Stochastic Modeling of a Computer System with Priority to PM over S/W Replacement Subject to Maximum Operation and Repair Times

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ABSTRACT
This paper focuses on the stochastic modeling of a computer system of two identical units- one is initially operative and other is kept as spare in cold standby. In each unit h/w and s/w components work together and fail independently. There is a single server who visits the system immediately as and when required. The server takes the unit under preventive maintenance after a maximum operation time at normal mode. The h/w components under go for repair at their failure and are replaced by new one in case these are not repaired up to a maximum repair time. However, s/w components are replaced by new one instead of repair. Priority is given to the preventive maintenance (PM) of the unit over replacement of the s/w components. The failure time distribution of the components follow negative exponential whereas the distributions of preventive maintenance, repair and replacement time are taken as arbitrary with different probability density functions. Several reliability and economic indices have been obtained using semi-Markov and regenerative point technique. The graphical study of the results has also been made.

General Terms
Stochastic Process

Keywords

1. INTRODUCTION
The remarkable progress in the field of computer technology has resulted in the widespread usage of computer applications in almost all academic, business and industrial sectors. A major challenge to the industrialists now a day is to provide reliable h/w and s/w components. Most of the academicians are also trying to explore new techniques for reliability improvement of the computer systems. In spite of these efforts, a little work has been dedicated to the reliability modeling of computer systems. And, most of the research work carried out so far in the subject of s/w and h/w reliability has been limited to the consideration of either h/w subsystem alone or s/w subsystem alone. But there are many complex systems in which h/w and s/w components work together to provide computer functionality. Friedman and Tran [1] and Welke et al.[2] tried to establish a combined reliability model for the whole system in which hardware and software components work together. Recently, Malik and Anand [4,7,8] and Malik and kumar [6] suggested reliability models of a computer system with independent failure of h/w and s/w components.

Further, the continued operation and ageing of these systems gradually reduce their performance, reliability and safety. And, a breakdown of such systems is costly, dangerous and may create confusion in our society. It is, therefore, of great importance to operate such systems with high reliability. It is proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Thus, the method of preventive maintenance can be used to improve the reliability and profit of system. Malik and Nandal [5] has proposed a reliability model for complex systems introducing the concept of preventive maintenance of the unit after a maximum operation time. Further, the reliability of a system can be increased by making replacement of the components by new one in case repair time is too long i.e., if it extends to a pre-specific time. Singh and Agrafoitis[3] analyzed stochastically a two-unit cold standby system subject to maximum operation and repair time.

In view of the above and considering the practical importance of computer systems in our daily lives, a stochastic model of a computer system of two identical units- one is initially operative and other is kept as spare in cold standby is developed. In each unit h/w and s/w components work together and fail independently. A single server is provided immediately to the system as and when required. The server takes the unit under preventive maintenance after a maximum operation time at normal mode. The h/w components under go for repair at their failure and are replaced by new one in case these are not repaired up to a maximum repair time. However, s/w components are replaced by new one instead of repair. Priority is given to the preventive maintenance (PM) of the unit over replacement of the s/w components. The failure time distribution of the components follow negative exponential whereas the distributions of preventive maintenance, repair and replacement time are taken as arbitrary with different probability density functions. Some reliability and economic measures of the system model such as mean time to system failure (MTSF), availability, busy period of the server due to h/w repair, busy period of the server due to s/w replacement, expected number of h/w replacements, expected number of s/w replacements, expected number of visits of the server and profit function are obtained using semi-Markov and regenerative point technique. The graphical behaviour of the results has also been shown for a particular case to make the study more effective.
2. NOTATIONS

E : The set of regenerative states
NO : The unit is operative and in normal mode
Cs : The unit is in cold standby
a/b : Probability that the system has hardware / software failure
λ₁/λ₂ : Constant hardware / software failure rate
α₀ : Maximum constant rate of Operation
β₀ : Maximum constant rate of Repair Time.
Pm/PM : The unit is under preventive Maintenance/under preventive maintenance continuously from previous state
Wp/MPM : The unit is waiting for PM / waiting for maintenance continuously from previous state
HFur/HFUR : The unit is failed due to hardware and is under repair / under repair continuously from previous state
HFurp/HFURP : The unit is failed due to h/w and is replacement / under replacement continuously from previous state
HFwr / HFWR : The unit is failed due to h/w and is repair/waiting for repair continuously from previous state
SFurp/SFURP : The unit is failed due to the software replacement/under replacement continuously from previous state
SFwrp/SFWRP : The unit is failed due to the software waiting for repair/waiting for repair continuously from previous state
h(t) / H(t) of unit due to software
pdf / cdf of replacement time
\[ g(t) / G(t) \] of hardware
pdf / cdf of repair time of the hardware
m(t) / M(t) : pdf / cdf of replacement time of the hardware
f(t) / F(t) : pdf / cdf of the time for PM of the unit
\[ Q_{ij}(t) / Q_{ij}(t) \] : pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in [0, t]
\[ q_{ij}^{kr}(t)/Q_{ij}(t) : \] pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in [0, t]
\[ \mu(t) : \] Probability that the system up initially in state Sᵢ ∈ E is up at time t without visiting to any regenerative state
\[ W_{ij}(t) : \] Probability that the server is busy in the state Sᵢ up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
\[ m_{ij} : \] Contribution to mean sojourn time (μᵢ) in state Sᵢ when system transit directly to state Sⱼ so that
\[ \mu_i = \sum_{j} m_{ij}, \text{ and } m_{ij} = \int_{0}^{\infty} q_{ij}^*(t) \, dt = -q_{ij}(0) \]

3. RELIABILITY INDICES

3.1 Transition Probabilities And Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ij}(x) = \int_{0}^{\infty} q_{ij}(t) \, dt \]

Using the notations

\[ a\lambda_1 + b\lambda_2 + \alpha_0 \text{ and } B = a\lambda_1 + b\lambda_2 + \alpha_0 \]

(1)

\[ p_{00} = \frac{\alpha_0}{A}, \quad p_{01} = \frac{\alpha_1 A}{A}, \quad p_{03} = \frac{b\lambda_2}{A}, \quad p_{10} = f^*(A), \quad p_{16} = \frac{a\lambda_1}{A} \]

\[ f^*(A) \]

\[ g^*(B) \]

\[ h^*(A) \]

\[ h^*(B) \]

\[ h^*(\beta_0) \]

\[ p_{3,16} = 1 - g^*(\beta_0), \quad p_{4,17} = \frac{\alpha_0}{A} [1 - m^*(A)] = p_{41,17}, \quad p_{52} = f^*(0), \quad p_{53} = f^*(0), \quad p_{53} = f^*(0), \quad p_{10,3} = h^*(0), \quad p_{11,3} = g^*(0), \quad p_{11,4} = g^*(\beta_0), \quad p_{11,5} = 1 - g^*(\beta_0), \quad p_{11,5} = 1 - g^*(\beta_0), \quad p_{12,2} = g^*(\beta_0), \quad p_{12,5} = 1 - g^*(\beta_0), \quad p_{13,1} = f^*(0), \quad p_{14,3} = m^*(0), p_{14,9} = \frac{a\lambda_1}{A} [1 - m^*(A)] = p_{42,19}, \quad p_{15,2} = m^*(0), \quad p_{16,1} = m^*(0), p_{17,3} = m^*(0), p_{18,3} = m^*(0), p_{19,2} = m^*(0), p_{21,5} = \frac{\alpha_0}{B} \]
where

\[ N_1 = \mu_0 + p_{03} \mu_1 + p_{05} \mu_2 + p_{03} \mu_3 + p_{24} p_{03} \mu_4 \quad \text{and} \quad D_1 = 1 - p_{03} p_{10} - p_{05} p_{20} - p_{03} p_{10} - p_{05} p_{24} \]

### 3.3 Steady State Availability

Let \( A_i(t) \) be the probability that the system is in up-state at instant \( t \) given that the system entered regenerative state \( i \) at \( t = 0 \). The recursive relations for \( A_i(t) \) are given as

\[ A_i(t) = M_i(t) + \sum_{j} q_{ji}(t) \cap A_j(t) \]

(9)

Where \( j \) is any successive regenerative state to which the regenerative state \( i \) can transit through

\[ n \geq 1 \text{ (natural number)} \]

\( M_i(t) \) is the probability that the system is up initially in state \( s \in E \) is up at time \( t \) without visiting to any other regenerative state, we have

\[ M_0(t) = e^{-a_1 t} b_{21} + a_{21} \]

\[ M_2(t) = e^{-a_2 t} b_{21} + a_{21} \]

\[ M_3(t) = e^{-a_3 t} b_{21} + a_{21} \]

(10)

Taking LT of above relations (9) and solving for \( A_i(s) \), the steady state availability is given by

\[ A_i(s) = \lim_{s \to 0} s A_i(s) = \frac{N_i}{D_i} \]

where

\[ N_i = \mu_0 + p_{03} \mu_1 + p_{05} \mu_2 + p_{03} \mu_3 + p_{24} p_{03} \mu_4 \quad \text{and} \quad D_1 = 1 - p_{03} p_{10} - p_{05} p_{20} - p_{03} p_{10} - p_{05} p_{24} \]

### 3.2 Mean Time to System Failure

Let \( \phi_i(t) \) be the c.d.f of first passage time from the regenerative state \( i \) to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for \( \phi_i(t) \):

\[ \phi_i(t) = \sum_{k=1}^{\infty} Q_{ik}(t) \phi_k(t) + \sum_{k=1}^{\infty} Q_{ik}(t) \]

(6)

Where \( j \) is an un-failed regenerative state to which the given regenerative state \( i \) can transit and \( k \) is a failed state to which the state \( i \) can transit directly. Taking LT of above relation (6) and solving for \( \tilde{\phi}_i(s) \)

We have

\[ \Phi(s) = \frac{1}{s} \tilde{\phi}_i(s) \]

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

\[ \text{MTSF} = \lim_{s \to 0} \frac{1 - \tilde{\phi}_i(s)}{s} = \frac{N_i}{D_i} \]

\[ N_i = \mu_0 + p_{03} \mu_1 + p_{05} \mu_2 + p_{03} \mu_3 + p_{24} p_{03} \mu_4 \quad \text{and} \quad D_1 = 1 - p_{03} p_{10} - p_{05} p_{20} - p_{03} p_{10} - p_{05} p_{24} \]

\[ \tilde{\phi}_i(s) = \frac{1}{s} \tilde{\phi}_i(s) \]

\[ \Phi(s) = \frac{1}{s} \tilde{\phi}_i(s) \]

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

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\[ N_i = \mu_0 + p_{03} \mu_1 + p_{05} \mu_2 + p_{03} \mu_3 + p_{24} p_{03} \mu_4 \quad \text{and} \quad D_1 = 1 - p_{03} p_{10} - p_{05} p_{20} - p_{03} p_{10} - p_{05} p_{24} \]

\[ \tilde{\phi}_i(s) = \frac{1}{s} \tilde{\phi}_i(s) \]

\[ \Phi(s) = \frac{1}{s} \tilde{\phi}_i(s) \]

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

\[ \text{MTSF} = \lim_{s \to 0} \frac{1 - \tilde{\phi}_i(s)}{s} = \frac{N_i}{D_i} \]
regenerative state or returning to the same via one or more
non-regenerative states and so

\[ W_i = e^{-\lambda_i(t)} \mathbb{F}(t) + (\alpha e^{-\lambda_i(t)} \mathbb{G}(t)) \mathbb{F}(t) + (b \lambda e^{-\lambda_i(t)} \mathbb{G}(t)) \mathbb{F}(t) \]

\[ W_q = \mathbb{F}(t) \]

\[ W_i = e^{-\lambda_i(t)} \mathbb{H}(t) + (\alpha e^{-\lambda_i(t)} \mathbb{H}(t)) \mathbb{H}(t) + (b \lambda e^{-\lambda_i(t)} \mathbb{H}(t)) \mathbb{H}(t) \]

\[ W_q = \mathbb{H}(t) \]

Taking LT of above relations (12) and solving for \( B_R^H(t) \), \( B_R^S(t) \) and \( B_R^{HRp}(t) \) the time for which server is busy
due to PM, h/w repair and h/w and s/w replacements
respectively is given by

\[ \begin{align*}
B_R^H(t) &= \lim_{s \to \infty} s B_R^{H}(s) = \frac{N_H}{D_2} , \\
B_R^S(t) &= \lim_{s \to \infty} s B_R^{S}(s) = \frac{N_S}{D_2} , \\
B_R^{HRp}(t) &= \lim_{s \to \infty} s B_R^{HRp}(s) = \frac{N_{HRp}}{D_2} 
\end{align*} \]

3.4 Busy Period of the Server

Let \( B_R^P(t), B_R^R(t), B_R^S(t) \) and \( B_R^{HRp}(t) \) be the probabilities
that the server is busy in Preventive maintenance of the system, repauring the unit due to hardware failure, replacement of the software and hardware components at an instant ‘t’ given that the system entered state i at \( t = 0 \). The recursive relations for \( B_R^P(t), B_R^R(t), B_R^S(t) \) and \( B_R^{HRp}(t) \) are as follows:

\[ B_R^P(t) = W_i(t) + \sum_{j} q_{i,j}^{(n)}(t) \mathbb{F}(j) \]

\[ B_R^R(t) = W_i(t) + \sum_{j} q_{i,j}^{(n)}(t) \mathbb{F}(j) \]

\[ B_R^S(t) = W_i(t) + \sum_{j} q_{i,j}^{(n)}(t) \mathbb{F}(j) \]

\[ B_R^{HRp}(t) = W_i(t) + \sum_{j} q_{i,j}^{(n)}(t) \mathbb{F}(j) \]

(12)

Where \( j \) is any successive regenerative state to which the regenerative state \( i \) can transit through \( n \geq 1 \)(natural number) transitions. \( W_i(t) \) be the probability that the server is busy in
state \( S_i \) due to preventive maintenance, hardware and software
failure up to time \( t \) without making any transition to any other

\[ D_2 = \mu_0 \{ (1- \mu_{11.1}) [ (1-\mu_{22.12-22.12-15.15-22-19}(1-\mu_{22.11-11.1}) \cdot \mu_{22.11-11.1} + \mu_{22.11-11.1} + \mu_{22.11-11.1} + \mu_{22.11-11.1} ] \cdot \mu_{22.11-11.1} + \mu_{22.11-11.1} + \mu_{22.11-11.1} + \mu_{22.11-11.1} ] \} + \mu_{12.12} \]

\[ \{ (\mu_{12.12} \mu_{22.12} + \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} + \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} + \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} \mu_{22.12} ] \} + \mu_{12.12} \]

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3.5 Expected Number of Visits by the Server

Let \( R^H(t) \) and \( R^S(t) \) the expected number of replacements of the failed hardware and software components by the server in \([0, t]\) given that the system entered the regenerative state \( i \) at \( t = 0 \). The recursive relations for \( R^H(t) \) and \( R^S(t) \) are given as

\[
R^H(t) = \sum_j q_{ij}(t) \delta_j + R^H(t)
\]

\[
R^S(t) = \sum_j q_{ij}(t) \delta_j + R^S(t)
\]

Where \( j \) is any regenerative state to which the given regenerative state \( i \) transits and \( \delta_j = 1 \), if \( j \) is the regenerative state where the server does job afresh, otherwise \( \delta_j = 0 \).

Taking LT of relations and, solving for \( R^H(s) \) and \( R^S(s) \).

The expected numbers of replacements per unit time to the hardware and software failures are respectively of given by

\[
R^H_{(s)} = \lim_{s \to 0} sR^H(s) = \frac{N^H}{D_2}
\]

\[
R^S_{(s)} = \lim_{s \to 0} sR^S(s) = \frac{N^S}{D_2}
\]

Where \( D_2 \) is already mentioned.

4. Economic Analysis

The profit incurred to the system model in steady state can be obtained as

\[
P = K_0 \cdot D_0 - K_1 \cdot D_1 - K_2 \cdot D_2 - K_3 \cdot D_3 - K_4 \cdot D_4 - K_5 \cdot D_5 - K_6 \cdot D_6 - K_7 \cdot D_7
\]
5. CONCLUSION
In the present study, the numerical results considering a particular case \( g(t) = \theta e^{-\alpha t} \), \( h(t) = \beta e^{-\alpha t} \), \( f(t) = \theta e^{-\alpha t} \) and \( m(t) = \beta e^{-\alpha t} \) are obtained for some reliability and economic measures of a computer system of two identical units having h/w and s/w components. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance rate (\( \alpha \)) for fixed values of other parameters as shown respectively in figs. 2 to 4. From these figures, it is revealed that MTSF, Availability and profit increase with the increase of PM rate (\( \alpha \)) and repair rate (\( \theta \)) of the hardware components. But the value of these measures increase with the increase of maximum operation time (\( \alpha_0 \)). Again if we increase the value of maximum constant rate of repair time (\( \beta_0 \)), the value of MTSF, availability and profit are increase. Thus on the basis on the results obtained for a particular case, it is suggested that the reliability and profit of a system in which chances of h/w failure are high can be improved by
(i) Reducing the repair time of the h/w components as well as conducting PM of the units after a pre-specific period of time.
(ii) Making replacement of the hardware components by new one in case repair time is too long.
(iii) Making replacement of s/w components by new one.

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7. REFERENCES
Fig.1 State Transition Diagram

- Operative State
- Failed State
- Regenerative Point

\[ f(t) \]  
\[ \alpha_0 \]  
\[ m(t) \]  
\[ HFURP \]  
\[ WPM \]  
\[ S17 \]  
\[ S0 \]  
\[ S9 \]  
\[ S10 \]  
\[ S12 \]  
\[ S14 \]  

\[ \beta_0 \]  
\[ \alpha_1 \]  
\[ \beta_1 \]  
\[ a \]  
\[ b \]  
\[ \lambda_0 \]  
\[ \lambda_1 \]  

\[ \alpha_0=5, \beta_0=10, \lambda_0=5, a=-7, b=-3, \gamma=-8, \lambda_1=-7, \lambda_2=-5 \]

\[ \alpha_0=5, \beta_0=10, \lambda_0=5, a=-7, b=-3, \gamma=-8, \lambda_1=-7, \lambda_2=-5 \]