizontal, vertical, and diagonal detail coefficients, and their neighbors according to the edge directions using the IEMBP algorithm in [10].

6- The enlarged image is obtained using the inverse wavelet transform.

## 4.2 The Proposed Algorithm

The steps of the proposed interpolation algorithm can be summarized as follows:

1- We start the proposed algorithm by taking $\hat{\mathbf{x}}_{\mathbf{in}}$, obtained by edge-based image interpolation using a symmetric biorthogonal wavelet transform algorithm; the initial estimate of the high-resolution image instead of zero detail sub-images as in [15].

2- The quality of the interpolated images can be improved , particularly in regions containing edges and contours, by iteratively enforcing the observation constraint as well as the sparseness constraint. Let $\hat{\mathbf{x}}_{\mathbf{k}}$ represent the estimate at the kth step. By combining (1) and (2), the new estimate $\hat{\mathbf{x}}_{\mathbf{k+1}}$ can then be obtained as:

$$\hat{\mathbf{x}}_{\mathbf{k+1}} = \mathbf{W}^{-1}(\mathbf{P'}\,\mathbf{WC}^{-1}\mathbf{D}_{\mathbf{T_k}}\mathbf{C}\hat{\mathbf{x}}_{\mathbf{k}} + \mathbf{PW}\hat{\mathbf{x}}_{\mathbf{in}}) \tag{5}$$

3- We gradually decrease the threshold value $T_k$ by subtracting a small amount δ in each iteration, i.e., $T_{k+1} = T_k - \delta$ [16]. This is effective in circumventing the non-convexity of the sparseness constraint.

4- We return to step 2 and keep iterating, until the generated image converge to a solution or a when the maximum number of iterations is reaches.

# 5. EXPRIMENTAL RESULTS

In order to verify the performance of the suggested interpolation algorithm, we do several experiments on different images. In this section, the proposed interpolation algorithm is compared with other existing interpolation approaches such as the simple bilinear interpolation, wavelet-based linear interpolation implemented by zero-padding of highpass sub-bands, and the image interpolation with the geometric representation algorithm.

In all experiments, the "symlet" wavelet of length 16 has been used for the wavelet transform, and the filters specified in reference [13] for the contourlet transform, whose redundancy ratio is around 2.33 have also been used. Meanwhile, the initial threshold in our algorithm has been chosen as $T_0 = 5$, and has been decreased by $\delta = 0.1$ in each iteration, with a maximum of 100 iterations.

We used several standard test images of size 512×512 for Lena, 256×256 for the medical, and Cameraman images. The true power of the interpolation algorithms can be observed by first down-sampling of each image by a factor of four, and then interpolating the results back to the original size. To evaluate the performance of any the interpolation scheme quantitatively, the Peak-Signal-to-Noise-Rate (PSNR) is the objective evaluation metric used.

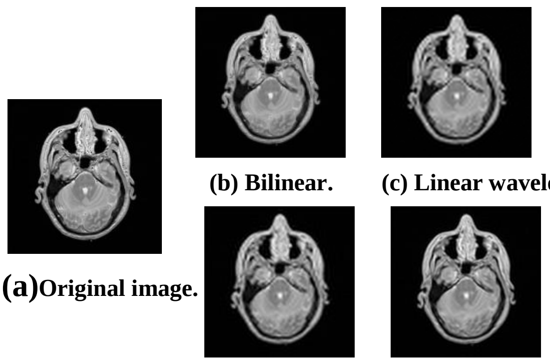
Table 1, shows the PSNR values for the proposed algorithm, the wavelet-based linear interpolation implemented by zero-padding of highpass sub-bands, the traditional bilinear interpolation, and the Image Interpolation with the multi-scale geometric representation algorithm. These results show that the proposed algorithm is superior to all these algorithms used for comparison. The PSNR of the proposed algorithm is 2 dB more than the IIGR algorithm, and is also better than the linear wavelet and the bilinear algorithms. Figures (4) and (5) show the interpolation results of two different images. These results reveal the visual superiority of the proposed interpolation algorithm.

**Table 1. The PSNR values for different algorithms.**

| Algorithm | Bilinear | Linear wavelet | Multi-scale geometric | Proposed |
|---|---|---|---|---|
| Lena | 28.68 | 29.9807 | 30.291 | 32.1131 |
| Medical | 22.9831 | 23.8441 | 23.9732 | 25.7236 |
| Cam | 22.3846 | 23.2347 | 23.3005 | 25.0257 |

**(a)Original image.** **(b) Bilinear.** **(c) Linear wavelet.** **(d) Multi-scale geometric.** **(e) Proposed.**

**Fig 4 Interpolation of Lena image. (a) Original image, (b) Bilinear interpolated image, (c) Linear wavelet interpolated image, (d) Image interpolated with a multi-scale geometric representation, and (e) Interpolated image with the proposed algorithm**.



**(a)Original image.** **(b) Bilinear.** **(c) Linear wavelet.** **(d) Multi-scale geometric.** **(e) Proposed.**

**Fig 5 Interpolation of the medical image. (a) Original image, (b) Bilinear interpolated image, (c) Linear wavelet interpolated image, (d) Image interpolated with a multi-scale geometric representation, and (e) Interpolated image with the proposed algorithm**.

## 6. CONCLUSION

In this paper, a new algorithm for obtaining a high-resolution image from a low-resolution image was presented and tested. This algorithm uses the new contourlet transform. The proposed algorithm improves the regularity along edges in the interpolated image by treating each successive approximation to the high-resolution image as a noisy approximation, and subsequently enforcing a sparsity constraint on the contourlet coefficients. Based on the given low-resolution image, we also enforce an observation constraint. Results have shown that the suggested algorithm is superior in performance to the bilinear, linear wavelet, and the image interpolation with a multi-scale geometric representation.

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