Quartic B-Spline Collocation Method for Fifth Order Boundary Value Problems

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ABSTRACT
A finite element method involving collocation method with quartic B-splines as basis functions have been developed to solve fifth order boundary value problems. The fifth order and fourth order derivatives for the dependent variable are approximated by the central differences of third order derivatives. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain. The proposed method is tested on four linear and two non-linear boundary value problems. The solution of a non-linear boundary value problem has been obtained as the limit of a sequence of solutions of linear boundary value problems generated by quasilinearization technique. Numerical results obtained by the present method are in good agreement with the exact solutions available in the literature.

Keywords
Collocation Method; Quartic B-spline; Basis Function; Fifth Order Boundary Value Problem; Absolute Error.

1. INTRODUCTION
In this paper we developed a collocation method with quartic B-splines as basis functions for getting the numerical solution of the general linear fifth order boundary value problem

$$a_o(x)y^{(5)}(x) + a_1(x)y^{(4)}(x) + a_2(x)y'''(x) + a_3(x)y''(x) + a_4(x)y'(x) + a_5(x)y(x) = b(x), \quad c < x < d \qquad (1)$$

subject to the boundary conditions

$$y(c) = A_0,\ y(d) = B_0,\ y'(c) = A_1,\ y'(d) = B_1,\ y''(c) = A_2, \qquad (2)$$

where A_0, B_0, A_1, B_1, A_2 are finite real constants and $a_0(x)$, $a_1(x)$, $a_2(x)$, $a_3(x)$, $a_4(x)$, $a_5(x)$ and $b(x)$ are all continuous functions defined on the interval [c,d].

Generally, these types of differential equations arise in the mathematical modelling of viscoelastic fluids [1, 2]. The existence and uniqueness of the solution for these types of problems have been discussed in Agarwal [3].

Over the years, there are several authors who worked on these types of boundary value problems by using different methods. For example, Caglar et al.[4] solved fifth order special case boundary value problems by collocation method with sixth degree B-splines. They divided the domain into n subintervals by means of n+1 distinct points. They approximated the solution using the given five boundary conditions and the residual is made equal to zero at the inner collocation points only. Ghajala and Shahid [5] solved a special fifth order boundary value problems using sixth degree spline curves. Lamini et al.[6] developed two methods for the solution of the special fifth order boundary value problems.

Further, the fifth order boundary value problems were investigated by Khan [7] using finite difference methods and Wazwaz [8] by means of adomian decomposition methods. Recently, Khan et al. [9] presented a class of methods based on non-polynomial sextic spline functions for the solution of a special fifth-order boundary-value problem . El-Gamel [10] employed the Sinc-Galerkin method to solve the fifth-order boundary value problems. Noor et al. [11] applied the homotopy perturbation method for solving the fifth-order boundary value problems. Viswanadham et al. used sextic and quintic B-splines to solve fifth order special case boundary value problems [12, 13]. The use of spline functions in the context of fifth-order boundary-value problems were studied by Fyfe [14], who used quintic polynomial spline functions to develop consistency relation connecting the values of solution with fifth-order derivative at the respective nodal points. Zhao-Chun Wu [15] and Juan Zhang [16] solved fifth order boundary value problems using variational iteration method.

The above studies are concerned to solve fifth order boundary value problems by using quintic or sextic B-splines. In this paper, quartic B-splines as basis functions have been used to solve the boundary value problems of the type (1)-(2).

In section 2 of this paper, the justification for using the collocation method has been mentioned. In section 3, the definition of quartic B-splines has been described. In section 4, description of the collocation method with quartic B-splines as basis functions has been presented and in section 5, solution procedure to find the nodal parameters is presented. In section 6, numerical examples of both linear and non-linear boundary value problems are presented. The solution of a nonlinear boundary value problem has been obtained as the limit of a sequence of solutions of linear boundary value problems generated by quasilinearization technique [17]. Finally, the last section is dealt with conclusions of the paper.

2. JUSTIFICATION FOR USING COLLOCATION METHOD
In finite element method (FEM) the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration. FEM involves variational methods such as Ritz's approach, Galerkin's approach, least squares method and collocation method etc. The collocation method seeks an approximate solution by requiring the residual of the differential equation to be identically zero at N selected points in the given space variable domain where N is the number of basis functions in the basis [18]. That means, to get an accurate solution by the collocation method, one needs a set of basis functions which in number match with the number of collocation points selected in the given space variable domain.

Further, the collocation method is the easiest to implement among the variational methods of FEM. When a differential equation is approximated by m^{th} order B-splines, it yields $(m+1)^{th}$ order accurate results [19]. Hence this motivated us to solve a fifth order boundary value problem of type (1)-(2) by collocation method with quartic B-splines as basis functions.

3. DEFINITION OF QUARTIC B-SPLINES

The cubic B-splines are defined in [20, 21]. In a similar analogue, the existence of the quartic spline interpolate s(x) to a function in a closed interval [c, d] for spaced knots (need not be evenly spaced)

$$c = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = d$$

is established by constructing it. The construction of s(x) is done with the help of the quartic B-Splines. Introduce eight additional knots x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_{n+1}, x_{n+2}, x_{n+3} and x_{n+4} such that

$x_{-4} < x_{-3} < x_{-2} < x_{-1} < x_0$ and $x_n < x_{n+1} < x_{n+2} < x_{n+3} < x_{n+4}$.

Now the quartic B-splines $B_i(x)$ are defined by

$$B_i(x) = \begin{cases} \sum_{r=i-2}^{i+3} \dfrac{(x_r - x)^4_+}{\pi'(x_r)} & x \in [x_{i-2}, x_{i+3}] \\ 0 & \text{otherwise} \end{cases}$$

where

$$(x_r - x)^4_+ = \begin{cases} (x_r - x)^4, & \text{if } x_r \geq x \\ 0, & \text{if } x_r \leq x \end{cases}$$

and $\pi(x) = \prod_{r=i-2}^{i+3} (x - x_r).$

It can be shown that the set $\{B_{-2}(x), B_{-1}(x), B_0(x), \ldots, B_n(x), B_{n+1}(x)\}$ forms a basis for the space $S_4(\pi)$ of quartic polynomial splines. The quartic B-splines are the unique non-zero splines of smallest compact support with knots at

$x_{-4} < x_{-3} < x_{-2} < x_{-1} < x_0 < \ldots < x_n < x_{n+1} < x_{n+2} < x_{n+3} < x_{n+4}$.

4. DESCRIPTION OF THE METHOD

To solve the boundary value problem (1)-(2) by the collocation method with quartic B-splines as basis functions, we define the approximation for y(x) as

$$y(x) = \sum_{j=-2}^{n+1} \alpha_j B_j(x) \tag{3}$$

where α_j's are the nodal parameters to be determined. In the present method, the internal mesh points $x_1, x_2, \ldots, x_{n-1}$ are selected as the collocation points. In collocation method, the number of basis functions in the approximation should match with the number of selected collocation points [18]. Here the number of basis functions in the approximation (3) is n+4, where as the number of selected collocation points is n-1. So, there is a need to redefine the basis functions into a new set of basis functions which in number match with the number of selected collocation points. The procedure for redefining the basis functions is as follows:

Using the quartic B-splines described in section 3 and the Dirichlet boundary conditions of (2), we get the approximate solution at the boundary points as

$$y(c) = y(x_0) = \sum_{j=-2}^{1} \alpha_j B_j(x_0) = A_0 \tag{4}$$

$$y(d) = y(x_n) = \sum_{j=n-2}^{n+1} \alpha_j B_j(x_n) = B_0. \tag{5}$$

Eliminating α_{-2} and α_{n+1} from the equations (3), (4) and (5), we get the approximation for y(x) as

$$y(x) = w_1(x) + \sum_{j=-1}^{n} \alpha_j P_j(x) \tag{6}$$

where

$$w_1(x) = \frac{A_0}{B_{-2}(x_0)} B_{-2}(x) + \frac{B_0}{B_{n+1}(x_n)} B_{n+1}(x)$$

and

$$P_j(x) = \begin{cases} B_j(x) - \dfrac{B_j(x_0)}{B_{-2}(x_0)} B_{-2}(x), & \text{for } j = -1,0,1 \\ B_j(x), & \text{for } j = 2,3,\ldots,n-3 \\ B_j(x) - \dfrac{B_j(x_n)}{B_{n+1}(x_n)} B_{n+1}(x) & \text{for } j = n-2,n-1,n. \end{cases}$$

Using the Neumann boundary conditions of (2) to the approximate solution y(x) in (6), we get

$$y'(c) = y'(x_0) = w_1'(x_0) + \alpha_{-1}P'_{-1}(x_0) + \alpha_0 P'_0(x_0) + \alpha_1 P'_1(x_0) = A_1 \tag{7}$$

$$y'(d) = y'(x_n) = w_1'(x_n) + \alpha_{n-2}P'_{n-2}(x_n) + \alpha_{n-1}P'_{n-1}(x_n) + \alpha_n P'_n(x_n) = B_1. \tag{8}$$

Now, eliminating α_{-1} and α_n from the equations (6), (7) and (8), we get the approximation for $y(x)$ as

$$y(x) = w_2(x) + \sum_{j=0}^{n-1} \alpha_j Q_j(x) \tag{9}$$

where

$$w_2(x) = w_1(x) + \frac{A_1 - w_1'(x_0)}{P'_{-1}(x_0)} P_{-1}(x) + \frac{B_1 - w_1'(x_n)}{P'_n(x_n)} P_n(x)$$

and

$$Q_j(x) = \begin{cases} P_j(x) - \dfrac{P_j'(x_0)}{P'_{-1}(x_0)} P_{-1}(x), & for \quad j = 0,1 \\ P_j(x) & for \quad j = 2,3,\ldots,n-3 \\ P_j(x) - \dfrac{P_j'(x_n)}{P_n'(x_n)} P_n(x), & for \quad j = n-2, n-1 \cdot \end{cases}$$

Using the boundary condition $y''(c) = A_2$ of (2) to the approximate solution y(x) in (9), we get

$$y''(c) = y''(x_0) = w''_2(x_0) + \alpha_0 Q''_0(x_0) + \alpha_1 Q''_1(x_0) = A_2. \tag{10}$$

Now eliminating α_0 from the equations (9) and (10), we get the approximation for y(x) as

$$y(x) = w(x) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j(x) \tag{11}$$

where

$$w(x) = w_2(x) + \frac{A_2 - w_2''(x_0)}{Q''_0(x_0)} Q_0(x)$$

and

$$\tilde{B}_j(x) = \begin{cases} Q_j(x) - \frac{Q_j''(x_0)}{Q_0''(x_0)} Q_0(x) \quad , \quad for \quad j=1 \\ Q_j(x), \quad for \quad j = 2,3,...,n-1. \end{cases}$$

Now the new basis functions for the approximation y(x) are $\left\{ \tilde{B}_j(x), \ j = 1,2,..., n-1 \right\}$ and they are in number match with the number of selected collocated points. Since the approximation for y(x) in (11) is a quartic approximation, let us approximate $y^{(4)}$ and $y^{(5)}$ at the selected collocated points with central differences as

$$y_i^{(4)} = \frac{y_{i+1}''' - y_{i-1}'''}{2h}$$

$$y_i^{(5)} = \frac{y_{i+1}''' - 2y_i''' + y_{i-1}'''}{h^2} \tag{12}$$

for $i = 1,2,\ldots,n-1$
where

$$y_i = y(x_i) = w(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j(x_i). \tag{13}$$

Now applying collocation method to (1), we get

$$a_0(x_i)y_i^{(5)} + a_1(x_i)y_i^{(4)} + a_2(x_i)y_i''' + a_3(x_i)y_i'' + a_4(x_i)y_i' + a_5(x_i)y_i = b(x_i) \quad \text{for } i = 1,2,\ldots,n-1. \tag{14}$$

Using (12) and (13) in (14) and after rearranging the terms, we get the system of equations which were written in the matrix form as

$$A\alpha = B \tag{15}$$

where

$$A = [a_{ij}];$$

$$a_{ij} = \tilde{B}_j'''(x_{i-1})\left(\frac{a_0(x_i)}{h^2} - \frac{a_1(x_i)}{2h}\right) + \tilde{B}_j'''(x_i)\left(-2\frac{a_0(x_i)}{h^2} + a_2(x_i)\right)$$
$$+ \tilde{B}_j'''(x_{i+1})\left(\frac{a_0(x_i)}{h^2} + \frac{a_1(x_i)}{2h}\right) + \tilde{B}_j''(x_i)a_3(x_i) + \tilde{B}_j'(x_i)a_4(x_i)$$
$$+ \tilde{B}_j(x_i)a_5(x_i)$$

for $i = 1,2,\ldots,n-1$, $\quad j = 1,2,\ldots,n-1$. (16)

$$B = [b_i];$$

$$b_i = b(x_i) - \Big[w'''(x_{i-1})\left(\frac{a_0(x_i)}{h^2} - \frac{a_1(x_i)}{2h}\right)$$
$$+ w'''(x_i)\left(-2\frac{a_0(x_i)}{h^2} + a_2(x_i)\right)$$
$$+ w'''(x_{i+1})\left(\frac{a_0(x_i)}{h^2} + \frac{a_1(x_i)}{2h}\right)$$
$$+ w''(x_i)a_3(x_i) + w'(x_i)a_4(x_i)$$
$$+ w(x_i)a_5(x_i)\Big]$$

for $i = 1,2,\ldots,n-1$. (17)
and $\alpha = [\alpha_1, \alpha_2, \ldots \alpha_{n-1}]^T$.

5. SOLUTION PROCEDURE TO FIND THE NODAL PARAMETERS

The basis function $\tilde{B}_i(x)$ is defined only in the interval $[x_{i-2}, x_{i+3}]$ and outside of this interval it is zero. Also at the end points of the interval $[x_{i-2}, x_{i+3}]$ the basis function $\tilde{B}_i(x)$ vanishes. Therefore, $\tilde{B}_i(x)$ is having non-vanishing values at the mesh points x_{i-1}, x_i, x_{i+1}, x_{i+2} and zero at the other mesh points. The first three derivatives of $\tilde{B}_i(x)$ also have the same nature at the mesh points as in the case of $\tilde{B}_i(x)$. Using these facts, we can say that the matrix A defined in (16) is a six band matrix. Therefore, the system of equations (15) is a six band system in α_i's. The nodal parameters α_i's can be obtained by using band matrix solution package. We have used the FORTRAN-90 programming to solve the boundary value problem (1)-(2) by the proposed method.

6. NUMERICAL EXAMPLES

To demonstrate the applicability of the proposed method for solving the fifth order boundary value problems of type (1)-(2), we considered six examples of which four are linear and two are non linear boundary value problems. Numerical results for each problem are presented in tabular forms and compared with the exact solutions available in the literature.

Example 1 Consider the linear boundary value problem

$$y^{(5)}(x) - y(x) = -(15 + 10x)e^x, \quad 0 < x < 1 \tag{18}$$

subject to the boundary conditions

$y(0) = 0$, $y(1) = 0$, $y'(0) = 1$, $y'(1) = -e$, $y''(0) = 0$.

The exact solution for the above problem is given by $y(x) = x(1-x)e^x$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 1. The maximum absolute error obtained by the proposed method is 5.871058×10^{-6}.

Table 1. Numerical results for Example 1

x	Exact Solution	Absolute error by proposed method
0.1	9.946539E-02	2.235174E-08
0.2	1.954244E-01	4.768372E-07
0.3	2.834704E-01	1.877546E-06
0.4	3.580379E-01	2.861023E-06
0.5	4.121803E-01	4.172325E-06
0.6	4.373085E-01	5.364418E-06
0.7	4.228881E-01	5.871058E-06
0.8	3.560865E-01	4.619360E-06
0.9	2.213642E-01	2.235174E-06

Example 2 Consider the linear boundary value problem

$$y^{(5)} - y^{(4)} = -e^x(2x + 7), \quad 0 < x < 1 \tag{19}$$

subject to the boundary conditions

$y(0) = 0$, $y(1) = 0$, $y'(0) = 1$, $y'(1) = -e$, $y''(0) = 0$.

The exact solution for the above problem is given by $y(x) = x(1-x)e^x$. The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 2. The maximum absolute error obtained by the proposed method is 8.165836×10^{-6}.

Table 2. Numerical results for Example 2

x	Exact Solution	Absolute error by proposed method
0.1	9.946539E-02	6.705523E-08
0.2	1.954244E-01	7.748604E-07
0.3	2.834704E-01	2.682209E-06
0.4	3.580379E-01	4.351139E-06
0.5	4.121803E-01	6.288290E-06
0.6	4.373085E-01	7.808208E-06
0.7	4.228881E-01	8.165836E-06
0.8	3.560865E-01	6.347895E-06
0.9	2.213642E-01	2.950430E-06

Example 3 Consider the linear boundary value problem

$$y^{(5)}(x) + y^{(4)}(x) + e^{-2x}\, y(x) = e^{-x}[-4e^{2x}(-3 + x) \cos x - \{1 - x + 4e^{2x} (5+2x)\}\sin x], \qquad 0 < x <1 \qquad (20)$$

subject to the boundary conditions

$$y(0) = 0,\ y(1) = 0,\ y'(0) = -1,\ y'(1) = e \sin 1,\ \ y''(0) = 0.$$

The exact solution for the above problem is given by $y(x) = e^x(x-1)\sin x$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 3. The maximum absolute error obtained by the proposed method is 8.851290×10^{-6}.

Table 3. Numerical results for Example 3

x	Exact Solution	Absolute error by proposed method
0.1	-9.929969E-02	1.862645E-07
0.2	-1.941242E-01	1.132488E-06
0.3	-2.792374E-01	3.367662E-06
0.4	-3.485664E-01	5.424023E-06
0.5	-3.952195E-01	7.539988E-06
0.6	-4.115383E-01	8.851290E-06
0.7	-3.891885E-01	8.493662E-06
0.8	-3.193011E-01	5.781651E-06
0.9	-1.926673E-01	2.011657E-06

Example 4 Consider the linear boundary value problem

$$y^{(5)}(x) + (x-2)\, y^{(4)}(x) + 2y'''(x) - (x^2+2x-1)y''(x) + (2x^2+4x)y'(x) - 2x^2y(x) = 4e^x\cos x - 2x^4 + 4x^3 + 6x^2 - 4x + 2, \qquad 0 < x <1 \qquad (21)$$

subject to the boundary conditions

$$y(0) = 0,\ y(1) = 1+2e \sin 1,\ y'(0) = 2,\ y'(1) = 2e(\sin 1+\cos 1)+2,\ y''(0) = 6.$$

The exact solution for the above problem is given by $y(x) = 2e^x\sin x+x^2$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 4. The maximum absolute error obtained by the proposed method is 2.670288×10^{-5}.

Table 4. Numerical results for Example 4

x	Exact Solution	Absolute error by proposed method
0.1	2.306660E-01	4.619360E-07
0.2	5.253106E-01	2.384186E-06
0.3	8.878211E-01	7.808208E-06
0.4	1.321888	1.180172E-05
0.5	1.830878	1.740456E-05
0.6	2.417691	2.288818E-05
0.7	3.084590	2.670288E-05
0.8	3.833011	1.764297E-05
0.9	4.663347	8.106232E-06

Example 5 Consider the nonlinear boundary value problem

$$y^{(5)} = e^x[y(x)]^4, \qquad 0<x<1 \qquad (22)$$

subject to the boundary conditions

$$y(0) = 1,\ \ y(1) = e^{-\frac{1}{3}},$$
$$y'(0) = -\frac{1}{3},\ \ y'(1) = -\frac{1}{3}e^{-\frac{1}{3}},$$
$$y''(0) = \frac{1}{9}.$$

This nonlinear boundary value problem is converted into a sequence of linear boundary value problems generated by quasi linearization technique [17] as

$$y^{(5)}_{(n+1)}(x) - [4e^x y^3_{(n)}]y_{(n+1)} = -3e^x y^4_{(n)}, \qquad n = 0,1,2,\ldots \qquad (23)$$

subject to the boundary conditions

$$y_{(n+1)}(0) = 1,\ \ y_{(n+1)}(1) = e^{-\frac{1}{3}},$$
$$y'_{(n+1)}(0) = -\frac{1}{3},\ \ y'_{(n+1)}(1) = -\frac{1}{3}e^{-\frac{1}{3}},$$
$$y''_{(n+1)}(0) = \frac{1}{9}.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for y. The domain [0,1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of problems (23). The exact solution for the problem (22) is not available in the literature. The numerical solutions can be obtained for this problem by refining the mesh size. Hussin and Kilicman [22] obtained the numerical solutions for this problem by refining the mesh size. Numerical results obtained by the proposed method are compared with the numerical results obtained by Hussin and Kilicman [22], and the results are presented in Table 5.

Table 5 Numerical results for Example 5

x	Numerical solutions obtained by Hussin and Kilicman [22]	Absolute error by the proposed method when compared with Hussin and Kilicman [22]
0.1	9.672161E-01	8.344650E-07
0.2	9.355070E-01	2.622604E-06
0.3	9.048374E-01	5.960464E-06
0.4	8.751733E-01	7.510185E-06
0.5	8.464817E-01	8.881092E-06
0.6	8.187308E-01	9.298325E-06
0.7	7.918895E-01	8.404255E-06
0.8	7.659283E-01	5.125999E-06
0.9	7.408182E-01	2.026558E-06

Example 6 Consider the nonlinear boundary value problem

$$\varepsilon y' y^{(5)} + y^{(4)} = -120x + 600\varepsilon(x^2-1/4)(x^2-1/20), \quad -1/2<x<1/2 \tag{24}$$

subject to the boundary conditions

$$y(-1/2)=0,\quad y(1/2)=0,$$
$$y'(-1/2)=0,\quad y'(1/2)=0,$$
$$y''(-1/2)=1.$$

The exact solution is $y = -x(x^2-1/4)^2$. This nonlinear boundary value problem is converted into a sequence of linear boundary value problems generated by quasi linearization technique [17] as

$$[\varepsilon y'_{(n)}]y^{(5)}_{(n+1)} + y^{(4)}_{(n+1)} + [\varepsilon y^{(5)}_{(n)}]y'_{(n+1)}$$
$$= -120x + 600\varepsilon(x^2-1/4)(x^2-1/20) + \varepsilon y'_{(n)}\, y^{(5)}_{(n)}$$
$$\text{for} \quad n = 0,1,2,\ldots \tag{25}$$

subject to the boundary conditions

$$y_{(n+1)}(-1/2)=0,\quad y_{n+1}(1/2)=0,$$
$$y'_{n+1}(-1/2)=0,\quad y'_{n+1}(1/2)=0,$$
$$y''_{n+1}(-1/2)=1.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for y. The domain [-1/2, 1/2] is divided into 10 equal subintervals and the proposed method is applied to the sequence of problems (25). Numerical results for this problem with $\varepsilon = 0.01$ are presented in Table 6. The maximum absolute error obtained by the proposed method is 1.774170×10^{-7}.

Table 6. Numerical results for Example 6

x	Exact Solution	Absolute error by proposed method
-0.4	3.240000E-03	9.313226E-08
-0.3	7.680000E-03	4.423782E-08
-0.2	8.820000E-03	2.793968E-08
-0.1	5.760000E-03	4.190952E-09
0.0	0.0000000000	1.049765E-07
0.1	-5.760001E-03	1.061708E-07
0.2	-8.820000E-03	1.648441E-07
0.3	-7.680000E-03	1.774170E-07
0.4	-3.240000E-03	8.032657E-08

7. CONCLUSIONS

In this paper, we have developed a collocation method with quartic B-splines as basis functions to solve fifth order boundary value problems. Here we have taken internal mesh points $x_1, x_2, \ldots, x_{n-1}$ as the selected collocation points. The quartic B-spline basis set has been redefined into a new set of basis functions which in number match with the number of selected collocation points. The proposed method is applied to solve several number of linear and non-linear problems to test the efficiency of the method. The numerical results obtained by the proposed method are in good agreement with the exact solutions available in the literature. The objective of this paper is to present a simple method to solve a fifth order boundary value problem and its easiness for implementation.

8. REFERENCES

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