On Strongly-$\alpha\delta$-Super-Irresolute Functions in Topological Spaces

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ABSTRACT
In this paper a new class of sets called $\alpha\delta$-closed set is introduced and its properties are studied. Further the notion of $\widetilde{T_{\alpha\delta}}$–Space and $\alpha\delta$–continuity, Super-$\alpha\delta$-continuity, $\alpha\delta$-irresoluteness, Strongly-$\alpha\delta$-super-irresoluteness are introduced. Further, we obtain some characterizations and some properties.

Keywords
$\alpha\delta$-closed set, $\widetilde{T_{\alpha\delta}}$–Space, $\alpha\delta$–continuity, Super-$\alpha\delta$-continuity, $\alpha\delta$-irresoluteness, Strongly-$\alpha\delta$-super-irresoluteness

1. INTRODUCTION
The importance of general topological spaces rapidly increases in many fields of applications such as data mining[17]. Information systems are basic tools for producing knowledge from data in any real-life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones.

Levine [10], Mashhour et al.[12], Njastad [13], Velicko [18] and Park JH et al. [15] introduced semi-open sets, pre-open sets, α-open sets, δ-closed sets and δ-semi-closed sets respectively. Levine [11] initiated the study of so-called g-closed sets, Bhattacharaya and Lahiri [3], Arya and Nour [2], R.Devi et al. [4, 5] introduced semi-generalized closed (briefly sg-closed) sets, generalized semi-closed (briefly gs-closed) sets, generalized α-closed (briefly $g\alpha$-closed) sets and α-generalized closed (briefly αg-closed) sets. Dontchev and Ganster [7], Dontchev et al. [6], Park JH et al. [16] and Jin Han Park et al. [8] introduced and studied the concept of δg-closed sets which is a slightly stronger form of g-closedness properly placed between δ-closedness and g-closedness and introduced the notion of $T_{3/4}$ –Spaces as the spaces where every δg-closed set is δ-closed set, $g\delta$-closed and δg^*-closed sets, δgs-closed sets, $g\delta s$-closed sets. Lellis Thivagar et al. [9] introduced and studied the concept of $\delta\hat{g}$-closed sets and notion of $\hat{T}_{3/4}$ –Space as the spaces where every $\delta\hat{g}$-closed set is δ-closed. M.E.Abd El-Monsef et al. [1] introduced $\alpha\hat{g}$-closed sets and notion of $T_{\alpha\hat{g}}$ –Space as the spaces where every $\alpha\hat{g}$-closed set is α-closed. The aim of this paper is to study the notion of $\alpha\delta$-Closed set and its various characterizations are given in this paper. Applying these sets, we obtain a new space which is called $\widetilde{T_{\alpha\delta}}$–Space. Further the notion of $\alpha\delta$–continuity and $\alpha\delta$–irresoluteness are introduced.

Throughout the present paper, spaces X and Y always mean topological spaces. Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$, The δ-interior [18] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $Int_\delta(A)$.The subset A is called δ-open [18] if $A = Int_\delta(A)$, *i.e.*, a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subset (X,\ \tau)$ is called δ-closed [18] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{\mathrm{x} / \mathrm{x} \in \mathrm{U} \in \tau \Rightarrow int\,(cl(U)) \cap A \neq \varphi\}$. The family of all δ-open (resp. δ-closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called semiopen [10] (resp. α-open [13], δ-semiopen [15]) if $A \subset cl(int(A))$ (resp.$A \subset int(cl(int(A)))$, $A \subset cl(Int_\delta(A))$) and the complement of a semiopen (resp. α-open, δ-semiopen) are called semiclosed (resp. α-closed, δ-semiclosed). The intersection of all semiclosed (resp. α-closed, δ-semiclosed) sets containing A is called the semi-closure (resp. α-closure, δ-semiclosure) of A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$, $\delta - scl(A)$). Dually, semi-interior (resp. α-interior, δ-semi-nterior) of A is defined to be the union of all semiopen (resp. α-open, δ-semiopen) sets contained in A and is denoted by $sint(A)$ (resp. $\alpha int\,(A)$, δ-$sint(A)$). Note that δ-$scl(A) = A \cup int(cl_\delta(A))$ and δ-$sint(A) = A \cup cl(Int_\delta(A))$.

We recall the following definition used in sequel.

Definition 1.1. Let (X,τ) be a Topological space then A is
(a) a generalized closed [11] (g-closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X,\ \tau)$.
(b) a semi-generalized closed [3] (sg-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in $(X,\ \tau)$.
(c) a generalized semi-closed [2] (gs-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X,\ \tau)$
(d) an α-generalized closed [5] (αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in $(X,\ \tau)$.
(e) an δ-generalized closed [7] (δg-closed) set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X,\ \tau)$.
(f) a δg^*-closed [6] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is δ-open in $(X,\ \tau)$.
(g) a δgs-closed [16] if δ-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is δ-open in $(X,\ \tau)$.

(h) a generalized δ-semiclosed [8] ($g\delta s$-closed) set if $\delta\text{-}scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in(X, τ).

(i) a $\hat{g}$-closed set [9] if $cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in (X, τ).

(j) a $\delta\hat{g}$-closed [9] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

(k) a $\alpha\hat{g}$-Closed[1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

(l) The complement of a g-closed set (resp. sg-closed, gs-closed, αg-closed, δg-closed, δg^*-closed, δgs-closed, $g\delta s$-closed, $\hat{g}$-closed, $\delta\hat{g}$-closed, $\alpha\hat{g}$-closed) is called a g-open (resp. sg- open, gs-open, αg-open , δg-open, δg^*-open, δgs-open, $g\delta s$-open, $\hat{g}$-open, $\delta\hat{g}$-open, $\alpha\hat{g}$-open)

Definition 1.2. A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be

(a) super-continuous [14] (resp. g-continuous [11], sg-continuous [3], gs-continuous [2], αg-continuous [5], δg-continuous [7], δg^*-continuous [6], δgs-continuous [16], $g\delta s$-continuous [8], $\hat{g}$-continuous [9], $\delta\hat{g}$-continuous [9], $\alpha\hat{g}$-continuous [1]) if $f^{-1}(V)$ is δ-closed set (resp. g-closed, sg-closed, gs-closed, αg-closed, δg-closed, δg^*-closed, δgs-closed, $g\delta s$-closed, $\hat{g}$-closed, $\delta\hat{g}$-closed, $\alpha\hat{g}$-closed) in (X, τ) for every closed set V of (Y, σ).

(b) δ-irresolute [14] if $f^{-1}(V)$ is δ-closed set in (X, τ) for every δ-closed set V of (Y, σ).

Definition 1.3. A space (X, τ) is called a

(a) $T_{3/4}$ –Space [7] if every δg-closed set in it is δ-closed.

(b) $T_{\alpha\hat{g}}$ –space [1] if every $\alpha\hat{g}$-closed set in it is α-closed.

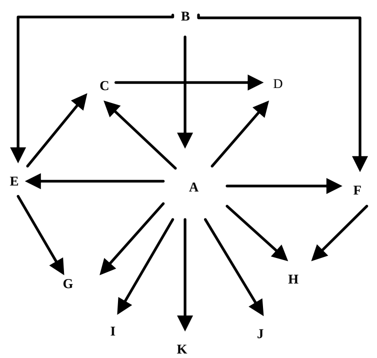
(c) $\hat{T}_{3/4}$ –space [9] if every $\delta\hat{g}$-closed set in it is δ-closed.

2. BASIC PROPERTIES OF αδ-CLOSED SETS

Definition 2.1. A subset A of a space (X, τ) is called $\alpha\delta$-closed set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X, τ).

Remark 2.2. For a subset of a space, from definitions stated above, we have the following diagram of implications:

A. $\alpha\delta$-closed, B. δ-closed, C. δg^*-closed,
D. δgs-closed, E. δg-closed, F. $\delta\hat{g}$-closed,
G. $g\delta s$-closed, H. $\alpha\hat{g}$-closed, I. αg-closed,
J. gs-closed, K. g-closed



None of these implications is reversible as shown by the following examples.

Example 2.3.

(a) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varphi, \{a\}, \{c\}, \{a,c\}, \{a,c,d\}\}$, set $A = \{a,b,c\}$ then A is g-closed (resp. gs-closed, αg-closed, δg-closed, δg^*-closed, δgs-closed, $g\delta s$-closed, $\delta\hat{g}$-closed, $\alpha\hat{g}$-closed) but not a $\alpha\delta$-closed in (X, τ).

(b) Let $X = \{a,b,c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}\}$, set $A = \{a\}$ then A is $g\delta s$-closed but not a δg-closed in (X, τ).

(c) Let $X = \{a,b,c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, set $A = \{b, c\}$ then A is $\alpha\delta$-closed but not a δ-closed in (X, τ).

Example 2.4.

(a) The converse of B implies F and F implies H is not true as shown in [9].

(b) The converse of B implies E is not true as shown by [7].

(c) The converse of C implies D is not true as shown by [16].

(d) The converse of E implies C is not true as shown by [8].

Proposition 2.5. If A is a $\alpha\delta$-closed set in a space (X, τ) and $A \subseteq B \subseteq cl_\delta(A)$ then B is also a $\alpha\delta$-closed.

Proof: Let U be a αg-open set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$ Since A is $\alpha\delta$-closed set, $cl_\delta(A) \subseteq U$ Also since $B \subseteq cl_\delta(A)$, $cl_\delta(B) \subseteq cl_\delta(cl_\delta(A)) = cl_\delta(A)$. Hence $cl_\delta(A) \subseteq U$. Therefore B is also $\alpha\delta$-closed.

Proposition 2.6. The finite union of $\alpha\delta$-closed set is $\alpha\delta$-closed set.

Proof: Let $\{A_i / i = 1,2,3 \dots n\}$ be a finite class of $\alpha\delta$-closed subsets of a space (X, τ). Then for each αg-open set U_i in X containing A_i, $cl_\delta(A_i) \subseteq U_i$, $i \in \{1,2,3 \dots n\}$. Hence $\cup_i A_i \subseteq \cup_i U_i = V$ since arbitrary union of αg-open set in (X, τ) is also αg-open set in (X, τ), V is αg-open set in (X, τ). Also $\cup_i cl_\delta(A_i) = cl_\delta(\cup_i A_i) \subseteq V$. Therefore $\cup_i A_i$ is $\alpha\delta$-closed set in (X, τ).

Proposition 2.7. Let A be a $\alpha\delta$-closed set of (X, τ) then $cl_\delta(A) - A$ doesn't contain a nonempty αg-closed set.

Proof: Suppose that A is $\alpha\delta$-closed set, Let U be a αg-closed set contained in $cl_\delta(A) - A$. Now U^c is αg-open set of (X, τ) such that $A \subseteq U^c$. Since A is $\alpha\delta$-closed set of (X, τ), then $cl_\delta(A) \subseteq U^c$. Thus $U \subseteq (cl_\delta(A))^c$. Also $U \subseteq cl_\delta(A) - A$. Therefore $U \subseteq (cl_\delta(A))^c \cap (cl_\delta(A)) = \varphi$. Hence $U = \varphi$.

Proposition 2.8. If A is αg-open and $\alpha\delta$-closed subset of (X, τ), then A is δ-closed subset of (X, τ).

Proof: Since A is αg-open and $\alpha\delta$-closed, $cl_\delta(A) \subseteq A$. Hence A is δ-closed.

Theorem 2.9. Let A be $\alpha\delta$-closed set of (X, τ). Then A is δ-closed iff $cl_\delta(A) - A$ is αg-closed set.

Proof: *Necessity:* Let A be a δ-closed subset of X. Then $cl_\delta(A) = A$ and so $cl_\delta(A) - A = \varphi$ which is αg-closed set.

Sufficiency: Since A is $\alpha\delta$-closed, by *Proposition 2.8*, $cl_\delta(A) - A$ doesn't contain a nonempty αg-closed set. But $cl_\delta(A) - A = \varphi$. That is $cl_\delta(A) = A$. Hence A is δ-closed.

Theorem 2.10. The intersection of $\alpha\delta$-closed set and a δ-closed set is always $\alpha\delta$-closed set.
Proof: Let A be a $\alpha\delta$-closed and B be δ-closed. If U be a αg-open set with $A \cap B \subseteq U$, then $A \subseteq U \cup B^c$, and so $cl_\delta(A) \subseteq U \cup B^c$. Now $cl_\delta(A \cap B) \subseteq cl_\delta(A) \cap B \subseteq U$. Hence $A \cap B$ is $\alpha\delta$-closed.

Theorem 2.11. A subset A of (X, τ) is $\alpha\delta$-open iff $U \subset Int_\delta(A)$ whenever U is αg-closed and $U \subset A$.

Proof: Obvious.

Theorem 2.12. If a subset A of (X, τ) is $\alpha\delta$-open, then $U = X$ whenever U is αg-open and $Int_\delta(A) \cup (X - A) \subset U$.

Proof: Let U be an αg-open set, such that $Int_\delta(A) \cup (X - A) \subset U$, Then $X - U \subset (X - Int_\delta(A)) \cap A$, *ie.*, $(X - U) \subset cl_\delta(X - A) - (X - A)$. Since $X - A$ is $\alpha\delta$-closed by *Theorem 2.10*, $X - U = \varphi$ and hence $U = X$.

Theorem 2.13. If A is a $\alpha\delta$-open subset of (X, τ) and $Int_\delta(A) \subset B \subset A$, whenever B is $\alpha\delta$-open.

Proof: Let $U \subset B$ and U be an αg-closed subset of X, Since A $\alpha\delta$-open and $U \subset A$, $U \subset Int_\delta(A)$ and then $U \subset Int_\delta(B)$. Hence B is $\alpha\delta$-open.

Theorem 2.14. If a subset A of (X, τ) is $\alpha\delta$-closed, then $cl_\delta(A) \setminus A$ is $\alpha\delta$-open.

Proof: Let $U \subset cl_\delta(A) \setminus A$, where U be $\alpha\delta$-closed in X. Then by *Proposition 2.7*, $U = \varphi$ and so $U \subset Int_\delta(cl_\delta(A) \setminus A)$. This show that $cl_\delta(A) \setminus A$ is $\alpha\delta$-open.

3. ON $\widetilde{T_{\alpha\delta}}$ -SPACES

Definition 3.1. A space (X, τ) is called $\widetilde{T_{\alpha\delta}}$-space if every $\alpha\delta$-closed set in it is an δ-closed.

Theorem 3.2.

(a) Every $T_{3/4}$ –Space is a $\widetilde{T_{\alpha\delta}}$–Space.
(b) Every $\widetilde{T_{\alpha\delta}}$–Space is a $T_{\alpha\hat{g}}$ –Space.
(c) Every $\hat{T}_{3/4}$ –Space is a $\widetilde{T_{\alpha\delta}}$–Space.

Proof: The proof is straight forward.

The converse of the above theorem is not true as shown in the following example.

Example 3.3.
(a) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}, \{a, c, d\}\}$. Then (X, τ) is a $\widetilde{T_{\alpha\delta}}$–Space but not a $T_{3/4}$ –Space and $\hat{T}_{3/4}$ –Space.
(b) Let $X = \{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then (X, τ) is a $T_{\alpha\hat{g}}$–Space but not a $\widetilde{T_{\alpha\delta}}$ –Space.

Theorem 3.4. For a topological space, (X, τ) the following conditions are equivalent,

(a) (X, τ) is a $\widetilde{T_{\alpha\delta}}$–Space.
(b) Every singleton $\{x\}$ is either αg-closed (or)δ-open.

Proof: (a) ⇒ (b) Let $x \in X$. Suppose $\{x\}$ is not a αg-closed set of (X, τ). Then $X - \{x\}$ is not a αg-open set. Thus $X - \{x\}$ is an αg-closed set of (X, τ). Since (X, τ) is a $\widetilde{T_{\alpha\delta}}$–Space, $X - \{x\}$ is an δ-closed set of (X, τ), *i.e.*, $\{x\}$ is δ-open set of (X, τ).

(b) ⇒ (a) Let A be an $\alpha\delta$-closed set of (X, τ). Let $x \in cl_\delta(A)$ by (ii), $\{x\}$ is either αg-closed (or)δ-open.

$Case\ (i)$: Let $\{x\}$ be αg-closed. If we assume that $x \notin A$, then we would have $x \in cl_\delta(A) - A$ which cannot happen according to Proposition 2.8. Hence $x \in A$.

$Case\ (ii)$: Let $\{x\}$ be δ-open. Since $x \in cl_\delta(A)$, then $\{x\} \cap A \neq \varphi$. This shows that $x \in A$. So in both cases we have $cl_\delta(A) \subseteq A$. Trivially $A \subseteq cl_\delta(A)$. Therefore $A = cl_\delta(A)$ or equivalently A is δ-closed. Hence (X, τ) is a $\widetilde{T_{\alpha\delta}}$–Space.

Theorem 3.5. In a $T_{\frac{3}{4}}$ –Space every $\alpha\delta$-closed set is δ-closed**.**

Proof: Let X be $T_{\frac{3}{4}}$ –Space. Let A be $\alpha\delta$-closed set in X. We know that $\alpha\delta$-closed set is δg-closed. Since X is $T_{\frac{3}{4}}$ –Space, A is δ-closed.

4. SUPER-$\alpha\delta$-CONTINUOUS AND STRONGLY-$\alpha\delta$-SUPER-IRRESOLUTE FUNCTIONS

Definition 4.1. A function $f: (X, \tau) \to (Y, \sigma)$ is called $\alpha\delta$-continuous if $f^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ) for every closed set V of (Y, σ).

Definition 4.2. A function $f: (X, \tau) \to (Y, \sigma)$ is called Super-αδ-continuous if $f^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ) for every δ-closed set V of (Y, σ).

Definition 4.3. A function $f: (X, \tau) \to (Y, \sigma)$ is called $\alpha\delta$-irresolute if $f^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ) for every $\alpha\delta$-closed set V of (Y, σ).

Definition 4.4. A function $f: (X, \tau) \to (Y, \sigma)$ is called Strongly-$\alpha\delta$-super-Irresolute (briefly, $S\alpha\delta$-super-Irr) if $f^{-1}(V)$ is δ-closed set in (X, τ) for every $\alpha\delta$-closed set V of (Y, σ).

Clearly, $f: (X, \tau) \to (Y, \sigma)$ is $\alpha\delta$-continuous (resp. $\alpha\delta$-irresolute) if and only if $f^{-1}(V)$is $\alpha\delta$-open in (X, τ) for every open (resp. $\alpha\delta$-open) set V of (Y, σ).

Theorem 4.4. For a function $f: (X, \tau) \to (Y, \sigma)$, we've the following implications.

(a) Every super-continuous is $\alpha\delta$-continuous
(b) Every $\alpha\delta$-continuous is $\delta\hat{g}$-continuous (resp.δg-continuous, δg^*-continuous, $g\delta s$-continuous, δgs-continuous, $\alpha\hat{g}$-continuous, αg-continuous, g-continuous, gs-continuous).

Proof: Obvious.

None of the implication is reversible as shown by the following example.

Example 4.5.

(a) Let $X = \{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be the identity. Then $f^{-1}(\{b, c\})$ is αδ-continuous but not a Super-continuous. Since $f^{-1}(\{b, c\})$ is not a δ-closed set in (X, τ).

(b) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{X, \varphi, \{d\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be the identity. Then $f^{-1}(\{a, b, c\})$ is $\delta\hat{g}$-continuous, δg-continuous, δg^*-continuous, $g\delta s$-continuous, δgs-continuous, $\alpha\hat{g}$-continuous, αg-continuous, g-continuous and gs-continuous but not a αδ-continuous. Since $f^{-1}(\{a, b, c\})$ is not a $\alpha\delta$-closed set in (X, τ).

Theorem 4.6. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is αg-continuous and δ-closed function, then $f(A)$ is $\alpha\delta$-closed set in (Y, σ) for every $\alpha\delta$-closed A of (X, τ).

Proof: Let A be $\alpha\delta$-closed in X. Let $f(A) \subset V$, where V be any αg-open in Y. Since f is αg-continuous, then $f^{-1}(V)$ is αg-open in X and $A \subset f^{-1}(V)$. Then we've $cl_\delta(A) \subset f^{-1}(V)$. and so $f(cl_\delta(A)) \subset V$. Since f is δ-closed, $f(cl_\delta(A))$ is δ-closed in Y and hence $cl_\delta(f(A)) \subset cl_\delta(f(cl_\delta(A))) \subset$ V. This shows that $f(A)$ is $\alpha\delta$-closed in Y.

Theorem 4.9. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (z, \eta)$ be two functions. Then

(a) If f is $\alpha\delta$-continuous and g is continuous, then $g \circ f$ is $\alpha\delta$-continuous.

(b) If f is Super-$\alpha\delta$-continuous and g is super-continuous , then $g \circ f$ is $\alpha\delta$-continuous.

(c) If f is $\alpha\delta$-irresolute and g is $\alpha\delta$-irresolute , then $g \circ f$ is $\alpha\delta$–irresolute

(d) If $f$is $\alpha\delta$-irresolute and g is super-$\alpha\delta$-continuous, then $g \circ f$ is $\alpha\delta$-continuous.

Proof:

(a) Let V be a closed set in (Z, η). Since g is continuous, $g^{-1}(v)$ is closed set in (Y, σ). Since f is $\alpha\delta$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ). Therefore $g \circ f$ is $\alpha\delta$-continuous.

(b) Let V be a closed set in (Z, η). Since g is Super-continuous, $g^{-1}(v)$ is δ-closed set in (Y, σ). Since f is Super-$\alpha\delta$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ). Therefore $g \circ f$ is $\alpha\delta$-continuous.

(c) Let V be $\alpha\delta$-closed set in (Z, η). Since g is $\alpha\delta$-irresolute, $g^{-1}(v)$ is $\alpha\delta$-closed set in (Y, σ). Since f is $\alpha\delta$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ). Therefore $g \circ f$ $\alpha\delta$-irresolute.

(d) Let V be δ-closed set in (Z, η). But every δ-closed set is $\alpha\delta$-closed set. Since g is Super-$\alpha\delta$-continuous, $g^{-1}(v)$ is $\alpha\delta$-closed set in (Y, σ). Since f is $\alpha\delta$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ). Therefore $g \circ f$ is $\alpha\delta$-irresolute.

Theorem 4.10.

(a) Every $S\alpha\delta$-super-Irr map is $\alpha\delta$-irresolute.

(b) Every $S\alpha\delta$-super-Irr map is Super-$\alpha\delta$ -continuous.

(c) Every $\alpha\delta$-Irresolute function is Super-$\alpha\delta$-Continuous.

Proof: (a) Let V be $\alpha\delta$-closed set in (Y, σ). Since f is $S\alpha\delta$-super-Irr, $f^{-1}(V)$ is δ-closed set in (X, τ). But every δ-closed set is $\alpha\delta$-closed. Hence $f^{-1}(V)$ is $\alpha\delta$-closed set in (X, τ). Thus the function f is $\alpha\delta$-irresolute.

(b) Let V be δ-closed set in (Y, σ). Since f is $S\alpha\delta$-super-Irr, $f^{-1}(V)$ is δ-closed set in (X, τ). But every δ-closed set is $\alpha\delta$-closed. Hence $f^{-1}(V)$ $\alpha\delta$-closed set in (X, τ). Thus the function f is Super-$\alpha\delta$-continuous map

(c) Let $f: X \rightarrow Y$ be $\alpha\delta$-Irresolute, Let V be any δ-closed set in Y. Then by *Remark 2.2.* V is $\alpha\delta$-closed set. Since f is $\alpha\delta$-Irresolute, Then inverse image $f^{-1}(V)$ is $\alpha\delta$-closed set in X. Therefore f is Super-$\alpha\delta$-continuous.

The converse of the above theorem is not true as shown in the following example.

Example 4.11. Let $X = \{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be the identity. Then $f^{-1}(\{b, c\})$ is $\alpha\delta$-irresolute (resp. Super-$\alpha\delta$-continuous) but not a $S\alpha\delta$-super-Irr. Since $f^{-1}(\{b, c\})$ is not a δ-closed set in (X, τ).

Example 4.11. Let $X = \{a, b, c\}$ and $\tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}, \{c, a\}\}$ and $\sigma = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be the identity. Then $f^{-1}(\{c\})$ is Super-$\alpha\delta$-continuous but not a $\alpha\delta$-Irresolute. Since $f^{-1}(\{c\})$ is not a $\alpha\delta$-closed set in (X, τ).

Theorem 4.12. If a map $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\alpha\delta$-Irresolute and δ-closed. Then A is $\alpha\delta$-closed in (X, τ) implies $f(A)$ is $\alpha\delta$-closed in (Y, σ).

Proof: Let A be a $\alpha\delta$-closed set in (X, τ). Let U be any αg-open subset of (Y, σ) such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$. Since f is an $\alpha\delta$-Irresolute map, then $f^{-1}(U)$ is a αg-open subset of (X, τ). Since A is $\alpha\delta$-closed and $f^{-1}(U)$ is a αg-open subset of (X, τ) containing A, then $cl_\delta(A) \subseteq f^{-1}(U)$.

That is $f(cl_\delta(A)) \subseteq U$. Now $cl_\delta(f(A)) \subseteq cl_\delta(f(cl_\delta(A))) = f(cl_\delta(A))\, cl_\delta(A) \subseteq U$, since f is a δ-closed map. Hence $f(A)$ is a $\alpha\delta$-closed subset of $(Y,\ \sigma)$.

Theorem 4.13. Let $f:(X,\ \tau) \to (Y,\sigma)$ be a function

(a) If $\alpha\delta$-Irresolute and X is a $\widetilde{T_{\alpha\delta}}$-Space, then f is δ-Irresolute.
(b) If $\alpha\delta$-Continuous and X is a $\widetilde{T_{\alpha\delta}}$-Space, then f is Super-Continuous.

Proof: (a) Let V be δ-closed in Y. Since f is $\alpha\delta$- Irresolute, $f^{-1}(V)$ is $\alpha\delta$-closed set in X. Since X is $\widetilde{T_{\alpha\delta}}$-Space, $f^{-1}(V)$ is δ-closed set in X. Hence f is δ-Irresolute.

(b) Let V be closed in Y. Since f is $\alpha\delta$-Continuous, $f^{-1}(V)$ is $\alpha\delta$-closed set in X. Since X is $\widetilde{T_{\alpha\delta}}$-Space, $f^{-1}(V)$ is δ-closed set in X. Hence f is Super-Continuous.

Theorem 4.14. Let $f: X \to Y$ be a function, then the following are equivalent,

(a) The function f is $\alpha\delta$-continuous
(b) For each point $x \in X$ and each open set V of Y with $f(x) \in V$, there exists a $\alpha\delta$-open set U of X such that $x \in U$, $f(U) \subset V$.

Proof:$(a) \Rightarrow (b)$ Let $f(x) \in V$ Then $x \in f^{-1}(V) \in \alpha\delta O(X)$, Since f is $\alpha\delta$-continuous. Let $U = f^{-1}(V)$ Then $x \in X$ and $f(U) \subset V$.

$(b) \Rightarrow (a)$ Let V be an open set of Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists an $\alpha\delta$-open set U_x of X such that $x \in U_x$ and $f(U) \subset V$. Now $x \in U_x \subset f^{-1}(V)$ and $f^{-1}(V) = \cup U_x$. Then $f^{-1}(V)$ is $\alpha\delta$-open set in X. Therefore, f is $\alpha\delta$-continuous.

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