Evaluation of the Performance of Ant Colony Optimization over Particle Swarm Optimization

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ABSTRACT

Traditional mathematical algorithms are incapable of solving real time engineering design problems because of its rigid procedure mainly due to discrete or random data and multiobjective functions in a problem. An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till the optimum or a satisfactory solution is found. There are two population based Swarm inspired methods in computational intelligence areas: Ant colony optimization (ACO) and Particle swarm optimization (PSO). This paper made an attempt to evaluate their performance of these two swarm intelligence techniques. A real engineering application of bevel gear design optimization is considered and results are analyzed with respect to the context.

General Terms

Ant Colony Optimization, Gear design, Particle Swarm Optimization.

Keywords

Ant Colony Optimization, Bevel Gear design, Multi Objective Optimization, Particle Swarm Optimization.

1. INTRODUCTION

One of the problems that obstruct the use of engineering optimization in design in the highly specific nature of commonly used numerical algorithms and their inborn limitations. For example, gradient based methods such as Newton Raphson, Bi-section and Secant methods which need differentiable objective function and constraints are the most striking examples of limitations. Also another important limitation is that they can hardly handle discrete variables which are commonly used in mechanical engineering design problems.

Majid Jaberipour and Esmaile Khorram [1] have described about two new harmony search meta-heuristic algorithms for engineering optimization problems with continuous design variables. Lin C et al [2] proposed a new immune algorithm makes use of the PSO advantages to improve the mutation mechanism in the immune algorithm. Wang et al [3] proposed an improved algorithm named bi-directional convergence ant colony optimization (ACO) algorithm based on the graphic definition of the job-shop problem. Davoud Sedighizadeh and Ellips Masehian [4] have surveyed PSO methods and its application. V. Savsani et al [5] have evaluated minimum weight of a spur gear train using particle swarm optimization (PSO) and simulated annealing (SA). Ruifeng Bo et al [6] have analyzed, concept optimization problem based on an Ant Colony System (ACS) and by analyzing the similarity between concept solving and Traveling Salesman Problem.

Zhou et al [7] proposed an ant colony algorithm to solve the prematurity and unsteadiness problem in GA for job shop scheduling with the objective of minimization make span. S. M. Kannan et al [8] have proposed a PSO for minimizing assembly variation in selective assembly problem. Shu-Kai S and Ju-Ming Chang [9] have developed a new proposal of using particle swarm optimization algorithms to solve multiobjective optimization problems was presented. Ju Seok Kang and Yeon-Sun Choi [10] proposed a new method is to optimize the helix angle of a helical gear to minimize the transmission error. Zhang Shaojun et al [11] have describe an optimized model is to formulate the maximization problem on the capacity of V-belt drive. Yin P. Y et al [12] derived a genetic PSO from the original PSO. It was incorporated with the genetic reproduction mechanisms, with crossover and mutation. Rania Hassan et al [13] attempted to examine the claim that PSO has the same effectiveness to finding the true global optimal solution as the GA.

The classical methods are inappropriate when the solution of the problem is computationally intensive. To get Pareto optimal solution for multi objective optimization, these methods have to be employed, many times with different weight or performance vectors [14]. Also these methods can handle either minimization or maximization only. The above difficulties of classical methods can be shorted out by using new Meta heuristics by adopting a Combined Objective Function.

Based on the above observation, this paper has made an attempt to use the potential of Particle swam optimization and Ant colony optimization to solve the bevel gear pair design problem.

2. PARTICLE SWARM OPTIMIZATION (PSO)

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. It has been applied successfully to wide variety of search and optimization problems. It can be applied to virtually any problem that can be expressed in terms of an objective function.

This PSO algorithm is initializing a number of particles randomly within the limits of the objective function. These particles are in a group known as the swarm. Each one particle represents a possible solution to the problem expressed by the objective function. While every time the objective function is evaluated to establish the fitness of each particle using its position as input. Fitness values are used to resolve which positions in the search space being attracted to both their personal best position as well as the best position found by the swarm so far. The PSO algorithm has shown its robustness and effectiveness in solving function value optimization problems in real engineering design. The attractiveness of the PSO algorithm is due to the features stochastic move, adaptively, and positive feedback.

2.1 Adaptation of PSO

There are two versions for keeping the neighbors best position, namely 1best and gbest. The flow diagram for this algorithm is shown in Fig. 1.[15]

In every iteration, each particle is updated by following two "best" values. The first one is best solution it has achieved; its value is called pbest. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called gbest. When particle takes part of the population as its neighbors, the best value is the local best and called lbest. In the local population, each particle keeps track of the best position lbest attained by its local neighboring particles. For the global population, the best position gbest is determined by any particles in the entire swarm. Thus the gbest model is a special case of the lbest model. Peng- Yeng Yin [16].

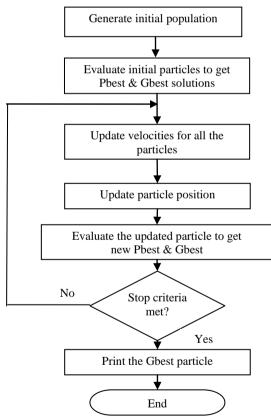


Fig 1: The algorithm for Particle Swarm Optimization

After finding the two best values, the particle updates its velocity and positions with following equation (1) and (2).[17] $v[] = \omega^*v[]+c 1^* rand()^* (pbest []- present[]) + c2^* rand()^* (gbest []- present[]) (1) present [] = present [] + v[] (2)$

Where,

- v [] : The velocity for the ith particle, represents the distance to be traveled by this particle from the current position.
- ω inertia weights usually 0.8 to 0.9 .
- rand () is a random number between (0,1)
- c1, c2 are learning factors .Usually c1 = c2 = 2.
- Present []: The location of the ith particle i.e., particle position.
- Pbest []: The best previous position of the ith particle is recorded and represented as pbest[].
- Gbest []: The index of the best particle among all the particles in the population is represented by gbest [].

3. ANT COLONY OPTIMIZATION (ACO)

The natural image on which ant algorithms are based is that of ant colony. The cooperative search performance of ants inspires the new algorithm for optimizing engineering design system and is especially suited for solving comprehensive optimization problems Dorigo et al [18]. A bi-level search procedure such as local and global have been introduced and this algorithm is applied for continuous function, it is called as Continuous Ants Colony Algorithm (CACO) Jayaram et al [19]. The distribution of ants is given in Fig. 2 and the flow chart of Ant colony algorithm is shown in Fig. 3. [15]

3.1. Global search

The initial solutions or initial regions for ACO are classified into superior and inferior solutions based on their fitness values. Global updates are applied to inferior regions only. The global search for ACO is fairly different from other nontraditional optimization algorithms. The following three steps are to be performed on the randomly generated initial solution. Baskar et al [20].

- Random walk (or) Crossover
- Mutation
- Trail diffusion

3.1.1. Random walk (or) Crossover

The crossover and mutation have been carried out in the "G" global ants, where 90% of the solutions are randomly selected in the inferior solutions and are replaced with randomly selected solutions from the superior solutions.

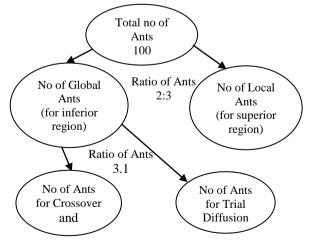


Fig 2: Distribution of Ants for Local and Global search

3.1.2. Mutation

Following to the random walk, at random, add or subtract the value to every variable of the newly created solutions in the inferior region with a probability equal to a properly defined mutation probability. The mutation size is reduced as per the relation

$$\Delta(T, R) = R (1 - r^{(1-T)b})$$
(3)

Where r is a random number from $\{0,1\}$, R is the maximum step size, T is the ratio of the current iteration number to that of the total number of iterations, and b is a positive parameter controlling the degree of non-linearity. The value of b is considered as 10 and determined on a trial basis.

$$R = X_{max} - X_i$$
(4)
There X_{max} is the maximum value in the region

 X_i is the current value

3.1.3. Trail Diffusion

W

In Trail diffusion conducted on 10% of inferior solutions, which were not measured during the random walk and mutation step. Here, two parents are selected at random from the present parent superior solutions.

The variables of the child's position vector can have either

- the value of the equivalent variable from the first parent;
- the equivalent value of the variable of the variable form second parent;
- or, a combination arrived from a weighted average of the above

 $x \text{ (child)} = (\alpha).x_i(\text{parent}_1) + (1 - \alpha).x_i(\text{parent}_2)$ (5) Where α is a uniform random number in the range [0,1].

The probability of selecting the third option is set equal to the mutation probability while allotting equal probability of selecting the first two steps. The trail value of the newly created child solutions is assigned a trail value lying between the values of the original parent solutions. The trail values and age of weakest regions are updated [20].

3.2. Local search

The second step in ACO is local search, which is applied as superior solution. The local (artificial) ants select a region i with a probability.

$$P_{i}(t) = \frac{\tau_{i}(t)}{\sum_{t} \tau_{k}(t)}$$
(6)

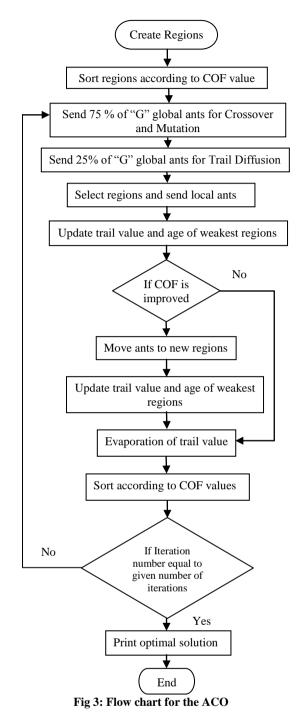
Where i is the region index and $\tau_i(k)$ is the pheromone trail on region i at time t.

The best function values are taken from the following steps. After selecting the region, the ant moves through a short distance, the direction of movement is retained if the fitness value progress is observed or else it is reversed. Likewise, the solutions position vector is updated and trail value is enhanced based on the fitness value.

In the ACO algorithm, the pheromone values are decreased after each iteration using the relation,

$$\tau_i \left(t + 1 \right) = \rho \cdot \tau_i \left(t \right) \tag{7}$$

Where ρ is the evaporation rate which is assumed to be 0.2 on a trial basis and $\tau_i(t)$ is the trail associated with solution at time t. The algorithm executes until the termination criteria is reached and it is the number of iterations by defined the user.



4. AN APPLICATION: GEAR DESIGN

The performance of the PSO and ACO are evaluated by taking Bevel gear design as the objective. The gear design problem are taken as, Design a Bevel gear drive to transmit 4kW with the Speed Ratio 4. The input shaft speed is 225 rpm, with nonreversible. This section contains design objectives, objective functions and design constraints are to be discussed.

4.1 Design Objectives

The objective functions considered in this application are given below:

• Maximization of power delivered by the bevel gear pair (f₁) Jain et al [21].

- Minimization of the overall weight which is indirectly related to the volume of the gears (f₂) Rao et al [22].
- Maximization of the efficiency of the gear pair (f₃) [23].
- Minimization of the cone distance (f₄).Kennady et al [24].

4.1.1 Objective functions

Maximization of power transmitted by bevel gear pair. Eqn. (8) represents this objective function.

$$f_I = P \quad \text{where , } P^{(L)} \le P \le P^{(U)} \tag{8}$$

Minimization of weight of the bevel gear pair. Eqn. (9) represents this objective function.

$$f_2 = \text{Weight} = 9.24 \text{ x } 10^{-6} [0.2838\text{b}^3 - 1.762\text{m}_{\text{t}} \text{Z}_1 \text{ b}^2 + 3.6369\text{m}_{\text{t}}^2 \text{Z}_1^2 \text{ b}]$$
(9)

Maximization of efficiency of gear pair. Eqn. (10) represents this objective function.

$$f_3 = 100 - P_L$$
(10)

 P_L = Power loss and it is expressed by the eqn. (11).

$$P_{\rm L} = 50 \text{ f } x \left(\frac{\cos \Theta + \cos \gamma}{\cos \Phi_{\rm n}} \right) x \frac{({\rm H}_{\rm s}^2 + {\rm H}_{\rm t}^2)}{({\rm H}_{\rm s} + {\rm H}_{\rm t})}$$
(11)
H_e and H_e are calculated by the eqns. (12) and (13)

 H_s and H_t are calculated by the eqns. (12) and (13) respectively.

$$H_{\rm S} = i + 1 \left(\sqrt{\left(\frac{R_0}{R}\right)^2 - \cos^2 \Phi_{\rm n}} \right) - \sin \Phi_{\rm n} \qquad (12)$$

$$H_{t} = \frac{i+1}{i} \left(\sqrt{\left(\frac{r_{0}}{r}\right)^{2} - \cos^{2} \Phi_{n}} \right) - \sin \Phi_{n}$$
(13)

$$R_{0} = R + \text{one addendum}$$

One addendum for 20° full depth involute system = one average module = m_{av}

Where,

 m_{av} = average module of gear and pinion $r_o = r + m_{av}$ $Ro = R + m_{av}$

$$r_o = \frac{d_1}{2} + m_{av}$$
$$r = \frac{d_1}{2}$$

 d_1 = Pitch diameter of the large end of bevel pinion in mm = m_t Z_1

$$R_o = \frac{d_2}{2} + m_{av}$$
$$R_2 = \frac{d_2}{2}$$

 d_2 = Pitch diameter of the large end of bevel gear in mm = $m_t Z_2$

Minimization of cone distance of gear pair. Eqn. (14) represents this objective function.

$$f_4 = R = 0.5 \text{ m}_{\text{t}} \text{ Z}_1 \sqrt{\text{i}^2 + 1} \tag{14}$$

4.1.2 Design Constraints

The constraints [25] considered in this gear design are given below:

The crushing stress constrain is represented by the expression (15)

Crushing stress: $\sigma_c < [\sigma_c]$ (15) The value of induced crushing stress is represented by the equation (16)

$$\sigma_{\rm c} = \frac{0.72}{\rm R-0.5b} \sqrt{\frac{\sqrt{(i^2 \pm 1)^3} \, \rm E[M_t]}{\rm ib}}$$
(16)

The bending stress constrain is represented by the expression (17)

Bending stress:
$$\sigma_{\rm b} < [\sigma_{\rm b}]$$
 (17)

The value of induced bending stress is represented by the equation (18)

$$\sigma_{\rm b} = \frac{R\sqrt{(i^2+1)} \ [M_t]}{(R-0.5b)^2 \ \rm bmy} \ X \ \frac{1}{\cos \Phi_{\rm n}}$$
(18)

Equation (19) represents the gear ratio constraint *Gear ratio:*

$$i = 4 = \frac{Z_2}{Z_1} (or) \frac{d_2}{d_1}$$
(19)

Equation (20) represents the cone distance constraint

Cone distance :
$$R \ge Rmin$$
 (20)
Where

 R_{min} = Minimum cone distance calculated by the formula based on surface Compressive stress.

The minimum cone distance is represented by the eqn. (21).

$$R_{min} = \psi_y \sqrt{(i^2 + 1)} \sqrt[3]{\left(\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right)^2 \frac{E[M_t]}{i}}$$
(21)

 Ψ_y = Ratio between the cone distance and face width to calculate the value of 'R_{min}'.

The number of teeth constraint is represented by the equation (22)

The number of teeth must be integer:

$$Z_i \in I$$
, for $i = 14, 15, 16, 17, 18, 19, 20$ (22)

The *module* constraint is represented by the expression (23) $m_{av} \ge m_{av \ min}$ (23)

Where, m_{av} = average module

 $m_{av min}$ = minimum average module calculated by the formula based on bending stress

The minimum average module $m_{av min}$ is represented by the eqn. (24).

$$m_{avmin} = 1.28 \sqrt[3]{\frac{[M_t]}{y[\sigma_b]\psi_m Z_1}}$$
(24)

 Ψ_m = Ratio between the face width and the average module to calculate the value of ' $m_{av min}$ '.

4.2. Design Objective Function

In this gear pair design problem has four different parameters: maximization of power, minimization of weight of material, maximization of efficiency and minimization of center distance. Since all this parameters are on different scales, in multiple-criterion objective function it is normalized to the same scale [26]. For maximizing criterion value, it is normalized by dividing its value with the normalizing factor, max; which is the maximum value of this criterion obtained from the solutions that have been explored by PSO / ACO so far and for a minimizing criterion value, it is normalized by dividing the normalizing factor, min_i, with its value. The maximum and minimum value of the criterion will be updated whenever the algorithm finds another feasible solution. In addition, to ensure the overall objective value will fall between 0 and 1, the weight of each criterion is also normalized. The normalized objective function can be obtained as follows:

$$COF = \sum_{i=1}^{n} NW_i * N(X_i)$$
⁽²⁵⁾

Where,

Where
$$NW_i = \frac{W_i}{(\sum_{i=1}^n W_i)}$$

 $N(X_i)$ normalized value of criterion i of solution X. Where,

$$N(X_i) = \frac{X_i}{\max_i} \text{ for maximizing criterion.}$$
(26)

$$N(X_i) = \frac{\min_i}{X_i} \text{ for minimizing criterion.}$$
(27)

X_i pre normalized value of criterion X.

 $\ensuremath{\mathsf{max}}_i$ best maximum $\ensuremath{\mathsf{ pre normalized}}$ value of criterion i of all solutions so far .

 $\min_i \;\; best \ minimum \;\; pre normalized \;\; value \; of \; criterion \; i \; of \; all solutions so far .$

N number of criteria.

Hence the COF for bevel gear design is, COF -

$$\left(\frac{\text{power}}{\text{max. power}}\right) \times \text{NW}_{1} + \left(\frac{\text{min. weight}}{\text{weight}}\right) \times \text{NW}_{2} + \left(\frac{\text{efficiency}}{\text{max. efficiency}}\right) \times \text{NW}_{3} + \left(\frac{\text{min. cent. dist}}{\text{cent. dist}}\right) \times \text{NW}_{4}$$
(28)

Where NW_1 , NW_2 , NW_3 and $NW_4 = 0.25$

5. RESULTS AND DISCUSSION

Mathematical models for the complete problem of bevel gear pair have been formulated in terms of design variables. The optimum values of the objective functions for Power, Weight, Efficiency and Center distance and the design variables (m, b, z and P) influencing the objective functions are obtained with respect to the minimum COF value by implementing the input data provided in Table 1.

Table.1. Da	ta considered	l in	design
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Table.1. Data considered in design			
Parameter/Constraint	Values for bevel gear pair		
Material of gear and pinion	40Ni2Cr1Mo 28		
Density of the material	8.836×10 ⁶ kg/mm ³		
Gear ratio	4		
Power delivered	4 to 6 kW		
Allowable bending stress	400N/mm ²		
Allowable crushing stress	1100N/mm ²		
Input speed	225 rpm		
Young's modulus of the material	$2.15 \times 10^5 \text{ N/mm}^2$		
Normal pressure Angle	20°		
Co-efficient of friction	0.08		
Ratio between cone distance and face width to calculate the value of ' R_{min} ' (Ψ_{v})	4		
Ratio between face width and average module to calculate the value of $m_{av min}$, (Ψ_m)	10		

The test bevel gear problem is carried for the module range 4mm to 6mm and the bevel gear design optimization carried by the tools PSO and ACO. The optimized values of required functions are tabulated in the Table 2. From the optimum results, power and weight are compared for ACO and PSO and the chart shown in the figures 4 and 5.

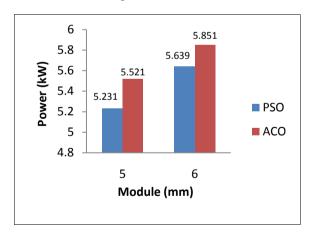


Fig 4: Comparison of Power Transmitted

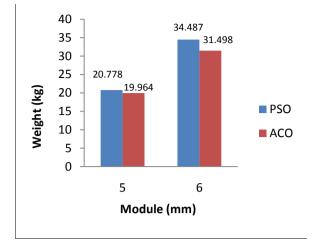


Fig 5: Comparison of Weight

It is clearly understood from the figure 4 and 5 that ACO gives the higher Power and lesser Weight while comparing with PSO. The Power gained by 5.5% and 3.5% with respect to 5mm and 6mm modules respectively. Similarly, Weight reduced by 3.9% and 8.7% with respect to 5mm and 6mm modules respectively. Both the tools give the same maximum Efficiency and minimum Cone distance.

6. CONCLUSION

In this paper an attempt has been made to evaluate the performance of Ant Colony Optimization and Particle Swarm Optimization with a real time engineering application. As part of this, bevel gear design is considered and to obtain optimal solution of bevel gear design problem. Within the various design variables available for a gear pair design, the power, weight, efficiency and center distance have been considered as objective functions and bending stress, crushing stress as vital constraints to get an efficient compact and high power transmitting drive. Among the algorithms ACO has produced convincing results for the test problems when compared with the PSO algorithms in the power transmitted and weight reduction of the bevel gear. As a future work, the above tools can be evaluated by optimizing various design application like springs, epicyclic gear train and gear box etc.

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Notations

PSO	: Particle Swarm Optimization
ACO	
CACO	: Continuous Ant Colony Optimization
Р	: Power transmitted in kW
Hs	: Specific sliding velocity at start of approach action
Ht	: Specific sliding velocity at end of recess action.
i	: Gear (or) transmission ratio
z1, z2	: Number of teeth in pinion, gear
d1, d2	: PCD of large end of pinion, gear in mm
Ro	: Outside radius of large end of bevel gear in mm
R_2	: Pitch radius of large end of bevel gear in mm
Ro	: Outside radius of large end of bevel Pinion in mm
r	: Pitch radius of large end of bevel Pinion in mm
ρ	: Density of the material in kg/mm ³
E	: Young's modulus in N/mm ²
mt	: Transverse Module in mm
σc	: Induced crushing stress in N/mm ²
[σc]	: Allowable crushing stress in N/mm ²
σb	: Induced bending stress in N/mm ²
[σb]	: Allowable bending stress in N/mm ²
b	: face with of gear and pinion in mm
R	: Cone distance in mm.
PL	: Percent power loss
[Mt]	: Design twisting moment in N mm
η	: Efficiency, %
у	: Form factor
f	: Average coefficient of friction
Φn	: Normal pressure angle in degrees
θ	: Pitch cone angle of bevel gear
γ	: Pitch cone angle of bevel pinion