Regular Weakly Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

P. Rajarajeswari
Assistant Professor, Department of Mathematics
Chikkanna Government Arts College
Tirupur-641 602

L. Senthil Kumar
Assistant Professor, Department of Mathematics
SVS College of Engineering
Coimbatore-642 109

ABSTRACT
This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper an intuitionistic fuzzy regular weakly generalized closed set and intuitionistic fuzzy regular weakly generalized open set are introduced. Some of its properties are studied. Also we have provided some applications of intuitionistic fuzzy weakly generalized closed set namely intuitionistic fuzzy \( r_wT_{1/2} \) space and intuitionistic fuzzy \( r_wT_{1/2} \) space.

Keywords
Intuitionistic fuzzy topology, Intuitionistic fuzzy regular weakly generalized closed set, Intuitionistic fuzzy regular weakly generalized open set, Intuitionistic fuzzy \( r_wT_{1/2} \) space and Intuitionistic fuzzy \( r_wT_{1/2} \) space.

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1. INTRODUCTION
Fuzzy set(FS), proposed by Zadeh [14] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang [3] in 1967. By adding the degree of non-membership to FS, Atanassov[1] proposed intuitionistic fuzzy set (IFS) in 1983 which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In the last few years various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce one of the concepts namely regular weakly generalized closed set which were introduced initially by N. Nagaveni[8] in General Topology in 1999. After this many researchers worked on this set and developed many interesting properties and applications. We have studied some of the basic properties regarding it. We also introduced the applications of intuitionistic fuzzy regular weakly generalized closed set namely intuitionistic fuzzy \( r_wT_{1/2} \) space, intuitionistic fuzzy \( r_wT_{1/2} \) space and obtained some characterizations and several preservation theorems of such spaces.

2. PRELIMINARIES
Definition 2.1: [1] Let \( X \) be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) \( A \) in \( X \) is an object having the form \( A = \{ ( x, \mu_A(x), \nu_A(x)) / x \in X \} \) where the functions \( \mu_A(x) : X \rightarrow [0, 1] \) and \( \nu_A(x) : X \rightarrow [0, 1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set \( A \) respectively and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \).

Definition 2.2: [1] Let \( A \) and \( B \) be IFS’s of the forms
\[
A = \{ ( x, \mu_A(x), \nu_A(x)) / x \in X \} \quad \text{and} \\
B = \{ ( x, \mu_B(x), \nu_B(x)) / x \in X \}.
\]

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)
(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
(c) \( A' = \{ ( x, \nu_A(x), \mu_A(x)) / x \in X \} \)
(d) \( A \cap B = \{ ( x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \lor \nu_B(x)) / x \in X \} \)
(e) \( A \cup B = \{ ( x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \wedge \nu_B(x)) / x \in X \} \)

For the sake of simplicity, we shall use the notation \( A = ( x, \mu_A(x), \nu_A(x)) \) instead of \( A = \{ ( x, \mu_A(x), \nu_A(x)) / x \in X \} \). Also for the sake of simplicity, we shall use the notation \( A = \{ ( x, \mu_A(x), \nu_A(x)) / x \in X \} \) instead of \( A = ( x, (A/\mu_A, B/\nu_B), (A/\nu_A, B/\mu_B)) \).

The intuitionistic fuzzy sets \( 0 = \{ ( x, 0, 1) / x \in X \} \) and \( 1 = \{ ( x, 1, 0) / x \in X \} \) are respectively the empty set and the whole set of \( X \).

Definition 2.3: [4] An intuitionistic fuzzy topology (IFTs in short) on a non empty \( X \) is a family \( \tau \) of IFS in \( X \) satisfying the following axioms:
(a) \( 0, 1 \in \tau \),
(b) \( G_1 \cap G_2 \in \tau \), for any \( G_1, G_2 \in \tau \),
(c) \( \cup G_i \in \tau \) for any arbitrary family \( \{ G_i / i \in J \} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS for short) in \( X \).

The complement \( A' \) of an IFOS \( A \) in an IFTS (\( X, \tau \)) is called an intuitionistic fuzzy closed set (IFCS for short) in \( X \).

Definition 2.4: [4] Let \( (X, \tau) \) be an IFTS and \( A = ( x, \mu_A, \nu_A) \) be an IFOS in \( X \). Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure of \( A \) are defined by
\[
\text{int}(A) = \cup \{ G / G \in \text{IFOS in } X \text{ and } G \subseteq A \},
\]
\[
\text{cl}(A) = \cap \{ K / K \in \text{IFCS in } X \text{ and } A \subseteq K \}.
\]

Result 2.5: [4] Let \( (X, \tau) \) be an IFTS and \( A = ( x, \mu_A, \nu_A) \) be an IFOS in \( X \). Then
(a) \( A \) is an intuitionistic fuzzy closed set in \( X \) \( \iff \) \( \text{cl}(A) = A \)
(b) \( A \) is an intuitionistic fuzzy open set in \( X \) \( \iff \) \( \text{int}(A) = A \)
(c) \( \text{cl}(A^c) = (\text{int}(A))^c \)
(d) \( \text{int}(A) = (\text{cl}(A))' \)

(e) \( A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B) \)

(f) \( A \subseteq B \Rightarrow \text{cl}(A) \subseteq \text{cl}(B) \)

(g) \( \text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) \)

(h) \( \text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B) \)

**Definition 2.6:** [13] Let \((X, \tau)\) be an IFTS and \(A = \langle x, \mu_A, \nu_A \rangle\) be an IFS in \(X\). Then the semi closure of \(A\) (scl\((A)\) in short) and semi interior of \(A\) (sint\((A)\) in short) are defined as

\[
\text{sint}(A) = \cup \{ G / G \text{ is an IFOS in X and } G \subseteq A \},
\]

\[
\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in X and } A \subseteq K \}.
\]

**Result 2.7:** [11] Let \(A\) be an IFS in \((X, \tau)\), then

(i) \( \text{scl}(A) = A \cup \text{int}(\text{cl}(A)) \),

(ii) \( \text{sint}(A) = A \cap \text{cl}(\text{int}(A)) \).

**Definition 2.8:** [9] Let \((X, \tau)\) be an IFTS and \(A = \langle x, \mu_A, \nu_A \rangle\) be an IFS in \(X\). Then the alpha closure of \(A\) (\(\text{cl}(A)\) in short) and alpha interior of \(A\) (\(\text{int}(A)\) in short) are defined as

\[
\text{cl}(A) = \cup \{ G / G \text{ is an IFROS in X and } G \subseteq A \},
\]

\[
\text{int}(A) = \cap \{ K / K \text{ is an IFRCS in X and } A \subseteq K \}.
\]

**Result 2.9:** [9] Let \(A\) be an IFS in \((X, \tau)\), then

(i) \( \text{cl}(A) = A \cup \text{int}(\text{cl}(A)) \),

(ii) \( \text{int}(A) = A \cap \text{cl}(\text{int}(A)) \).

**Definition 2.10:** An IFS \(A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \} \) in an IFTS \((X, \tau)\) is called an

(a) intuitionistic fuzzy semi open set (IFOS) if \(\text{int}(\text{cl}(A)) \subseteq A[5]\)

(b) intuitionistic fuzzy alpha-closed set (IFACS) if \(\text{cl}(\text{int}(A)) \subseteq A[5]\)

(c) intuitionistic fuzzy pre-closed set (IFPCS) if \(\text{cl}(\text{int}(A)) \subseteq A[5]\)

(d) intuitionistic fuzzy regular closed set (IFRCS) if \(\text{cl}(\text{int}(A)) = A[5]\)

(e) intuitionistic fuzzy generalized closed set (IFGCS) if \(\text{cl}(A) \subseteq U \) whenever \(A \subseteq U \) and \(U \) is an IFOS [12]\)

(f) intuitionistic fuzzy generalized semi closed set (IFGsCS) if

\[
\text{scl}(A) \subseteq U \text{ whenever } A \subseteq U \text{ and } U \text{ is an IFOS}[10]
\]

(g) intuitionistic fuzzy alpha generalized closed set (IFGACS) if

\[
\text{cl}(A) \subseteq U \text{ whenever } A \subseteq U \text{ and } U \text{ is an IFOS}[9]
\]

An IFS \(A\) is called intuitionistic fuzzy semi open set, intuitionistic fuzzy alpha-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy \(\alpha\) generalized open set (IFOS, IFACS, IFPCS, IFRCS, IFGCSS and IFACS) if the complement of \(A^c\) is an IFSCS, IFPCS, IFRCS, IFGCSS and IFACS respectively.

### 3. INTUITIONISTIC FUZZY REGULAR WEAKLY GENERALIZED CLOSED SET

In this section we introduce intuitionistic fuzzy regular weakly generalized closed set and have studied some of its properties.

**Definition 3.1** An IFS \(A\) in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRGCWS) if \(\text{cl}(\text{int}(A)) \subseteq U \) whenever \(A \subseteq U \), \(U \) is IFROS in \(X\).

The family of all IFRGCSs of an IFTS \((X, \tau)\) is denoted by IFRGCWS\((X)\).

**Example 3.2:** Let \(X = \{a, b\}\) and let \(\tau = \{\emptyset, T, 1\}\) be an IFT on \(X\), where \(T = \langle x, (0.3, 0.3), (0.7, 0.7) \rangle\). Then the IFS \(A = \langle x, (0.2, 0.2), (0.8, 0.7) \rangle\) is an IFRGCWS in \(X\).

**Result 3.3:** Every IFROS is an IFOS.

**Proof:** Let \(A\) be an IFROS in \(X\). Therefore \(\text{int}(\text{cl}(A)) = A\). Which implies \(\text{int}(\text{int}(A)) = \text{int}(A)\) and hence \(\text{int}(A) = \text{int}(A)\). That is \(A = \text{int}(A)\). Hence \(A\) is an IFOS in \(X\).

**Theorem 3.4:** Every IFCS is an IFRGCWS but not conversely.

**Proof:** Let \(A\) be an IFCS in \((X, \tau)\). Let \(U\) be an intuitionistic fuzzy regular open set in \((X, \tau)\) such that \(A \subseteq U\). Since \(A\) is an intuitionistic fuzzy closed, \(\text{cl}(A) = A\) and hence \(\text{cl}(A) \subseteq U\). But \(\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U\). Therefore \(\text{cl}(\text{int}(A)) \subseteq U\). Hence \(A\) is an IFRGCWS in \(X\).

**Example 3.5:** Let \(X = \{a, b\}\) and let \(\tau = \{\emptyset, T, 1\}\) be an IFT on \(X\), where \(T = \langle x, (0.2, 0.4), (0.7, 0.6) \rangle\). Then the IFS \(A = \langle x, (0.2, 0.2), (0.8, 0.7) \rangle\) is an IFRGCWS in \(X\) but not an IFCS in \(X\).

**Theorem 3.6:** Every IFRCS is an IFRGCWS but not conversely.

**Proof:** Let \(A\) be an IFRCS in \((X, \tau)\). By hypothesis, \(\text{cl}(\text{int}(A)) = A\). Therefore \(\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U\). Therefore \(\text{cl}(\text{int}(A)) \subseteq U\). Hence \(A\) is an IFRGCWS in \(X\).

**Example 3.7:** Let \(X = \{a, b\}\) and let \(\tau = \{\emptyset, T, 1\}\) be an IFT on \(X\), where \(T = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle\). Then the IFS \(A = \langle x, (0.3, 0.3), (0.7, 0.6) \rangle\) is an IFRGCWS but not an IFCS in \(X\) since \(\text{cl}(\text{int}(A)) = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle \not\subseteq A\).

**Theorem 3.8:** Every IFGCS is an IFRGCWS but not conversely.

**Proof:** Let \(A\) be an IFGCS in \((X, \tau)\). By hypothesis, \(\text{cl}(\text{int}(A)) \subseteq A\). Therefore \(\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U\). Therefore \(\text{cl}(\text{int}(A)) \subseteq U\). Hence \(A\) is an IFRGCWS in \(X\).

**Example 3.9:** Let \(X = \{a, b\}\) and let \(\tau = \{\emptyset, T, 1\}\) be an IFT on \(X\), where \(T = \langle x, (0.2, 0.4), (0.8, 0.6) \rangle\). Then the IFS \(A = \langle x, (0.1, 0.3), (0.8, 0.7) \rangle\) is an IFRGCWS but not an IFGCS in \(X\) since \(A \subseteq T\) but \(\text{cl}(A) = \langle x, (0.8, 0.6), (0.2, 0.4) \rangle \not\subseteq T\).

**Theorem 3.10:** Every IFRCS is an IFRGCWS but not conversely.
Example 3.11: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.3, 0.5), (0.7, 0.5))\}$. The IFS $A = \{(x, (0.2, 0.4), (0.8, 0.6))\}$ is an IFRWGCS but not an IFSCS in $X$ since $\text{cl}(\text{int}(A)) \neq 0$. Hence $A$ is an IFRWGCS in $X$.

Example 3.12: Every IFPCS is an IFRWGCS but not conversely.

Proof: Let $A$ be an IFPCS in $X$ and let $A \subseteq U$ and $U$ is an IFROS in $(X, \tau)$. Since $A$ is IFPCS, $\text{cl}(\text{int}(A)) = A \subseteq U$. This implies $\text{cl}(\text{int}(A)) \subseteq U$. Hence $A$ is an IFRWGCS in $X$.

Example 3.13: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.2, 0.3), (0.6, 0.7))\}$. The IFS $A = \{(x, (0.4, 0.5), (0.6, 0.7))\}$ is an IFRWGCS but not an IFPCS in $X$ since $\text{cl}(\text{int}(A)) = 1$, $\subseteq A$.

Theorem 3.14: Every IFαGCS is an IFRWGCS but not conversely.

Proof: Let $A$ be an IFαGCS in $X$ and let $A \subseteq U$ and $U$ is an IFROS in $(X, \tau)$. By Definition, $\text{cl}(\text{int}(A)) \subseteq A$ and $A \subseteq U$. Therefore $\text{cl}(\text{int}(A)) \subseteq U$. Hence $A$ is an IFRWGCS in $X$.

Example 3.15: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.5, 0.5), (0.4, 0.6))\}$. Then the IFS $A = \{(x, (0.5, 0.3), (0.6, 0.7))\}$ is an IFRWGCS but not an IFαGCS in $X$ since $\text{cl}(\text{int}(A)) = 1$, $\subseteq A$.

Proposition 3.16: IFCS and IFRWGCSs are independent to each other which can be seen from the following example.

Example 3.18: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.5, 0.4), (0.2, 0.2))\}$. Then the IFS $A = \{(x, (0.8, 0.7), (0.2, 0.2))\}$ is an IFRWGCS but not an IFCS in $X$ since $\text{int}(A) = \{(x, (0.5, 0.6), (0.5, 0.2))\}$.

Example 3.19: IFGSCS and IFRWGCSs are independent to each other.

Example 3.20: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.5, 0.4), (0.5, 0.6))\}$. Then the IFS $A = T$ is an IFGSCS but not an IFRWGCS in $X$ since $A \subseteq T$ but $\text{cl}(\text{int}(A)) = \{(x, (0.5, 0.6), (0.5, 0.4))\}$.

Example 3.21: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.7, 0.9), (0.3, 0.1))\}$. Then the IFS $A = \{(x, (0.6, 0.7), (0.4, 0.3))\}$ is an IFRWGCS but not an IFGSCS in $X$ since $\text{cl}(\text{int}(A)) = \{(x, (0.6, 0.7), (0.4, 0.3))\}$.

Remark 3.22: The union of any two IFRWGCSs’s need not be an IFRWGCS in general as seen in the following example.

Example 3.23: Let $X = \{a, b\}$ be an IFTS and let $T_1 = \{(x, (0.4, 0.4), (0.6, 0.6))\}$ and $T_2 = \{(x, (0.4, 0.3), (0.6, 0.7))\}$. Then $\tau = \{0, T_1, T_2, 1\}$ is an IFT on $X$ and the IFS’s $A = \{(x, (0.4, 0.2), (0.6, 0.8))\}$ and $B = \{(x, (0.3, 0.3), (0.7, 0.7))\}$ are IFRWGCS’s but $A \cup B$ is not an IFRWGCS in $X$.

The following implications are true:

In this diagram by “$A \rightarrow B$” we mean $A$ implies $B$ but not conversely and “$A \leftrightarrow B$” means $A$ and $B$ are independent of each other.

None of them is reversible.

4. INTUITIONISTIC FUZZY REGULAR WEAKLY GENERALIZED OPEN SETS

In this section we introduce intuitionistic fuzzy regular weakly generalized open set and have studied some of its properties.

Definition 4.1: An IFS $A$ is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS) in short) in $(X, \tau)$ if the complement $A'$ is an IFRWGCS in $X$.

Example 4.2: Let $X = \{a, b\}$ and let $\tau = \{0, T, 1\}$ be an IFT on $X$, where $T = \{(x, (0.3, 0.3), (0.7, 0.7))\}$. Then the IFS $A = \{(x, (0.8, 0.7), (0.2, 0.2))\}$ is an IFRWGOS in $X$.

Theorem 4.3: For any IFTS $(X, \tau)$, we have the following:

(i) Every IFOS is an IFRWGOS.
(ii) Every IFPOS is an IFRWGOS.
(iii) Every IFαOS is an IFRWGOS.
(iv) Every IFGPCS is an IFRWGOS. But the converses are not true in general.

Proof: Straightforward.
The converse of the above statement need not be true in general which can be seen from the following examples.

**Example 4.4:** Let \( X = \{a, b\} \) and let \( \tau = \{0., T, 1.\} \) be an IFT on \( X \), where \( T = (x, (0.2, 0.4), (0.7, 0.6)) \). Then the IFS \( A = (x, (0.7, 0.7), (0.2, 0.2)) \) is an IFRWGCS in \( X \) but not an IFOS in \( X \).

**Example 4.5:** Let \( X = \{a, b\} \) and let \( \tau = \{0., T, 1.\} \) be an IFT on \( X \), where \( T = (x, (0.3, 0.7), (0.5, 0.1)) \). Then the IFS \( A = (x, (0.6, 0.5), (0.4, 0.5)) \) is an IFRWGCS but not an IFOS in \( X \).

**Example 4.6:** Let \( X = \{a, b\} \) and let \( \tau = \{0., T, 1.\} \) be an IFT on \( X \), where \( T = (x, (0.2, 0.4), (0.8, 0.6)) \). Then the IFS \( A = (x, (0.7, 0.6), (0.3, 0.3)) \) is an IFRWGCS but not an IFOS in \( X \).

**Example 4.7:** Let \( X = \{a, b\} \) and let \( \tau = \{0., T, 1.\} \) be an IFT on \( X \), where \( T = (x, (0.4, 0.3), (0.6, 0.7)) \). Then the IFS \( A = (x, (0.3, 0.3), (0.6, 0.7)) \) is an IFRWGCS but not an IFOS in \( X \).

**Theorem 5.5:** Let \( X \) be an IFTS and \( A \) be an IFCS in \( X \). Then \( \text{int}(\text{cl}(A)) \subseteq \text{int}(\text{cl}(A)) \) whenever \( F \) is an IFCS and \( F \subseteq A \).

**Proof:** Suppose \( A \) is an IFRWGCS in \( X \). Let \( F \) be an IFCS and let \( F \subseteq A. \) Then \( F \) is an IFCS in \( X \) such that \( A' \subseteq F' \). Since \( A' \) is an IFRWGCS, \( \text{int}(\text{cl}(A')) \subseteq F' \). Hence \( \text{int}(\text{cl}(A)) \subseteq F' \). Hence \( A \) is an IFRWGCS of \( X \).

5. APPLICATIONS OF INTUITIONISTIC FUZZY REGULAR WEAKLY GENERALIZED CLOSED SETS

In this section, we introduce intuitionistic fuzzy regular weakly generalized closed set and its characteristics are proved.

**Definition 5.1:** An IFTS \( (X, \tau) \) is called intuitionistic fuzzy regular weakly generalized closed set if it is an IFCS in \( X \).

**Definition 5.2:** An IFTS \( (X, \tau) \) is called intuitionistic fuzzy regular weakly generalized closed set if it is an IFCS in \( X \).

**Theorem 5.3:** Every \( \nu_\tau \) space is an IFCS in \( \nu_\tau \) space. But the converse is not true in general.

**Proof:** Let \( X \) be a regular \( \nu_\tau \) space and let \( A \) be an IFRWGCS in \( X \). By hypothesis \( A \) is an IFCS in \( X \). Since every IFCS is an IFCS, \( A \) is an IFCS in \( X \). Hence \( A \) is an IFCS in \( X \). The converses need not be true which can be seen from the following examples.

**Example 5.4:** Let \( X = \{a, b\} \) and \( \tau = \{(0.9, 0.9), (0.1, 0.1)\} \) and \( \text{and} \) let \( \tau = \{0., T, 1.\} \). Then \( (X, \tau) \) is an IFCS in \( \nu_\tau \) space. But it is not a regular \( \nu_\tau \) space since the IFS \( A = (x, (0.2, 0.2), (0.8, 0.7)) \) is IFRWGCS but not IFCS in \( X \).

**Theorem 5.5:** Let \( (X, \tau) \) be an IFTS and \( A \) be an IFCS in \( X \). Then \( \text{int}(\text{cl}(A)) \subseteq \text{int}(\text{cl}(A)) \) whenever \( F \) is an IFCS and \( F \subseteq A \).

**Proof:** According to a previous theorem, it is true.

(i) Any union of IFRWGCS is an IFRWGCS.
(ii) Any intersection of IFRWGOS is an IFRWGCS.

6. REFERENCES


