Polynomial based Design of CIC Compensation
Filter used in Software Defined Radio for Multirate
Signal Processing

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ABSTRACT
Software defined radio has find important place in modern
communication systems where majority of signal processing is
performed in the digital domain using programmable DSPs. Digital down conversion (DDC) is one of the core
technologies in SDR, as well as an important component of
digital intermediate frequency receiver system. DDC use CIC
decimation filters for sample rate decimation. CIC decimation
filters require less computation but large passband droop
occurs in the frequency response. This paper presents a simple
design of compensation FIR filter for CIC decimation filter
which will correct the passband droop. Compensation filter
design consists of polynomial based design of FIR filter which
are used in cascade with CIC filter. As proposed filter depends
on simple transformation less approach and computation is
required. The resulting structure is multiplier less and exhibits
small passband droop in comparison to CIC filter. It modifies
the frequency response of CIC decimation filter while
maintaining the linear phase.

General Terms
Digital Signal Processing

Keywords
CIC, Decimation, Multirate filter, Linear Phase

1. INTRODUCTION
The power consumption in context of software defined radio
can be reduced by designing a system that has low rate of
multiplication and addition operations. Sampling rate reduction
(decimation) from a high ADC sampling rate to a small
multiple of the symbol rate is a key functionality in a digital
radio receiver. The standards to be supported by a software
radio platform are often based on incommensurate
clock/symbol rates. This makes the sample rate interpolation
and decimation a critical functionality in multi standard radio
design [9] and [10]. The sampling rate conversion (SRC) can
be seen as a process of resampling, thus is spectral domain
repetitions of the input signal spectrum are expected. If the
original signal is not band-limited, the different spectral
replicas will overlap after downsampling. This overlapping in
the spectral domain, known as aliasing, sometimes changes the
signal irreversibly. Similarly, after interpolation (upsampling)
the spectrum repeats itself at the multiples of the sampling rate.
These are called image spectra, and the phenomenon itself
imaging [11]. These two phenomena must be avoided by the
SRC system in order to preserve the signal content. This paper
presents a specific power efficient method for decimation with
appropriate frequency response. This concept has been
developed for the first stage of the decimation chain of a
multistandard radio receiver previously also [3]-[6].

The structures presented here are cascades of CIC decimation
and simple polynomial-based filter finite-impulse response
(FIR) filter [12]. The key point is to design the FIR filter so that
the frequency response droop remain minimum and filtering
performance of the overall structure remain at the same level
[8]. The FIR filter is generally implemented in a non-recursive
way which guarantees a stable filter. FIR filter design
essentially consists mainly of two parts approximation problem
and realization problem. The approximation stage takes the
specification and gives a transfer function through few steps
[26]. In this method a desired or ideal response is chosen,
usually in the frequency domain, then an allowed class of
filters is chosen (e.g. the length N for a FIR filters). A measure
of the quality of approximation is selected. Finally a method or
algorithm is selected to find the best filter transfer function.
The realization part deals with choosing the structure to
implement the transfer function which may be in the form of
circuit diagram or in the form of a program. There are
essentially three well-known methods for FIR filter design
namely, window method, frequency sampling technique and
optimal filter design methods [26]-[30]. In this paper we will
present an approach to realize compensation FIR filters using
polynomials. The polynomial approach to design filters had
been discussed in various papers [15]-[25], but here, a special
type of transformation is used along with polynomial approach
to design compensation FIR filter. This method is very simple
in terms of computation and approach. Also this method has lot
of variations so that we can define more than one type of FIR
filter from one set of specification.

The paper is organized as follows. In Section 2, the
fundamentals of CIC filters for sampling rate conversion (SRC)
are discussed. The novel compensation filter design method is
presented in Section 3. The novel structure, which is the main
contribution of this paper, consists of a fixed polynomial-based
filter FIR filter and that works at low output sampling rate.
An example application using this novel structure is discussed in
Section 4. In Section 5, we show the performance and
advantages of the proposed method by means of illustrative
examples. Finally, we draw some conclusions in Section 6.

2. CIC FILTER FOR SAMPLING RATE
CONVERSION
The cascaded integrated comb filter is an efficient linear phase
finite impulse response (FIR) digital filter. CIC filters achieve
decimation and interpolation without using multipliers [1]. The
CIC filter is used as an anti aliasing filter when the data rate is
high since the operation in the CIC filter only consists of
addition operation. The transfer function of the CIC filter can be written in equation 1.

\[ H(z) = \left( \frac{1-z^{-R}}{1-z^{-1}} \right)^N \]  

(1)

where, \( R \) and \( N \) are the decimation rate and order of the CIC filter. CIC filter has lot of advantages mainly of being multiplierless device including only simple adder and delay model. VLSI implementation is very easy as same section is used repetitively. CIC decimation filter make the circuit to work at low frequencies as well as low power dissipation.

In case of large stop band attenuation the number of stages also increases as a result CIC filter frequency response does not have a wide, flat passband. To achieve the frequency response correction a FIR filter that has a magnitude response, inverse of the CIC filter can be applied. Such filters are called compensation filters [2]. The frequency response of an

\[ |H(f)| = \left| \frac{\sin(\pi M f)}{\sin(\pi f)} \right|^N \]  

(2)

In case of large \( R \), the compensation filter response can be approximated by the inverse sinc function.

\[ \approx \frac{\pi M f}{\sin(\pi M f)} \]  

(3)

\[ \approx |\text{sinc}^{-1}(M f)|^N \]  

(4)

The frequency response of CIC filter is shown above in fig.2. The compensation filter will correct the passband droop caused by CIC filter. Its magnitude response will be given as inverse of the CIC filter response as shown in equation 3 [1], [2].

\[ G(f) = \left| MR \left( \frac{\sin(\pi f/R)}{\sin(\pi M f)} \right) \right|^N \]  

(3)

\[ \approx \frac{\pi M f}{\sin(\pi M f)} \]  

(4)

In case of large \( R \), the compensation filter response can be approximated by the inverse sinc function.

\[ \approx |\text{sinc}^{-1}(M f)|^N \]  

(5)
3. PROPOSED COMPENSATION FILTER

Proposed compensation filter design has two main design steps:

1. Compensation filter order and other specifications are decided according to passband droop which occurs due to CIC filter.
2. Design compensation FIR filter using polynomial based method.

3.1 Compensation Filter Specification Decision

Passband droop due to CIC filter can be given as

\[
d_c^k = \frac{\sin \left( \frac{nNf_m}{f_s} \right)}{N \sin \left( \frac{nNf}{f_s} \right)}
\]

(6)

where \( F_m \) is maximum input signal frequency of decimator and \( F_s \) is input signal sampling frequency. Passband droop shown in fig.5 will be used to decide passband ripple and order of compensation filter.

\[\text{Figure 5: Ideal frequency response of CIC filter}\]

3.2 Compensation FIR Filter Design

Compensation filter is designed using proposed method [8]:
1. First transform the required filter characteristics to a function, which we call as object function, by using a special transformation.
2. This object function is thereafter realized by using a previously defined set of polynomials.
3. The realized object function is then converted back to filter characteristics using inverse of the transform used in step 1.

The transformation and its inverse is used to design compensation filter in the present discussion [21]. Suppose we need to design a filter with characteristics \( H(\omega) \). First, we apply the inverse of the transformation on the filter characteristics \( H(\omega) \) and it results in an object function \( f(x) \) as shown in fig.6. Object function thus calculated and the desired filter definition is inverse of each other. Let us consider a polynomial as an object function where the values of \( x^n \) are all equal to 0; that is, all the zeros of the object function lie on the origin. In this case the polynomial considered as object function is

\[f(x) = x^6\]

(7)

The value of the function, where we want our stop band to start, can be taken at \( x = 1 \). Therefore, at the start of the stop band

\[x^6 = 1\]

(8)

Also the transformation \( x = x_0 \cos (\omega/2) \) is chosen here and \( x_0 \) is the maximum value of \( x \). Looking at the object function and transformation it is clear that \( x = 0 \) transforms to \( \omega = \pi \) and \( x = 1 \) transforms to \( \omega = 2 \cos^{-1}(1/x_0) \) which is the frequency where the stop-band starts.

\[\text{Fig 6: Graphical representation of polynomial } x^6\]

3.2.1 Calculation Of Stopband Frequency, \( \omega_s \), And Passband Frequency, \( \omega_p \) of Compensation Filter

Using the method given in [8] we can calculate the stopband frequency, passband frequency of the desired compensation filter. It is desired that at \( x = x_0 \) the value of the function \( f(x_0) \) should be \( b \) times its value than it has at the stopband. Thus,

\[(x_0)^b = b\]

(9)

So,

\[x_0 = (b)^{1/b}\]

(10)

As it is assumed that at the stop band frequency \( x=1 \) so putting these values in transformation equation

\[1 = (b)^{1/b}\]

(11)

and stopband frequency will be to

\[\omega_s = 2 \cos^{-1}(1/b^{1/b})\]

(12)

Similarly passband frequency will occur when function is at 3db down of its peak value than that at stop band so

\[x_p^b = b^{1/\sqrt{2}}\]

(13)

where \( x_p \) is corresponds to passband frequency in object function. Calculation gives the value of passband frequency

\[\omega_p = 2 \cos^{-1}(1/2^{1/2b})\]

(14)

Stopband frequency of compensation filter can be given by

\[\omega_s = 2 \cos^{-1}(1/b^{1/b})\]

(15)
Value of constant $b$ will be given as function of $p$ where $p$ is stopband attenuation value in dB. Stopband attenuation $p$ of compensation filter is known in advance from the CIC filter magnitude response. The maximum value of the object function is $b$ times its value than that at the stopband. So value of $b$ will be

$$b = 10^{p/20} \quad (16)$$

### 4. EXAMPLE APPLICATION

As an example we will consider a decimator system with specifications as listed in Table 1. In the first step a decimator is designed which decimates by 10 using CIC filter implements the rate change with differential delay $M = 1$ and number of stages $N = 9$.

**Table 1: Specification of a Decimator system**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Sampling Frequency</td>
<td>80Mhz</td>
</tr>
<tr>
<td>Output sampling frequency</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Passband edge</td>
<td>4 MHz</td>
</tr>
<tr>
<td>Passband ripple</td>
<td>0.05 dB</td>
</tr>
<tr>
<td>Stopband attenuation</td>
<td>80 dB</td>
</tr>
</tbody>
</table>

Large numbers of stages are required to achieve the stopband attenuation as specified. The frequency response of CIC filter is plotted using FDATOOL on MATLAB as shown in figure 7. It is clear from the frequency response that passband has a significant droop. Fig.8 shows the zoomed view of CIC filter response which shows the unavailability of well defined transition width. To overcome this droop and to define proper transition width a compensation filter is designed using proposed method.

Compensation filter order is decided by considering small passband ripple and narrow transition width given in table 1. Using proposed method passband frequency and stopband frequency are calculated. If $n = 110$ and value of $b = 10000$ if passband attenuation is 60 dB, corresponding values of stopband frequency and passband frequencies are of $\omega_s$ is 2.17622 radians and $\omega_p$ is 0.6733 radians respectively. Frequency response of cascaded structure can be plotted using normalized value of frequencies which will give less passband droop.

### 5. SIMULATION RESULT

**Figure 7: Frequency response of CIC filter for given specifications**

**Figure 8 : Zoomed view of frequency response of CIC filter for given specifications**
The compensation filter is a single rate FIR filter operating at 10 MHz. Its passband edge is decided to be 4.10 MHz by proposed method; slightly larger than the required passband edge 4.0 MHz. The filter order \( L \) is chosen to be 110. The large filter order is necessary to meet the small pass band ripple requirement and the narrow transition bandwidth requirement. Frequency responses are plotted using FDA Tools on MATLAB. Fig.9 shows the frequency response of compensation filter. Fig 10 shows the improved frequency response of cascade of CIC filter and compensation filter. Fig. 10 shows the improvement in stop band attenuation. Comparing the responses shows that compensation filter improvement is around 30dB in stopband attenuation. It is also evident from the responses that proposed compensation filter produces well defined transition width. Here polynomial based filter design is used, so less computation is required. Also as passband droop is calculated theoretically we can practically see that using proposed method better results are achieved [7].

6. CONCLUSION
As novel simple design is used for compensation filter less computation is required to design cascade of CIC and compensation filter now. On using proper polynomial passband droop can be reduced at maximum level and without much increase in complexity in comparison to previous proposed methods. In case of more passband droop we can use the polynomial of higher order. Proper selection of compensation filter also help to define frequency response more close to ideal values. Also multistage filtering can be done in order to reduce the sampling rate at first stage. VLSI implementation is also possible and implementation can be checked and verified on popular FPGA series as VIRTEX and SPARTAN etc.

7. REFERENCE


