On the state estimation for dynamic power system

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ABSTRACT
In this contribution we provide a simple and useful state estimation approach for a general class of non linear models that describe dynamic power systems. At first we show, through a small power network, that this class of systems is modeled by non linear differential-algebraic equations that we may always transform to a system of ordinary differential equations. After, we investigate a state estimator based on the EKF technique as well as the local stability analysis. High performances are illustrated through a simulation study applied on 3 and 5 buses test systems.

Keywords
Power system dynamics, State Estimation, Extended Kalman Filter, convergence analysis.

Nomenclature

M	Inertia constant of the generator
D	Damping constant of the generator
δ	mechanical rotor angle of the rotating machine
ω	mechanical angular velocity
ω_s	electrical angular velocity
P_M	Mechanical power input
P_j, Q_j	Nodal active and reactive power
$P_{c,d}$	Transit power
Y_{bus}	Nodal admittance matrix
G_{ij}, B_{ij}	real and imaginary terms of bus admittance matrix corresponding to *ith* row and *jth* column
N	Total number of system buses
n_g	Number of generator buses
n_l	Number of load buses
P_{Gi}	Electrical power supplied by the generator
θ_i, V_i	Phase and voltage at bus *i*

1. INTRODUCTION
Control design, diagnosis and monitoring of power systems become a major concern and an important economic issue during the last decades, in particular, in industrialized countries. Indeed, the main reasons are not only to optimize energy consumption but also to reduce the "Black Out" in vulnerable regions. It is important to observe, preferably in real-time, relevant variables of the power network. To achieve this goal it is necessary to use state estimation techniques or state observers from a database using physical sensors. Indeed, it is very difficult or impossible (for accessibility, technical and/or cost reasons) to measure through hardware sensors the excessive number of state variables in large scale systems. It is therefore important to develop software sensors that can produce a reliable estimate of the necessary variables. Unfortunately, these techniques are often based on a static model of the network around an operating point, and are off line due to large computational requirements. Therefore, the Energy Management Systems (EMS) based on these techniques are, in general, inefficient and lead to poor management of the networks [1].

State estimation in power system has mainly focused on Static State Estimation (SSE) from redundant measurement [2].

However, to oversee an electrical power system in efficient, economic and secure manner, it is most important to be acquainted with the different dynamics states, then, it is Dynamic State Estimation (DSE) in electric power system, which apprises of the aforesaid information.

In designing a DSE, it is important to model the dynamics of the complex bus voltages/phases (algebraic state) and generators (dynamic state). The existing models not only fail to depict the true behavior of the power system dynamics but also some of the parameters appearing in these models do not bear any physical meaning, reducing the size of model [3] or linearization form [4]. To override the limitations of the existing models, a relevant model has been considered in this paper to model the dynamics of the power system based on the nonlinear DAE models proposed in [5]. Where we show that we can always rewrite the system with a nonlinear DAE form with explicit ODE.

After the step of modeling, it is extremely important to consider a robust estimator which reflects a reliable image in the terms of capacity as for estimation, robustness and stability. The most important objective is the possibility of real-time application with the development of the Digital Signal Processor devices [6]. The existing methods are based on:

- The power system is considered as a quasi-static system and then applying a tracking estimator [7].
- Definition of spaces of linear combinations and their algebraic complement for the calculation of the observer gain [8].
- Consideration of linear approximation to extract the algebraic states [9] or to consider a linear dynamic power system model and applying the classical Kalman Filter [1] (by varying the algorithm of resolution such as Square Root Filter Algorithm [10] or changing the weight vector on measurement in objective function such as in the exponential form used in [11]).

We consider in this paper, an estimator based on a modified version of EKF while using new numerical approximations for the calculation of the Jacobian matrix.

The purpose of this work is to provide a generic dynamic model of balanced power systems and to design state estimation techniques to overcome the problems raised above. Indeed, description of the network by a dynamic model leads us to have an idea about the transient behavior that plays a central role for monitoring and control design. For doing so, through a 3 buses IEEE example, we show that the dynamic model is always written as a system of coupled dynamic nonlinear equations and algebraic ones. To develop useful and simple state estimators, we transform the obtained DAE model to an augmented one written as a system of ordinary differential equations. Thus, we propose a state estimator based on the Extended Kalman Filter, we also investigate stability analysis. In the last section, numerical simulations of two networks 3 and 5 buses test systems show the relevance and efficiency of the proposed approaches.

2. DYNAMIC POWER SYSTEM MODEL

The dynamics of a power system can be modeled with a combination of nonlinear differential equations and nonlinear algebraic equations. These sets of equations are often solved separately in different analysis techniques. The solution is accomplished in an iterative way, with the important feature that all the desired system characteristics are included. The general form of the DAE model is given as:

$$\begin{cases} \dot{\boldsymbol{x}}_d(t) = F_d(\boldsymbol{x}_d(t), \boldsymbol{x}_a(t), \boldsymbol{u}(t)) \\ 0 = g(\boldsymbol{x}_d(t), \boldsymbol{x}_a(t)) \\ \boldsymbol{y}(t) = h(\boldsymbol{x}_d(t), \boldsymbol{x}_a(t)) \end{cases} \quad (1)$$

With: $x_d(t) \in \mathbb{R}^{n_d}$ and $x_a(t) \in \mathbb{R}^{n_a}$ are respectively dynamic and algebraic states, $F_d(t) \in \mathbb{R}^{n_d}$ a function representing the nonlinear differential equations, $g(.) \in \mathbb{R}^{n_a}$ represents the nonlinear algebraic constraints (equations), $u(t) \in \mathbb{R}^p$ the control and $y(t) \in \mathbb{R}^m$ the output system. The problem with the system (1) is that $\dot{x}_a(t)$ does not appear explicitly.

2.1 Problem Formulation

To put out, in details, the physical dynamic power model, we will treat the case of the 3 buses test system given in Fig. 1 (with $n_g=2$ and $n_l=1$):

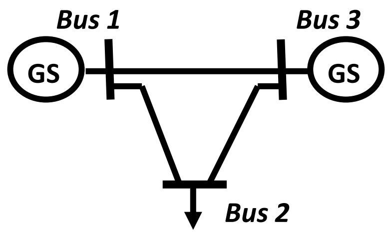

Figure 1. 3 buses test system

We consider these assumptions [5]:

- The internal field currents are constant, providing the representation of the machine as a constant voltage behind the direct axis transient reactance.
- The mechanical power provided by the prime mover is constant and all dynamics of the prime mover are neglected.
- All generators are rotating at synchronous speed (steady state) and are round rotors.
- All generators in the system are identical, and therefore the inertia constant (M_i) along with the damping constant (D_i) of each generator have the same value.
- The mechanical rotor angle is the same as the electrical phase angle of the voltage therefore δ now refers to the electrical angle. To further simplify the notation, the transient reactance is incorporated into the system Y_{bus}, resulting in θ_i as generator terminal voltage phase and V_i as the terminal voltage magnitude.

If we take node 1 as reference, the set of equation of this network is given by [8]:

$$\begin{cases} f_i^I : \dot{\delta} - \omega_i + \omega_s = 0 \\ f_i^{II} : \dot{\omega}_i = \dfrac{\omega_s}{2M}(P_{M_i} - P_{G_i}(\delta, \theta, V) - D\omega_i) \\ g_i^I : P_j - P_j(\delta, \theta, V) = 0 \\ g_i^{II} : Q_j - Q_j(\delta, \theta, V) = 0 \\ y_q : P_{c,d} = P_{c,d}(\delta, \theta, V) \end{cases} \quad (2)$$

With:
$i = 1...n_g - 1; j = (n_g + 1)...(n_g + n_l); q = 1...m; c, d = 1...N$,
the node 1 is taken as the reference and :

$$P_{G_i} = \sum_{j=1}^{N} |V_i||V_j|[G_{ij}\cos(\delta_i - \theta_j) + B_{ij}\sin(\delta_i - \theta_j)]$$

Therefore the model (2) can be rewritten under this form:

$$F(\dot{x}, x, \beta) = u$$
$$y = h(x, \beta)$$

with:

$$x = [\delta_i, \omega_i, \theta_i, V_i]^T, u = \frac{P_{M_i}}{M}, \beta = \{Y_{bus}\}, F(.) = [f_i, g_j]^T$$

and $y = P_{c,d}$ where u and y will be respectively the control and the output of the system. The choice of transit power as output which is based on this measure is used by the Tunisian Company of Electricity and Gas. Thus for this network, the state vector and the system equations are given by (3) and (4).

$$x = [x_1\, x_2\, x_3\, x_4]^T = [\delta_3\, \omega_3\, \theta_2\, V_2]^T \qquad (3)$$

$$\begin{cases} f^I: & \dot{x}_1 = x_2 - \omega_s \\ f^{II}: & \dot{x}_2 = \dfrac{\omega_s}{2M}(P_{M_3} - P_{G_3}(x_1, x_3, x_4) - Dx_2) \\ g^I: & P_2 - P_2(x_1, x_3, x_4) = 0 \\ g^{II}: & Q_2 - Q_2(x_1, x_3, x_4) = 0 \\ y_1: & P_{3,2}(x_1, x_3, x_4) \end{cases} \qquad (4)$$

with x_1 and x_2 are the dynamic variables, x_3 and x_4 are the algebraic variables. While using (1) the system is rewritten as:

$$\begin{cases} \dot{x}_d = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = F_d(x_1, x_2, x_3, x_4, u) = [f^I, f^{II}]^T \\ g(x_1, x_2, x_3, x_4) = [g^I, g^{II}]^T = 0 \\ y(t) = P_{3,2}(x_1, x_3, x_4) \end{cases} \qquad (5)$$

A simple diagram for the simulation of power system with model (1) is proposed, which: for the dynamic states we use a block of integration with nonlinear function ($F_d(t)$) with algebraic constraints resolver under a package *SIMULINK* of *MATLA®B*. The simulation diagram is as follows (Fig. 2):

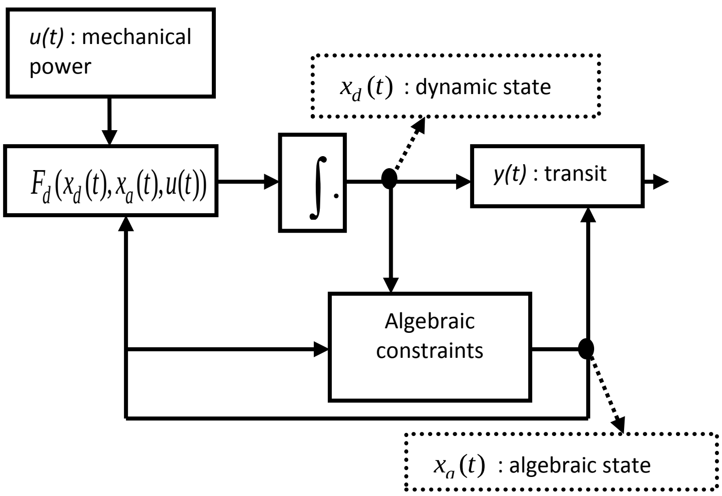

Figure 2. Diagram of simulation

2.2 Semi-explicit DAE index 1

If at an equilibrium point, the system (1) is called semi-explicit [9], index-1 property requires that $g(x_d, x_a)$ is solvable for x_a and $\det(g_{x_a}(x_d, x_a)) \neq 0$ (to simplify $x_d(t) = x_d, x_a(t) = x_a$):

$$\begin{cases} 0 = g_{x_d}(x_d, x_a)\dot{x}_d + g_{x_a}(x_d, x_a)\dot{x}_a \\ 0 = g_{x_d}(x_d, x_a)F_d(x_d, x_a, u) + g_{x_a}(x_d, x_a)\dot{x}_a \end{cases} \qquad (6)$$

Where $g_{x_a}(x_d, x_a) = \dfrac{\partial g(x_d, x_a)}{\partial x_a}$ and

$g_{x_d}(x_d, x_a) = \dfrac{\partial g(x_d, x_a)}{\partial x_d}$ In other words, the differentiation index is 1, if, by differentiation of the algebraic equations with respect to time, an implicit ODE system results [12]:

$$\begin{cases} \dot{x}_d = F_d(x_d, x_a, u) \\ \dot{x}_a = -g_{x_a}{}^{-1}(x_d, x_a)g_{x_d}(x_d, x_a)F_d(x_d, x_a, u) \end{cases} \qquad (7)$$

Where $g_{x_a}{}^{-1}(x_d, x_a) \in \mathbb{R}^{n_a \times n_a}$ and $g_{x_d}(x_d, x_a) \in \mathbb{R}^{n_a \times n_d}$.

A study of nature and stability of DAE system is given by [13]. It should be noted that:

$$g_{x_a}(x_d, x_a) = \frac{\partial g(x_d, x_a)}{\partial x_a} = \begin{pmatrix} g_{x_a 1} & g_{x_a 2} \\ g_{x_a 3} & g_{x_a 4} \end{pmatrix} \equiv [J] \qquad (8)$$

With ***J*** is the Jacobian matrix used in the Load Flow calculation excepted for generators terms, which allows us to verify that this $\det(g(x_d, x_a)) \neq 0$ and g is solvable for any x_a (the elements of this matrix are the components of the diagonal Jacobian matrix used in load flow).

Finally, the complete model in form ODE is as follows:

$$\dot{x} = \begin{pmatrix} \dot{x}_d \\ \dot{x}_a \end{pmatrix} = \bar{f}(x_d, x_a, u)$$

$$= \begin{pmatrix} F_d(x_d, x_a, u) \\ -g_{x_a}^{-1}(x_d, x_a)g_{x_d}(x_d, x_a)F_d(x_d, x_a, u) \end{pmatrix} \qquad (9)$$

$$\bar{y} = \begin{pmatrix} 0 \\ y \end{pmatrix} = \bar{h}(x_d, x_a) = \begin{pmatrix} g(x_d, x_a) \\ h(x_d, x_a) \end{pmatrix}$$

In the expression of $\bar{h}(x_d, x_a)$, the purpose of adding the algebraic constraint $g(x_d, x_a)$ is to check it permanently. It should be noted that the assumptions and the propositions given can be generalized for the other forms of dynamic power system models (models including a characteristic of the static/dynamic loads [14] and generators with exciter model [5]).

3. DYNAMIC STATE ESTIMATION

The main problem in dynamic state estimation of power system is that few methods are applicable. Effectively, the numerous and strong nonlinearities in presence lead generally to the use of Extended Kalman Filter to resolve the state estimation problem. We propose here the Extended Kalman Estimator to increase the precision as well as the robustness of the estimation. A study of the convergence of EKF will be presented.

3.1 Extended Kalman Filter

The Kalman filter is a recursive estimator. It means that to consider the running state, only preceding state and current measurements are necessary. The history of the observations and the estimates is; thus; not necessary. In the extended Kalman filter (EKF), the state transition and observation models need not be linear functions of the state but may instead be differentiable functions [15]. The considered nonlinear discrete system is given by (10):

$$\begin{cases} x_{k+1} = f(x_k, u_k) + v_k \\ y_k = h(x_k, u_k) + w_k \end{cases} \tag{10}$$

Where v_k and w_k are the system and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance Q_k and R_k respectively.

In this paper, we have used the classical form of EKF (we have used Euler discretization with a step size *Te*, $x_{k+1} = x_k + T_e \overline{f}(x_k, u_k) = f(x_k, u_k)$ to discretize the continuous model (09)) given by:

$$\begin{aligned} \hat{x}_{k+1} &= f(\hat{x}_k, u_k) + K_k e_k \\ K_k &= F_k P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \\ P_{k+1} &= (F_k - K_k H_k) P_k F_k^T + Q_k \\ e_k &= y_k - h(\hat{x}_k, u_k) \end{aligned} \tag{11}$$

With:

$$F_k = F(\hat{x}_k, u_k) = \frac{\partial(x_k + T_e \overline{f}(x_k, u_k))}{\partial x_k}\Big|_{x_k = \hat{x}_k} \text{ and}$$

$$H_k = H(\hat{x}_k, u_k) = \frac{\partial \overline{h}(x_k, u_k)}{\partial x_k} = \begin{pmatrix} \dfrac{\partial g(x_k)}{\partial x_k} \\ \dfrac{\partial h(x_k)}{\partial x_k} \end{pmatrix}\Big|_{x_k = \hat{x}_k}.$$

There are some attempts to apply Kalman Filter on linearized D.A.E system [16], but our proposition is to apply the E.K.E in the classic general form with some numerical approximations that we propose for the Jacobian calculation.

Initially, it should be noted that due to the difficulty of finding F_k (following the transformation of the algebraic variables in ODE model), we will make the following numerical approximation:

$$F_k = F(\hat{x}_k, u_k) = \frac{\partial(x_k + T_e \overline{f}(x_k, u_k))}{\partial x_k}\Big|_{x_k = \hat{x}_k}$$
$$= \begin{Bmatrix} \dfrac{\partial(x_{d_k} + T_e F_d(x_{d_k}, x_{a_k}, u_k))}{\partial(x_{d_k}, x_{a_k})} \\ \dfrac{\partial(x_{a_k} + T_e(-g_{x_a}^{-1}(x_{d_k}, x_{a_k}) g_{x_d}(x_{d_k}, x_{a_k}) F_d(x_{d_k}, x_{a_k}, u_k)))}{\partial(x_{d_k}, x_{a_k})} \end{Bmatrix} \tag{12}$$

The numerical approximation is used on the second term of F_k (since it is very difficult to determine) as follows:

$$\begin{aligned} &\frac{\partial(x_{a_k} + T_e(-g_{x_a}^{-1}(x_{d_k}, x_{a_k}) g_{x_d}(x_{d_k}, x_{a_k}) F_d(x_{d_k}, x_{a_k}, u_k)))}{\partial(x_{d_k}, x_{a_k})} \\ &\approx (I_{n_a} + T_e(-g_{x_a}^{-1}(x_{d_k}, x_{a_k}) g_{x_d}(x_{d_k}, x_{a_k}) \frac{\partial F_d(x_{d_k}, x_{a_k}, u_k)}{\partial(x_{d_k}, x_{a_k})})) \end{aligned} \tag{13}$$

For $x_{d_k} = \hat{x}_{d_k}, x_{a_k} = \hat{x}_{a_k}$. The terms $g_{x_a}^{-1}$ and g_{x_d} are calculated numerically.

3.2 Convergence Analysis

In this section, we present a convergence analysis of EKF based on the method of [17] [18] and [19] by including an unknown diagonal matrix to model linearization errors and a Lyapunov function. This is leads to the resolution of a LMI which depends on the choice on R_k and Q_k.

Initially, the error vector is defined: $\tilde{x}_k = x_k - \hat{x}_k$ and the candidate Lyapunov function: $V_{k+1} = \tilde{x}_{k+1}^T P_{k+1}^{-1} \tilde{x}_{k+1}$, where :

$$\begin{cases} \tilde{x}_{k+1} = \alpha_k (F_k - K_k H_k) \tilde{x}_k = \alpha_k \tilde{F}_k \\ P_{k+1}^{-1} = (\tilde{F}_k P_k F_k^T + Q_k)^{-1} \\ \alpha_k = diag(\alpha_{1k}, ..., \alpha_{(n_d + n_a)k}) \end{cases}$$

We have then:

$$\begin{aligned} V_{k+1} &= (\alpha_k \tilde{F}_k \tilde{x}_k)^T P_{k+1}^{-1} (\alpha_k \tilde{F}_k \tilde{x}_k) \\ &= \tilde{x}_k^T \tilde{F}_k^T \alpha_k (\tilde{F}_k P_k F_k^T + Q_k)^{-1} \alpha_k \tilde{F}_k \tilde{x}_k \end{aligned} \tag{14}$$

A decreasing sequence $\{V_k\}_{k=1,...}$ means that there exists a positive scalar $0 < \xi < 1$ so that: $V_{k+1} - (1-\xi) V_k \le 0$.
Therefore, this gives us this LMI:

$$\tilde{F}_k^T \alpha_k (\tilde{F}_k P_k F_k^T + Q_k)^{-1} \alpha_k \tilde{F}_k - (1-\xi) P_k^{-1} \le 0 \tag{15}$$

With the same reasoning used in [18], we determine domains in which (15) is satisfactory. Under the following assumption:

$$|\alpha_{jk}| \le \overline{\alpha}_k = sup_j |\alpha_{jk}| \le \left(\frac{(1-\xi) \underline{\sigma}(\tilde{F}_k P_k F_k^T + Q_k)}{\overline{\sigma}(F_k^T) \overline{\sigma}(P_k) \overline{\sigma}(\tilde{F}_k)} \right)^{\frac{1}{2}} \tag{16}$$

$\{V_k\}_{k=1,...}$ is a decreasing sequence. With $\underline{\sigma}$ and $\overline{\sigma}$ denoting the maximum and minimum singular values respectively, and as α_k is a diagonal matrix then:

$$\begin{aligned} [\overline{\sigma}(\alpha_k)]^2 &\le \frac{(1-\xi) \underline{\sigma}(\tilde{F}_k P_k F_k^T + Q_k)}{\overline{\sigma}(F_k^T) \overline{\sigma}(P_k) \overline{\sigma}(\tilde{F}_k)} \\ &\le \frac{(1-\xi) \underline{\sigma}(P_k^{-1})}{\overline{\sigma}(\tilde{F}_k^T) \overline{\sigma}((\tilde{F}_k P_k F_k^T + Q_k)^{-1}) \overline{\sigma}(\tilde{F}_k)} \end{aligned} \tag{17}$$

We have then:

$$\begin{aligned} &\overline{\sigma}(\tilde{F}_k^T \alpha_k (\tilde{F}_k P_k F_k^T + Q_k)^{-1} \alpha_k \tilde{F}_k) \\ &\le [\overline{\sigma}(\alpha_k)]^2 \overline{\sigma}(\tilde{F}_k^T) \overline{\sigma}((\tilde{F}_k P_k F_k^T + Q_k)^{-1}) \overline{\sigma}(\tilde{F}_k) \\ &\le (1-\xi) \underline{\sigma}(P_k^{-1}) \end{aligned} \tag{18}$$

When (18) is satisfied, V_k is a strictly decreasing sequence.

This last equation gives us an idea on the choice of Q_k and for R_k, we proceed as follows:

$$\begin{aligned}\bar{\sigma}(\tilde{F}_k) &= \bar{\sigma}(F_k - K_k H_k) \\ &= \bar{\sigma}(F_k - F_k P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} H_k) \\ &= \bar{\sigma}[F_k (I_{n_a+n_d} - P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} H_k)]\end{aligned} \quad (19)$$

by replacing $P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} H_k$ by A_k, we obtain:

$$\begin{aligned}\bar{\sigma}(\tilde{F}_k) &\le \bar{\sigma}(F_k)\bar{\sigma}[A_k (A_k^{-1} + I_{n_a+n_d})] \\ &\le \bar{\sigma}(F_k)\bar{\sigma}(A_k)\bar{\sigma}(A_k^{-1} + I_{n_a+n_d})\end{aligned} \quad (20)$$

with:

$$\begin{aligned}&\bar{\sigma}(A_k) \le \bar{\sigma}(P_k H_k^T)\bar{\sigma}((H_k P_k H_k^T + R_k)^{-1} H_k)) \\ &\le \bar{\sigma}(P_k H_k^T)\bar{\sigma}((H_k P_k H_k^T + R_k)^{-1})\bar{\sigma}(H_k) \\ &\le \frac{\bar{\sigma}(P_k H_k^T)\bar{\sigma}(H_k)}{\underline{\sigma}(H_k P_k H_k^T + R_k)}\end{aligned} \quad (21)$$

We obtain finally:

$$\bar{\sigma}(\tilde{F}_k) \le \frac{\bar{\sigma}(F_k)\bar{\sigma}(P_k H_k^T)\bar{\sigma}(H_k)\bar{\sigma}(A_k^{-1} + I_{n_a+n_d})}{\underline{\sigma}(H_k P_k H_k^T + R_k)} \quad (22)$$

However, in order to ensure $\lim_{k\to\infty}(x_k - \hat{x}_k) = 0$ and since V_k is a strictly decreasing sequence and P_k is a bounded matrix, it follows that:

$$0 \le \mu \tilde{x}_k^T \tilde{x}_k \le V_k \le (1-\xi)^k V_0$$

$$\Rightarrow 0 \le \mu \lim_{k\to\infty}(\tilde{x}_k^T \tilde{x}_k) \le \lim_{k\to\infty}(V_k) \le V_0 \lim_{k\to\infty}((1-\xi)^k) = 0$$

with $0 \le \mu I_{n_d+n_a} \le P_k^{-1}$.

Consequently, in the same reasoning of [18] and [19], and to guarantee that the EKE ensures local asymptotic convergence, we must verify the following conditions:

- System (10) is A-locally uniformly rank observable, there exists $k \ge A-1$ where the observability matrix:

$$rank(O(k\text{-}A+1,k)) = (n_d + n_a) \quad (23)$$

where: $O(k-A+1,k) = \begin{bmatrix} H_{k-A+1} \\ H_{k-A+2}F_{k-A+1} \\ \cdots \\ H_k F_{k-1}\cdots F_{k-A+1}\end{bmatrix}$

In practice, we use a numerical rank test on $O(k-A+1,k)$.

- F_k, H_k are uniformly bounded matrices and F_k^{-1} exist.
- The matrices Q_k and R_k are chosen as follows:

$$\begin{aligned}Q_{k+1} &= \gamma e_{k+1}^T e_{k+1} I_{n_d+n_a} + \lambda I_{n_d+n_a} \\ R_{k+1} &= \varsigma H_{k+1} P_{k+1/k} H_{k+1}^T + \tau I_m\end{aligned} \quad (24)$$

where γ and λ have to be chosen large and positive and ς and τ a positive scalar fixed by the user.

4. SIMULATION RESULTS

Studies are carried out on the IEEE 3 and 5 buses test system to evaluate the performance of the proposed dynamic model and the EKF. The transit power is considered as measurements. For the discretization of the model (09), we have used Euler discretization with a step size $T_e = 10^{-3}s$.

4.1 Results of simulation of 3 buses test system

The measurement values are generated by adding low variance noise (±5% of real value) to the generated measurements (transit power $P_{3,2}$).

Firstly, to verify that $\det(g_{x_a}(x_d, x_a)) \ne 0$, Fig. 3 presents the evolution of $\det(g_{x_a}(x_d, x_a))$:

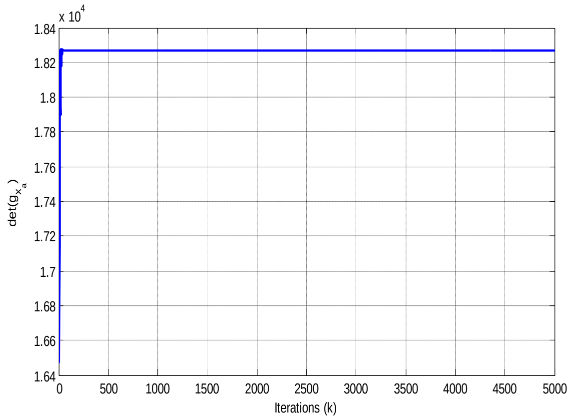

Figure 3. Evolution of $\det(g_{x_a}(x_d, x_a))$.

Fig. 3 demonstrates in a clear way that the transformation of DAE model to ODE is applicable for power system.

Now, Fig. 4 shows the evolution of the rank of the observability matrix (numerical calculation with *A=4*).

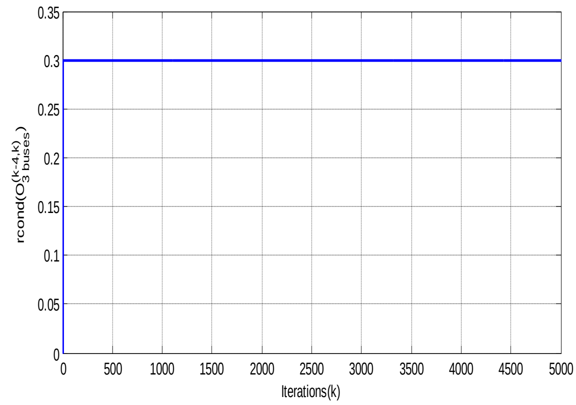

Figure 4. Evolution of $rcond(O_{3buses}^{(k-4,k)})$.

After the verification of the observability, $O_{3buses}^{(k-4,k)}$ is well conditioned ($rcond(O_{3buses}^{(k-4,k)})$ >0), we present the variation of $\| x^{real} - \hat{x}\|$ in Fig. 5 (based on the choice of Q_k and R_k) to validate the convergence of the proposed estimator. We consider the measurement values are generated with the diagram of simulation (Fig. 2) and by adding high variance noise to the calculated measurements (±15 % of real value of transit power $P_{3,2}$) with:

- Standard Choice of Q_k and R_k (Standard EKF):

$$Q_k^{EKF} = 4.989 * 10^{-2} I_4$$
$$R_k^{EKF} = 0.915$$

- Proposed choice (Modified EKF):

$$Q_k^{EKF} = 10^{10} e_k^T e_k I_4 + 10^{-3} I_4$$
$$R_k^{EKF} = 10 H_k P_k H_k^T + 10^{-3}$$

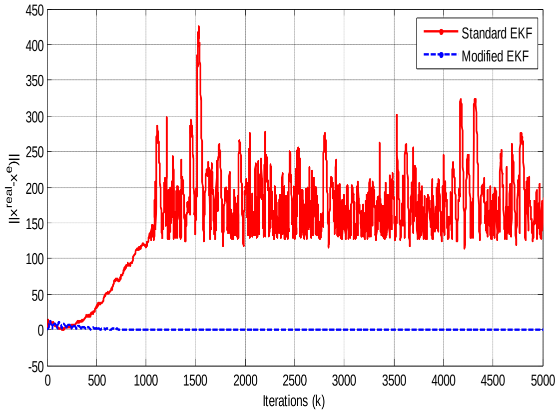

Figure 5. Evolution of $\left\| x^{real} - \hat{x} \right\|$.

Fig. 5 shows in a clear way, the interest of the appropriate choice of the expressions of R_k and Q_k to ensure stability and convergence. Another problem connected to the stability of DSE methods is the choice of initial values for various states to estimate. We tested the Standard and Modified EKF for 100 simulations while varying the initial values in a random way (variation of ±20% with respect to the actual initial values). Table 1 shows the % of convergence by applying a disturbance to the system parameters:

- Case 1: Adding a low variance noise to the system (variation of ±5% applied on G_{ij} and B_{ij}).
- Case 2: Adding a high variance noise to the system (variation of ±15% applied on G_{ij} and B_{ij}).

Table 1. (%) of Convergence with random initials values

Estimator	Case 1	Case 2
Standard EKF	54 %	49%
Modified EKF	97%	94%

In the general case, the studied algorithms converge to the good values only when are initialized ± near to their actual values (the voltages are selected close to the values of the generators voltages and the phases equal to 0). Table 1 shows clearly that the Modified EKF converges in the majority of the cases compared with the Standard version.

4.2 Results of simulation of 5 buses test system

The network includes:

- 2 generators node: bus 1 and with node 1 is the reference bus and 3 static load nodes: 3, 4 and 5.
- The output is ($P_{2,3}$) with a state vector composed by 8 variables: $[x] = [\delta_2\ \omega_2\ \theta_3\ V_3\ \theta_4\ V_4\ \theta_5\ V_5]^T$. All variables/sizes are given in p.u).

First, we present the evolution of the reciprocal condition estimator ($rcond(O_{5buses}^{(k-8,k)})$):

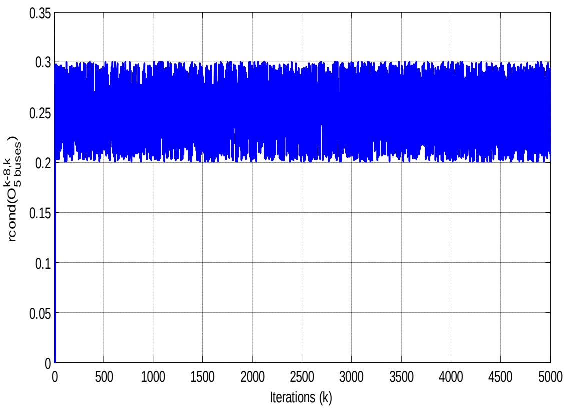

Figure 6. Evolution of $rcond(O_{5buses}^{(k-8,k)})$

After the verification of the observability $rcond(O_{5buses}^{(k-8,k)})$ is well conditioned ($rcond(O_{5buses}^{(k-8,k)}) > 0$). We consider now, the measurement values are generated with the diagram of simulation (Fig. 2) and by adding high variance noise to the calculated measurements (±15% of real value of transit power $P_{2,3}$) with :

- Standard Choice of Q_k and R_k (Standard EKF):

$$Q_k^{EKF} = 9.615 * 10^{-5} I_8$$
$$R_k^{EKF} = 2.7 * 10^{-3}$$

- Proposed choice (Modified EKF):

$$Q_k^{EKF} = 10^{10} e_k^T e_k I_{24} + 10^{-3} I_8$$
$$R_k^{EKF} = 10 H_k P_k H_k^T + 10^{-3}$$

and we present the evolution of the norm of error estimation $\| x^{real} - \hat{x}\|$ in Fig. 7:

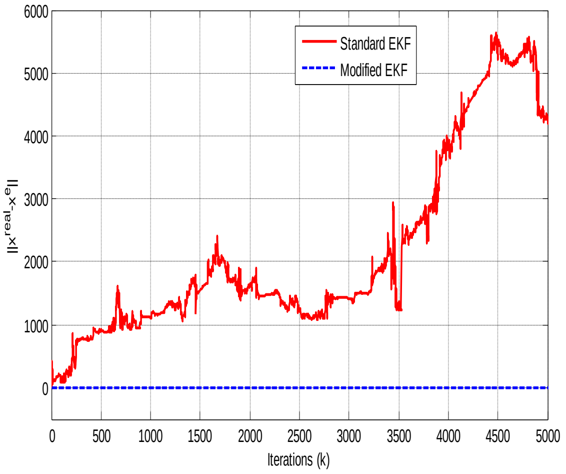

Figure 7. Evolution of $\| x^{real} - \hat{x} \|$.

Fig. 7 shows that the appropriate choice of matrices Q_k and R_k with the proposed equation insures the convergence of the estimated states to the real values. This shows, very clearly, that the proposed estimator (including proposed numerical approximations for the calculation of Jacobian matrix) gives a reliable image in the terms of capacity as for estimation and robustness.

This is reflected by the existence of ξ (given in Fig. 8) where the condition: $0 < \xi < 1$ is verified (section *Convergence Analysis*).

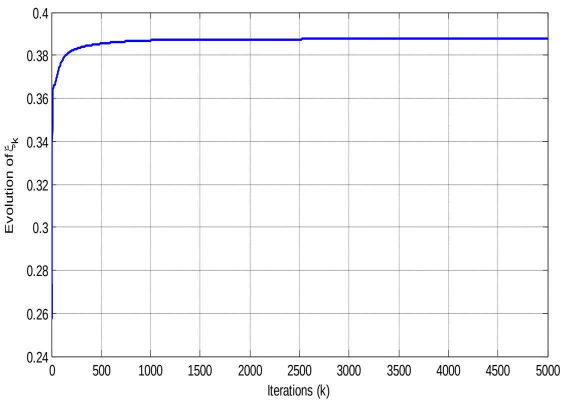

Figure 8. Evolution of ξ_k .

Fig. 8 shows the existence of ξ_k which satisfies the condition already presented in the convergence analysis and is bounded between 0 and 1 ($\xi = 0.39$).

We present in Table 2 the % of convergence for 100 random initial values with Standard and Modified EKF (we consider the same two cases in 3 buses test system):

Table 2. (%) of Convergence with random initial values

Estimator	Case 1	Case 2
Standard EKF	53 %	46%
Modified EKF	96%	93%

The values obtained in Table 2 confirm that the Modified EKF increases the estimation quality. In fact, the rate of convergence is multiplied by 2 times in comparison with Standard EKF.

It should be noted that the results of simulation of dynamic model given by the diagram (Fig. 2) are validated and compared with those generated by the Toolbox *SimPowerSystems* of *MATLAB®* (we obtained the same results). In addition, the use of this Toolbox facilitates the real time implementation in DSP device.

I one word, many results are omitted.

5. CONCLUSION

An efficient dynamic power system model has been described and investigated while based on introducing a transformation of ordinary DAE model. We also used the classical method of EKF to dynamic state estimation of power system while including some new numerical approximation for the calculation of Jacobian matrix and which was preceded by a convergence analysis. The results show well the appropriate choice of the dynamic model used in terms of robustness and, in a very clear way, the high quality of estimation offered by the Modified EKF. Experimental verification is then a necessity to testify the practical performance of this approach in the near future with real-time application for the monitoring and the diagnosis of large electrical network.

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