Acyclic Coloring of Central Graphs

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ABSTRACT
The purpose of this study is to discuss the acyclic coloring and acyclic chromatic number of central graph of cycles(C_n), complete bipartite graphs(K_{m,n}), and complete graphs(K_n), denoted respectively by C(C_n), C(K_{m,n}) and C(K_n).

KEYWORDS
Central Graph, Acyclic Coloring and Acyclic Chromatic Number.

1. INTRODUCTION
Let G be a finite undirected graph with no loops and multiple edges. The Central Graph C(G) [3] of a graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

By definition, |E(G)| = p+q. For any (p, q) - graph there exists actually p vertices of degree (p-1) and q vertices of degree 2 in C(G).

A proper vertex coloring of a graph is acyclic [1] if every cycle has at least three colors. The acyclic chromatic number of G, denoted by a(G), is the minimum k such that G admits an acyclic k – Coloring.

2. THE ACYCLIC COLORING OF C [C_n]
2.1 Theorem
For any cycle C_n of length n ≥ 5, a[C(C_n)] = n-1.

Proof:
Let C_n be any cycle of length n with vertices V_1, V_2 . . . V_n.
Let $V^i_{ij}$ represents the newly introduced vertices in the edge connecting V_i and V_j. In C(C_n), the vertex V_i is adjacent with all the vertices of C_n except V_{i-1} and V_{i+1}, i = 2, 3, . . . , n-1.
V_i is also adjacent to the newly added vertices $V^i_{i+1}$ and $V^i_{i-1}$. Consider the acyclic coloring of C(C_n) as follows:

Since all the newly added vertices in C(C_n) forms an independent set, we can assign color C to all these vertices.

(i-e) assign C to V_{ij}, 1 ≤ i, j ≤ n.
Assign C_1 to V_1 and V_2 as they are non – adjacent in C(C_n).
Assign C_2 to V_3 and V_4 as they are non – adjacent.
Assign C_i to V_{i+1} , 3 ≤ i ≤ n-2.

Suppose if we assign C_1 to V_3 and V_6, then the cycle V_1, V_3, V_2, V_6, V_1 will form a bichromatic (C_1 – C_6) cycle. By the same argument, we can not assign the same color to the non – adjacent vertices V_i and $V^i_{i+1}$, 5 ≤ i ≤ n.

We will discuss the following cases.

Case (i): C and C_1
The induced subgraph of these color classes contains a bichromatic Path, V_1, V_12, V_2, V_23.
Since V_1 and V_2 as well as V_1 and V_23 are non–adjacent, there is no (C – C_1) Cycle in C(C_n).

Case (ii): C and C_2
By the same argument as in case (i), there is no (C – C_2) cycle in C(C_n).

Case (iii): C_1 and C_2
The color class of C_1 is {V_1, V_2} and that of C_2 is {V_3, V_4}. The induced sub graph of C_1 and C_2 contains a bichromatic path V_3, V_1, V_4, V_2 and since V_2, V_3 are non–adjacent, there is no bichromatic (C_1 – C_2) cycle in C(C_n).

Case (iv):
Since V_1 and V_2 are non–adjacent, there exists no bichromatic (C_1 – C_i) (3 ≤ i ≤ n-2) cycle in C(C_n). Similarly, there exists no bichromatic (C_2 – C_i) (3 ≤ i ≤ n-2) cycle in C(C_n). Thus, C(C_n) has no bichromatic cycle in the above coloring and hence, a(C(C_n)) = n-1, n ≥ 7.

Remark:
a(C(C_n)) = n-1 for any odd n > 3 and
= n-1 for all even n except 6

In fact, a(C(C_6)) = 4

Fig 1: a[C(C_7)] = 6
3. THE ACRYCLIC COLORING OF C (Km,n):

3.1 Theorem

For any complete bipartite graph C (Km,n) (m ≤ n and n ≥ 3), a [C (Km,n)] = n.

Proof

Consider the complete bipartite Graph Km,n (m ≤ n, n ≥ 3) with bipartition (X, Y) where X = {V1, V2, ..., Vm} and Y = {U1, U2, ..., Un}. Let Wij represents the newly introduced vertex in the edge joining Vi and Uj in C (Km,n). The < Vi, i = 1 to m > is a complete sub graph in C (Km,n). Similarly, < Uj, j = 1 to n > is also a complete sub graph in C (Km,n). The Set {Wij, 1 ≤ i ≤ m, 1 ≤ j ≤ n} form an independent set in C (Km,n).

Now, Assign a Coloring to the Vertices of C (Km,n) as follows:
Assign Cj to Vi, i = 1 to m and Cj to Uj, j = 1 to n.

For the newly added vertices Wij, assign colors such that Cj to Wij where i,j,k ≤ n and Cj to Wij and Ck to Wij where k ≤ n, 1 ≤ i,j ≤ n.
Suppose we assign Cj to Wij and Cj to Wij, then the cycle Vj,Wj,Ui,Wj,Vi,Vj becomes a bichromatic (Cj – Cj) cycle in C (Km,n). Hence, assign Cj (k=2) to Wij when Cj is assigned to Wij, 1 ≤ i,j,k ≤ n. C (Km,n) has no bichromatic Cycle. Therefore, a [C (Km,n)] = n,m ≤ n and n ≥ 3.

4. THE ACRYCLIC COLORING OF C (Kn)

4.1 Theorem

For any complete graph Kn (n ≥ 3), a (C (Kn)) = \( \left\lceil \frac{n}{2} \right\rceil + 1 \) if n is even
= \( \left\lceil \frac{n}{2} \right\rceil \) + 2 if n is odd.

Proof

Consider a complete Graph Kn on n vertices (n ≥ 3). Let Wij represents the newly introduced vertex in the edge joining Vi and Vj. The vertices Wij and Wij represents the same vertex). The Set {Wij, 1 ≤ i ≤ n, 1 ≤ j ≤ n} is an independent set in C (Kn).

Now, assign a coloring to the vertices of C (Kn) as follows.
Assign C to Wij for all i and j, 1 ≤ i,j ≤ n. Assign same color to exactly two vertices of {Vj, 1 ≤ i ≤ n} Suppose if we assign same color, Say C’, to three Vertices Vj, Vi and Vj, then the cycle Vj,Wj,Vj,Wj,Vj becomes a bichromatic (C’ – C’) cycle. Hence, assign the same color to exactly two vertices of {Vi, 1 ≤ i ≤ n} So, there is no bichromatic cycle in C (Kn). Therefore, a [C (Kn)] = \( \left\lceil \frac{n}{2} \right\rceil + 1 \) if n’ is even.
= \( \left\lceil \frac{n}{2} \right\rceil \) + 2, if n are odd.
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6. CONCLUSION:
In this present study, we have provided new results concerning the acyclic chromatic number of different family of graphs. In particular, we have provided exact results for central graph of cycles, complete bipartite graphs and complete graphs. Now, we are working on the central graph of Bistar graph Families.

7. REFERENCES: