Least Square based Curve Fitting in Internet Access Traffic Sharing in Two Operator Environment

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ABSTRACT
In Internet service market, at many places the competition occurs among internet service providers. The network suffers from congestion and users have priority to prefer that network having lowest congestion. During the repeated connecting attempt process the user has to maintain call-by-call effort. In this paper Markov chain model is used to establish a relationship between internet access traffic sharing and blocking probability of the network. The model based relationship has been simplified into the linear relationship. The accuracy of the fitting is examined through coefficient of determination. This process has eased up the multiple parameter based complicated relationship into a much simplified form. An average linear relationship is predicted for handy applications.

Keywords
User behavior, Transition Probability Matrix (TPM), Markov Chain Model (MCM), Coefficient of Determination (COD), Confidence Interval.

1. INTRODUCTION
The broadband infrastructure of computer network is growing fast but even in rural and remote areas the dominant methodology of accessing internet is through public switch telecommunication network (PSTN) in many countries. The modem is used and dial-up-setup is the key of technology model of user’s behavior with the following assumptions:

1. The user initially chooses one of the two operators (indicated as $O_1$ and $O_2$) with probability $p$ and $1-p$ (initial share) respectively.
2. The probability ($p$) can take into account all the factors that may lead the user to choose one of the two operators as his first choice, including the range of services it offers and past experience.
3. After each failed attempt the user has two choices, he can either abandon (with probability $P_A$) or switch to the other operator for a new attempt.
4. Switching between the two operators is performed on a call-by-call basis and depends just on the latest attempt.
5. During the repeated call attempt process the blocking probabilities $L_1$ and $L_2$ (i.e. the probabilities that the call attempt through the operator $O_1$ or $O_2$ fails), and the probability of abandonment $P_A$ stay constant.

2. A REVIEW
The stochastic process has been used by many scientists and researchers for the purpose of statistical modelling whose detailed description is in Medhi [1], [2]. Chen and Mark [3] discussed the fast packet switch shared concentration and output queueing for a busy channel. Hambali and Ramani [4] evaluated multicast switch with a variety of traffic patterns. Newby and Dagg [6] have a useful contribution on the optical inspection and maintenance for stochastically deteriorating system. Dorea et al. [8] used Markov chain for the modelling of a system and derived some useful approximations. Yeian and Lygeres [10] presented a work on stabilization of class of stochastic different equations with Markovian switching. Shukla et al. [11] advocated for model based study for space division switches in computer network. Francini and Chiussi [7] discussed some interesting features for QoS guarantees to the unicast and multicast flow in multistage packet switch. On the reliability analysis of network a useful contribution is by Shukla et al. [13] whereas Paxson [9] introduced some of their critical experiences while measuring the internet traffic. Shukla et al. [14], [15], [16], [17], [18], [19] and [20] presented different dimensions of Internet traffic sharing in the light of share loss analysis. Shukla et al. [21], [22], [23], [24], [25] and [26] have given some Markov Chain model applications in view to disconnectivity factor, multi marketing and crime based analysis. Shukla and Thakur [27] presented Index based internet traffic analysis of users by a Markov chain model. Shukla et al. [28], [29], [30], [31] and [32] discussed cyber crime analysis for multidimensional effect in computer network and internet traffic sharing. Shukla et al. [33], [34], [35], [36], [37], [38], [39] and [40] discussed the elasticity property and its impact on parameters of internet traffic sharing in presence blocking probability of computer network specially when two operators are in business competitions with each other in a market. Shukla et al. [41] presented analysis of user web browsing using Markov chain model for iso-browser share probability. Shukla et al. [42] studied least square curve fitting for Iso-failure in web browsing using Markov chain model.

3. DEFINING A SYSTEM
Naldi [5] has suggested state discrete time Markov chain model containing

- O1: The first operator
- O2: The Second operator
- Z: Success
- A: Abandon state

Both state Z and A are absorbing state because completions of a call or the abandonment terminates the user attempt process.(see fig 1)
Fig. 1 Markov chain model of the user’s behavior
[Naldi [5]]

Let \(X^{(n)}\), \(n=0\) be a Markov chain over four state \(O_1, O_2, Z, A\), the \(X^{(n)}\) is the position of user at the \(n\)th call attempt. The starting conditions are as describe by [Naldi [5]]

\[
\begin{align*}
P[X^{(0)} = O_1] &= p \\
P[X^{(0)} = O_2] &= 1 - p \\
P[X^{(0)} = Z] &= 0 \\
P[X^{(0)} = A] &= 0
\end{align*}
\]

Assume that \(L_1\) and \(L_2\) are blocking probabilities of operator \(O_1\) and \(O_2\), \(p_A\) is the probability of abandoning attempt process. Naldi [5] has obtained the following transition probability matrix and some interesting results. Since the user switches from \(O_1\) and \(O_2\) only if his call fails and if he does not abandon, the transition probability from \(O_1\) to \(O_2\) is \(L_1(1 - p_A)\). Consequently, the probability of a call places through \(O_1\) being completed is a single attempt is \(1-L_1\).

The one-step transition probabilities matrix as stands by [Naldi [5]] is:

\[
\begin{bmatrix}
O_1 & O_2 & Z & A \\
0 & L_2(1 - p_A) & 1 - L_2 & L_2p_A \\
L_2(1 - p_A) & 0 & 1 - L_2 & L_2p_A \\
Z & 0 & 1 & 0 \\
A & q & 1 - q & 0 & 0
\end{bmatrix}
\]

...(3.2)

Following results are derived by [Naldi [5]]:

\[
\begin{align*}
P[X^{(n)} = O_1] &= P[X^{(n-1)} = O_2]L_2(1 - p_A) \\
P[X^{(n)} = O_2] &= P[X^{(n-1)} = O_1]L_2(1 - p_A) \\
P[X^{(n)} = Z] &= (1 - p)\frac{L_2}{L_1}(1 - p_A)^n, \quad n \text{ even} \\
P[X^{(n)} = A] &= p\frac{L_2}{L_1}(1 - p_A)^n, \quad n \text{ odd}
\end{align*}
\]

for \(O_2\)

\[
\begin{align*}
P[X^{(n)} = O_1] &= (1-p)\frac{L_2}{L_1}(1 - p_A)^n, \quad n \text{ even} \\
P[X^{(n)} = O_2] &= p\frac{L_2}{L_1}(1 - p_A)^n, \quad n \text{ odd}
\end{align*}
\]

...(3.5)

4. TRAFFIC SHARING

Traffic sharing [Naldi [5]] presents how the users traffic is shared between the two operators. This computation is based on general assumptions that:

(i) if user fail to connect \(O_1\) he switches to operator \(O_2\)
(ii) if he fail to \(O_2\) then comes back to \(O_1\) for connection attempts
(iii) if the call is well connected the attempts process terminates immediately.

For infinitely large number of attempts Naldi [5] derived following expressions related to traffic sharing between two operators

\[
\begin{align*}
\overline{P}_1 &= \lim_{n \to 0} P[X^{(n)} = (1-L_1)\frac{L_2}{L_1}(1-p_A)^2 \ldots (1-L_2) (1-p_A)^2 \ldots} \\
\overline{P}_2 &= \lim_{n \to 0} P[X^{(n)} = (1-L_2)\frac{L_2}{L_1}(1-p_A)^2 \ldots (1-L_2) (1-p_A)^2 \ldots}
\end{align*}
\]

...(4.1)

The expressions \(\overline{P}_1\) and \(\overline{P}_2\) are functions of \(p, L_1, L_2\) and \(p_A\) .

We express (4.1) and (4.2) as

\[
\begin{align*}
\overline{P}_1 &= f_1(L_1, L_2, p, p_A) \\
\overline{P}_2 &= f_2(L_1, L_2, p, p_A)
\end{align*}
\]

...(4.3)

...(4.4)

These variables \(\overline{P}_1\) and \(\overline{P}_2\) highly depend on blocking probability \(L_1\) and \(L_2\) as suggested by Naldi [5]. Therefore it is as interesting thought to established a direct relationship between \(\overline{P}_1\) and \(L_1\), \(\overline{P}_2\) and \(L_2\). We assume

\[
\overline{P}_1 = g_1(L_1) \quad \text{when } L_2, p, p_A \text{ are constant} \quad \ldots (4.5)
\]

\[
\overline{P}_2 = g_2(L_2) \quad \text{when } L_2, p, p_A \text{ are constant} \quad \ldots (4.6)
\]

5. LEAST SQUARE CURVE FITTING BETWEEN TRAFFIC SHARING AND BLOCKING PROBABILITY

In particular, let us suggest a linear relationship with constants \(a\) & \(b\)

\[
\overline{P}_i = a + b L_i 
\]

...(5.1)

Let \(\overline{P}_i, L_i\), \(i = 1, 2, 3, \ldots n\) be \(n\) observations generated from equation (4.1) keeping values fixed for \(p, p_A\) and \(L_2\). Suppose \(n=9\) and blocking probabilities for \(L_1\) are \(0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\) then using (4.1), the generated data of \(\overline{P}_1\) is in table (4, 5, 6) varying over \(p, L_1\) and \(p_A\). The \(\overline{P}_1\) is obtained using line equation (5.1) along with least square estimates \(\hat{a}, \hat{b}\) using (6.1), (6.2).
### Table 1

<table>
<thead>
<tr>
<th>S.NO</th>
<th>Fixed Parameter</th>
<th>( \hat{P}_1 = \hat{a} + \hat{b} \cdot L_1 )</th>
<th>( \hat{a} )</th>
<th>( \hat{b} )</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p=0.3, L_2=0.2, p_A=0.1 )</td>
<td>0.438-0.424.L_1</td>
<td>0.438</td>
<td>-0.424</td>
<td>0.998453</td>
</tr>
<tr>
<td>2</td>
<td>( p=0.3, L_2=0.2, p_A=0.2 )</td>
<td>0.429-0.411.L_1</td>
<td>0.429</td>
<td>-0.411</td>
<td>0.999047</td>
</tr>
<tr>
<td>3</td>
<td>( p=0.3, L_2=0.2, p_A=0.3 )</td>
<td>0.417-0.416.L_1</td>
<td>0.417</td>
<td>-0.416</td>
<td>0.996851</td>
</tr>
<tr>
<td>4</td>
<td>( p=0.3, L_2=0.4, p_A=0.1 )</td>
<td>0.586-0.541.L_1</td>
<td>0.586</td>
<td>-0.541</td>
<td>0.995151</td>
</tr>
<tr>
<td>5</td>
<td>( p=0.3, L_2=0.4, p_A=0.2 )</td>
<td>0.549-0.517.L_1</td>
<td>0.549</td>
<td>-0.517</td>
<td>0.996795</td>
</tr>
<tr>
<td>6</td>
<td>( p=0.3, L_2=0.4, p_A=0.3 )</td>
<td>0.514-0.493.L_1</td>
<td>0.514</td>
<td>-0.493</td>
<td>0.998023</td>
</tr>
<tr>
<td>7</td>
<td>( p=0.3, L_2=0.6, p_A=0.1 )</td>
<td>0.740-0.645.L_1</td>
<td>0.740</td>
<td>-0.645</td>
<td>0.991689</td>
</tr>
<tr>
<td>8</td>
<td>( p=0.3, L_2=0.6, p_A=0.2 )</td>
<td>0.682-0.621.L_1</td>
<td>0.682</td>
<td>-0.621</td>
<td>0.994351</td>
</tr>
<tr>
<td>9</td>
<td>( p=0.3, L_2=0.6, p_A=0.3 )</td>
<td>0.627-0.584.L_1</td>
<td>0.627</td>
<td>-0.584</td>
<td>0.996412</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>S.NO</th>
<th>Fixed Parameter</th>
<th>( \hat{P}_1 = \hat{a} + \hat{b} \cdot L_2 )</th>
<th>( \hat{a} )</th>
<th>( \hat{b} )</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p=0.6, L_2=0.2, p_A=0.1 )</td>
<td>0.692-0.672.L_2</td>
<td>0.692</td>
<td>-0.672</td>
<td>0.999094</td>
</tr>
<tr>
<td>2</td>
<td>( p=0.6, L_2=0.2, p_A=0.2 )</td>
<td>0.684-0.662.L_2</td>
<td>0.684</td>
<td>-0.662</td>
<td>0.999431</td>
</tr>
<tr>
<td>3</td>
<td>( p=0.6, L_2=0.2, p_A=0.3 )</td>
<td>0.668-0.655.L_2</td>
<td>0.668</td>
<td>-0.655</td>
<td>0.999664</td>
</tr>
<tr>
<td>4</td>
<td>( p=0.6, L_2=0.4, p_A=0.1 )</td>
<td>0.789-0.729.L_2</td>
<td>0.789</td>
<td>-0.729</td>
<td>0.996674</td>
</tr>
<tr>
<td>5</td>
<td>( p=0.6, L_2=0.4, p_A=0.2 )</td>
<td>0.763-0.726.L_2</td>
<td>0.763</td>
<td>-0.726</td>
<td>0.997894</td>
</tr>
<tr>
<td>6</td>
<td>( p=0.6, L_2=0.4, p_A=0.3 )</td>
<td>0.738-0.707.L_2</td>
<td>0.738</td>
<td>-0.707</td>
<td>0.998749</td>
</tr>
<tr>
<td>7</td>
<td>( p=0.6, L_2=0.6, p_A=0.1 )</td>
<td>0.891-0.769.L_2</td>
<td>0.891</td>
<td>-0.769</td>
<td>0.993186</td>
</tr>
<tr>
<td>8</td>
<td>( p=0.6, L_2=0.6, p_A=0.2 )</td>
<td>0.849-0.767.L_2</td>
<td>0.849</td>
<td>-0.767</td>
<td>0.995602</td>
</tr>
<tr>
<td>9</td>
<td>( p=0.6, L_2=0.6, p_A=0.3 )</td>
<td>0.811-0.755.L_2</td>
<td>0.811</td>
<td>-0.755</td>
<td>0.997362</td>
</tr>
</tbody>
</table>
Table 3

<table>
<thead>
<tr>
<th>S.NO</th>
<th>Fixed Parameter</th>
<th>Line</th>
<th>( \hat{a} )</th>
<th>( \hat{b} )</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p=0.8, L_2=0.2, p_A=0.1 )</td>
<td>( P_1 = \hat{a} + \hat{b} \cdot L_1 )</td>
<td>0.861</td>
<td>-0.832</td>
<td>0.999283</td>
</tr>
<tr>
<td>2</td>
<td>( p=0.8, L_2=0.2, p_A=0.2 )</td>
<td>0.852-0.831.L_1</td>
<td>0.852</td>
<td>-0.831</td>
<td>0.999554</td>
</tr>
<tr>
<td>3</td>
<td>( p=0.8, L_2=0.2, p_A=0.3 )</td>
<td>0.843-0.826.L_1</td>
<td>0.843</td>
<td>-0.826</td>
<td>0.999739</td>
</tr>
<tr>
<td>4</td>
<td>( p=0.8, L_2=0.4, p_A=0.1 )</td>
<td>0.926-0.854.L_1</td>
<td>0.926</td>
<td>-0.854</td>
<td>0.997157</td>
</tr>
<tr>
<td>5</td>
<td>( p=0.8, L_2=0.4, p_A=0.2 )</td>
<td>0.906-0.854.L_1</td>
<td>0.906</td>
<td>-0.854</td>
<td>0.998228</td>
</tr>
<tr>
<td>6</td>
<td>( p=0.8, L_2=0.4, p_A=0.3 )</td>
<td>0.887-0.850.L_1</td>
<td>0.887</td>
<td>-0.850</td>
<td>0.998965</td>
</tr>
<tr>
<td>7</td>
<td>( p=0.8, L_2=0.6, p_A=0.1 )</td>
<td>0.991-0.856.L_1</td>
<td>0.991</td>
<td>-0.856</td>
<td>0.993803</td>
</tr>
<tr>
<td>8</td>
<td>( p=0.8, L_2=0.6, p_A=0.2 )</td>
<td>0.961-0.868.L_1</td>
<td>0.961</td>
<td>-0.868</td>
<td>0.996079</td>
</tr>
<tr>
<td>9</td>
<td>( p=0.8, L_2=0.6, p_A=0.3 )</td>
<td>0.933-0.869.L_1</td>
<td>0.933</td>
<td>-0.869</td>
<td>0.997697</td>
</tr>
</tbody>
</table>

6. FITTING STRAIGHT LINE

Since, we have suggested an approximate the relationship between parameter \( P_1 \) and \( L_1 \) like a straight line \( \hat{P}_1 = \hat{a} + \hat{b} \cdot L_1 \) normal equations are

\[
\begin{align*}
\sum_{i=1}^{n} P_i &= n \cdot \hat{a} + \hat{b} \cdot \sum_{i=1}^{n} L_i \\
\sum_{i=1}^{n} P_i \cdot L_i &= \hat{a} \sum_{i=1}^{n} L_i + \hat{b} \sum_{i=1}^{n} L_i^2 
\end{align*}
\]

...(6.1)

By solving (6.1) the least squares estimates of \( a \) and \( b \) are (denoted as \( \hat{a}, \hat{b} \) :

\[
\hat{a} = \frac{\sum_{i=1}^{n} P_i - \hat{b} \sum_{i=1}^{n} L_i}{n}
\]

...(6.2)

\[
\hat{b} = \frac{n \sum_{i=1}^{n} P_i L_i - (\sum_{i=1}^{n} P_i)(\sum_{i=1}^{n} L_i)}{n \sum_{i=1}^{n} L_i^2 - (\sum_{i=1}^{n} L_i)^2}
\]

...(6.3)

Where \( n \) is the number of observations in sample \( n \) and the resultant straight line is

\[
\hat{P}_1 = \left\{a + b \cdot L_1 \right\}
\]

...(6.4)

The coefficient of determination (COD) is defined as

\[
\text{COD} = \frac{\sum_{i=1}^{n} (\hat{P}_i - \bar{P})^2}{\sum_{i=1}^{n} (P_i - \bar{P})^2}
\]

...(6.5)

where \( \bar{P} = \frac{1}{n} \sum_{i=1}^{n} P_i \) is mean of original data of \( P_1 \) obtained through Markov chain model. The term \( \hat{P}_1 = \hat{a} + \hat{b} \cdot L_1 \) is the estimated value give observation \( L_1 \). The COD lies between 0 to 1. If the line is good fit then it is near to 1. We generate pair of value \((L_1, P_1)\) from express tables (4, 5 and 6) by providing few fixed input parameters.
\( \hat{a} = 0.4052; \quad \hat{b} = -0.3974; \quad \hat{P}_1 = a + bL_1; \quad \hat{P}_1 = 0.4052 - 0.3974(L_1) \quad \text{......(6.1.1)} \)

**Table 5**

<table>
<thead>
<tr>
<th>Fixed parameter</th>
<th>L_1</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0.5</td>
<td>P_1</td>
<td>0.6096</td>
<td>0.5564</td>
<td>0.5004</td>
<td>0.4411</td>
<td>0.3784</td>
<td>0.3119</td>
<td>0.2412</td>
<td>0.1659</td>
<td>0.0857</td>
<td>0.997633</td>
</tr>
<tr>
<td>L_x=0.4, p_x=0.2</td>
<td>\hat{P}_1</td>
<td>0.6266</td>
<td>0.5613</td>
<td>0.4961</td>
<td>0.4309</td>
<td>0.3656</td>
<td>0.3004</td>
<td>0.2351</td>
<td>0.1699</td>
<td>0.1046</td>
<td></td>
</tr>
</tbody>
</table>

\( \hat{a} = 0.6918; \quad \hat{b} = -0.6524; \quad \hat{P}_1 = a + bL_1; \quad \hat{P}_1 = 0.6918 - 0.6524(L_1) \quad \text{......(6.1.2)} \)

**Table 6**

<table>
<thead>
<tr>
<th>Fixed parameter</th>
<th>L_1</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0.7</td>
<td>P_1</td>
<td>0.6909</td>
<td>0.6204</td>
<td>0.5484</td>
<td>0.4750</td>
<td>0.4000</td>
<td>0.3234</td>
<td>0.2451</td>
<td>0.1652</td>
<td>0.0835</td>
<td>0.997000</td>
</tr>
<tr>
<td>L_x=0.4, p_x=0.2</td>
<td>\hat{P}_1</td>
<td>0.6982</td>
<td>0.6223</td>
<td>0.5464</td>
<td>0.4705</td>
<td>0.3946</td>
<td>0.3187</td>
<td>0.2428</td>
<td>0.1670</td>
<td>0.0911</td>
<td></td>
</tr>
</tbody>
</table>

\( \hat{a} = 0.7714; \quad \hat{b} = -0.7588; \quad \hat{P}_1 = a + bL_1; \quad \hat{P}_1 = 0.7741 - 0.7588(L_1) \quad \text{......(6.1.3)} \)

### 7. CONFIDENCE INTERVALS (COI)

The 100(1- \( \alpha \) ) percent confidence interval for a and b are

\[
\hat{a} \pm \left\{ t_{(n-2)} \frac{\alpha}{2} \right\} s \left[ \frac{1}{n} \sum_{i=0}^{n} L_i \right] \quad \text{...(7.1)}
\]

where \( L_i = \frac{1}{n} \sum_{i=0}^{n} L_i \). The \( \overline{L}_i = 4.5 \text{ from table (4, 5 and 6) } \)

\[
\hat{b} \pm \left\{ t_{(n-2)} \frac{\alpha}{2} \right\} s \left[ \sqrt{\frac{1}{n} \sum_{i=0}^{n} (L_i - \overline{L}_i)^2} \right] \quad \text{...(7.2)}
\]

where \( s = \sqrt{\frac{\sum (P_i - \hat{P}_i)^2}{n-2}} \) and \( t_{(n-2)} \frac{\alpha}{2} \) is obtained from standard table. Take \( \alpha = 0.05 \), n=9 then \( t_5 = 0.025 = 2.365 \)

**Table: 7 Confidence interval for constant a and b**

<table>
<thead>
<tr>
<th>Fixed parameter</th>
<th>Constant (a)</th>
<th>Constant (b)</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0.3, L_x=0.2, p_x=0.3</td>
<td>( \hat{a} = 0.4052 )</td>
<td>( \hat{b} = -0.3974 )</td>
<td>for a: (a=0.3989, b=0.4114) for b: (b=-0.4024, b=-0.3924)</td>
</tr>
<tr>
<td>p=0.5, L_x=0.4, p_x=0.2</td>
<td>( \hat{a} = 0.6918 )</td>
<td>( \hat{b} = -0.6524 )</td>
<td>for a: (a=0.6624, b=0.7212) for b: (b=-0.0758, b=-0.6290)</td>
</tr>
</tbody>
</table>
9. CONCLUSION

The expression showing the simple relationships between variables \( \hat{P}_1 \) and \( L_4 \), as derived by Naldi [5] are complicated and having many input parameters like \( p, p_0, L_2 \) etc. If we keep all these three fixed, the expression of relationship is even not simple. By using of method least square the linear approximate relationship is displayed in table 1, 2 and 3. The similar are also in table 4, 5 and 6. The coefficients of determination (COD) have values near to 1 showing the best fitting of straight line between \( \hat{P}_1 \) and \( L_4 \). The confidence interval for estimated value \( \hat{a} \) and \( \hat{b} \) are indication for good fitting of line. We define \( \hat{P}_1 = a + b(L_4) \) in table 7 where \( \hat{a}, \hat{b} \) are average estimate obtain through all tables. We found that average linear relationship which is best fitted for prediction is \( \hat{P}_1 = 0.6237 + 0.6029(L_4) \)

<table>
<thead>
<tr>
<th>Average Estimates</th>
<th>( \hat{a} ) = 0.6237</th>
<th>( \hat{b} ) = 0.6029</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=0.7, L_2=0.4, p_0=0.5 )</td>
<td>( \hat{a} = 0.7741 )</td>
<td>( \hat{b} = 0.7588 )</td>
</tr>
</tbody>
</table>

10. REFERENCES


