On Intuitionistic Fuzzy - Generalized Semi Continuous Mappings

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ABSTRACT
In this paper we have introduced intuitionistic fuzzy generalized semi continuous mappings and some of their basic properties are studied.

Keywords
Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi closed sets, intuitionistic fuzzy generalized semi continuous mappings, intuitionistic fuzzy topology, intuitionistic fuzzy topology, intuitionistic fuzzy continuous mappings.

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1. INTRODUCTION
The concept of fuzzy sets was introduced by Zadeh [13] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduce intuitionistic fuzzy generalized semi continuous mappings and studied some of their basic properties. We arrive at some characterizations of intuitionistic fuzzy generalized semi continuous mappings.

2. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = { (x, μA(x), νA(x)) / x ∈ X} where the functions μA(x): X → [0, 1] and νA(x): X → [0, 1] denote the degree of membership (namely μA(x)) and the degree of non-membership (namely νA(x)) of each element x ∈ X to the set A, respectively and 0 ≤ μA(x) + νA(x) ≤ 1 for each x ∈ X. Denote the set of all intuitionistic fuzzy sets in X by IFS(X).

Definition 2.2: [1] Let A and B be IFS’s of the form A = { (x, μA(x), νA(x)) / x ∈ X} and B = { (x, μB(x), νB(x)) / x ∈ X}. Then
(a) A ⊆ B if and only if μA(x) ≤ μB(x) and νA(x) ≥ νB(x) for all x ∈ X.
(b) A = B if and only if A ⊆ B and B ⊆ A.
(c) A' = { (x, μA(x), νA(x)) / x ∈ X}.
(d) A \ B = { (x, μA(x) ∧ μB(x), νA(x) ∨ νB(x)) / x ∈ X}.
(e) A ∪ B = { (x, μA(x) ∨ μB(x), νA(x) ∧ νB(x)) / x ∈ X}.

For the sake of simplicity, we shall use the notation A = (x, μA(x), νA(x)) for any G / G is an IFS in X satisfying the following axioms:

(a) 0, 1, ∈ τ
(b) G1 \ G2 ∈ τ, for any G1, G2 ∈ τ
(c) τ ∈ τ for any arbitrary family (Gi / i ∈ J) ⊆ τ.

In this case the pair (X, τ) is called an intuitionistic fuzzy topology (IFT in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

The complement A’ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.3: [3] Let (X, τ) be an IFTS and A = { (x, μA(x), νA(x)) / x ∈ X} be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

int(A) = ∪ { G / G is an IFOS in X and G ⊆ A },

cl(A) = ∩ { K / K is an IFCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have cl(A') = [int(A)]' and int(A') = [cl(A)].

Definition 2.4: [3] An IFS A = { (x, μA(x), νA(x)) / x ∈ X} is said to be an
(i) intuitionistic fuzzy semi open set (IFSOS in short) if A ⊆ cl(int(A)),

(ii) intuitionistic fuzzy semi closed set (IFscS in short) if A ⊆ int(cl(A))).
(iii) intuitionistic fuzzy regular open set (IFROS in short) if
$$\text{A} = \text{int}(\text{cl}(\text{A})).$$

The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS \((X, \tau)\) is denoted by  
\(\text{IFO}(X)\) (respectively IF SOS(X), IFαO(X), IFRO(X)).

**Definition 2.6:** [7] An IFS \(A = \langle x, \mu_A, \nu_A \rangle\) in an IFTS \((X, \tau)\) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if \(\text{int}(\text{cl}(\text{A})) \subseteq A\).

(ii) intuitionistic fuzzy \(\alpha\)-closed set (IFαCS in short) if \(\text{cl}(\text{int}(\text{cl}(\text{A}))) \subseteq A\).

(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if \(A = \text{cl}(\text{int}(\text{cl}(\text{A}))\).

The family of all IFCS (respectively IFSCS, IFGRCS) of an IFTS \((X, \tau)\) is denoted by IFC(X) (respectively IFSC(X), IFGC(X), IFRC(X)).

**Definition 2.7:** [12] Let \(A\) be an IFS in an IFTS \((X, \tau)\). Then

\[
\text{sint}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},
\]

\[
\text{scl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.
\]

Note that for any IFS \(A\) in \((X, \tau)\), we have \(\text{scl}(A^c) = (\text{sint}(A))^c\) and \(\text{sint}(A^c) = (\text{scl}(A))^c\).

**Definition 2.8:** [11] An IFS \(A\) in an IFTS \((X, \tau)\) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X\).

(ii) intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if \(\text{cl}(\text{int}(\text{cl}(A))) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFROS in \(X\).

**Definition 2.9:** [10] An IFS \(A\) in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \((X, \tau)\).

**Definition 2.10:** [10] An IFS \(A\) is said to be an intuitionistic fuzzy generalized semi open set (IFGOS in short) in \(X\) if the complement \(A^c\) is an IFGCS in \(X\).

The family of all IFGCSs (IFGOSs) of an IFTS \((X, \tau)\) is denoted by IFGCS(X) (IFGOS(X)).

**Definition 2.11:** [8] An IFS \(A\) in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy generalized open set (IFGOS in short) if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFPGCS in \((X, \tau)\).

Result 2.12: [8] Every IFCS, IFGCS, IFRCs, IFαCS, IFGSC, is an IFγGCS but the converses may not be true in general. Every IFαCS is IFGCS but the converse is need not be true.

Definition 2.13: [9] An IFS \(A\) is said to be an intuitionistic fuzzy alpha generalized open set (IFαGOS in short) in \(X\) if the complement \(A^c\) is an IFαGCS in \(X\).

The family of all IFαGCSs (IFαGOSs) of an IFTS \((X, \tau)\) is denoted by IFαGC(X) (IFαGO(X)).

**Definition 2.14:** [5] Let \(f\) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \(f\) is said to be intuitionistic fuzzy continuous (IF continuous in short) if \(f^{-1}(B) \in \text{IFO}(X)\) for every \(B \in \sigma\).

**Definition 2.15:** [7] Let \(f\) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \(f\) is said to be

(i) intuitionistic fuzzy semi continuous (IFS continuous in short) if \(f^{-1}(B) \in \text{IFS}(X)\) for every \(B \in \sigma\).

(ii) intuitionistic fuzzy \(\alpha\)-continuous (IFα continuous in short) if \(f^{-1}(B) \in \text{IFαO}(X)\) for every \(B \in \sigma\).

(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if \(f^{-1}(B) \in \text{IFPO}(X)\) for every \(B \in \sigma\).

Result 2.16: [7] Every IF continuous mapping is an IFγ-continuous mapping and every IFα-continuous mapping is an IFG continuous mapping.

Definition 2.17: [6] A mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy \(\gamma\) continuous (IFγ continuous in short) if \(f^{-1}(B) \in \text{IFγOS in } (X, \tau)\) for every \(B \in \sigma\).

Definition 2.18: [10] Let \(f\) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \(f\) is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if \(f^{-1}(B) \in \text{IFGCS}(X)\) for every IFCS \(B\) in \(Y\).

Result 2.19: [10] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.20: [9] A mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy generalized semi continuous (IFG continuous in short) if \(f^{-1}(B) \in \text{IFGCS in } (X, \tau)\) for every IFCS \(B\) of \((Y, \sigma)\).

**Definition 2.21:** [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \(\gamma\) generalized semi continuous (IFγT1/2 in short) space if every IFγGCS in \(X\) is an IFCS in \(X\).

**Definition 2.22:** [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \(\gamma\) generalized semi open set (IFγT1/2 in short) space if every IFγGOS in \(X\) is an IFGCS in \(X\).
3. INTUITIONISTIC FUZZY \( \pi \) - GENERALIZED SEMI CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy \( \pi \) - generalized semi continuous (IF\( \pi \)GS continuous in short) if \( f^{-1}(B) \) is an IF\( \pi \)GS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

**Definition 3.1:** A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \pi \) - generalized semi continuous (IF\( \pi \)GS continuous in short) if \( f^{-1}(B) \) is an IF\( \pi \)GS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

**Example 3.2:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = (x, (0.4, 0.2), (0.6, 0.7)) \), \( G_2 = (y, (0.7, 0.8), (0.3, 0.2)) \). Then \( \tau = \{0., \ G_1, 1.\} \) and \( \sigma = \{0., \ G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \pi \)GS continuous mapping.

**Theorem 3.3:** Every IF continuous mapping is an IF\( \pi \)GS continuous mapping but not conversely.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF continuous mapping.

Let \( A \) be an IFCS in \( Y \). Since \( f \) is an IFG continuous mapping, \( f^{-1}(A) \) is an IFGCS in \( X \). Hence \( f \) is an IF\( \pi \)GS continuous mapping.

**Example 3.4:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = (x, (0.1, 0.1), (0.8, 0.6)) \), \( G_2 = (y, (0.8, 0.6), (0.1, 0.1)) \). Then \( \tau = \{0., \ G_1, 1.\} \) and \( \sigma = \{0., \ G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \pi \)GS continuous mapping.

**Theorem 3.5:** Every IF\( \alpha \) continuous mapping is an IF\( \pi \)GS continuous mapping but not conversely.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \alpha \) continuous mapping.

Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IF\( \alpha \)CS in \( X \). Since every IF\( \alpha \)CS is an IF\( \pi \)GS, \( f^{-1}(A) \) is an IF\( \pi \)GS in \( X \). Hence \( f \) is an IF\( \pi \)GS continuous mapping.

**Example 3.6:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = (x, (0.7, 0.8), (0.3, 0.1)) \), \( G_2 = (y, (0.2, 0.3), (0.5, 0.4)) \). Then \( \tau = \{0., \ G_1, 1.\} \) and \( \sigma = \{0., \ G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( G_2 = (y, (0.5, 0.4), (0.2, 0.3)) \) is IFCS in \( Y \). Then \( f^{-1}(G_2) \) is IF\( \pi \)GS in \( X \) but not IF\( \alpha \)CS in \( X \). Then \( f \) is IF\( \pi \)GS continuous mapping but not an IF\( \alpha \) continuous mapping.
Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)G continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF\( \pi \)GCS in \( X \). Since every IF\( \pi \)GCS is an IF\( \pi \)GCS and every IF\( \pi \)GCS is an IF\( \pi \)GCS, \( f^{-1}(A) \) is an IF\( \pi \)GSCS in \( X \). Hence \( f \) is an IF\( \pi \)G continuous mapping.

Example 3.14: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ x, (0.4, 0.6), (0.2, 0.2) \} \), \( G_2 = \{ y, (0.6, 0.2), (0.4, 3) \} \). Then \( \tau = \{ 0, G_1 \} \) and \( \sigma = \{ 0, G_2 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( G_2^c = \{ y, (0.4, 0.3), (0.6, 0.2) \} \) is IFCS in \( Y \).

Then \( f^{-1}(G_2^c) \) is IF\( \pi \)GCS in \( X \) but not IF\( \pi \)GCS in \( X \). Then \( f \) is IF\( \pi \)G continuous mapping but not an IF\( \pi \)G continuous mapping.

Remark 3.15: IFP continuous mapping and IF\( \pi \)G continuous mapping are independent to each other.

Example 3.16: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ x, (0.2, 0.4), (0.5, 0.4) \} \), \( G_2 = \{ y, (0.1, 0.3), (0.3, 0.4) \} \), \( G_3 = \{ x, (0.1, 0.3), (0.5, 0.4) \} \), \( G_4 = \{ x, (0.2, 0.4), (0.3, 0.4) \} \), \( G_5 = \{ x, (0.4, 0.4), (0.3, 0.4) \} \), \( G_6 = \{ y, (0.5, 0.4), (0, 0.3) \} \) and \( \sigma = \{ 0, G_1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is IFP continuous mapping but not an IFP continuous mapping since \( G_6^c = \{ y, (0, 0.3), (0.5, 0.4) \} \) is an IFCS in \( Y \) but \( f^{-1}(G_6^c) = \{ x, (0, 0.3), (0.5, 0.4) \} \) is not IFPCS in \( X \).

Remark 3.18: IF\( \pi \) continuous mapping and IF\( \pi \)G continuous mapping are independent to each other.

Example 3.19: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ x, (0.1, 0.3), (0.4, 0.3) \} \), \( G_2 = \{ x, (0.2, 0.3), (0.2, 0.3) \} \), \( G_3 = \{ x, (0.2, 0.3), (0.4, 0.3) \} \), \( G_4 = \{ x, (0.1, 0.3), (0.2, 0.3) \} \), \( G_5 = \{ x, (0.3, 0.3), (0.2, 0.3) \} \) and \( \sigma = \{ 0, G_1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is IF\( \pi \) continuous mapping but not an IF\( \pi \)G continuous mapping since \( G_6^c = \{ x, (0, 0.1), (0.3, 0.4) \} \) is an IFCS in \( Y \) but \( f^{-1}(G_6^c) = \{ x, (0, 0.1), (0.3, 0.4) \} \) is not IFGSCS in \( X \).

Example 3.20: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ x, (0.5, 0.1), (0.5, 0.9) \} \), \( G_2 = \{ y, (0.2, 0.1), (0.7, 0.8) \} \). Then \( \tau = \{ 0, G_1 \} \) and \( \sigma = \{ 0, G_2 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is IF\( \pi \)G continuous mapping but not an IF\( \pi \)G continuous mapping since \( G_6^c = \{ y, (0.7, 0.8), (0.2, 0.1) \} \) is an IFCS in \( Y \) but \( f^{-1}(G_6^c) = \{ x, (0, 0.7), (0.2, 0.1) \} \) is not IF\( \pi \)CS in \( X \).

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram ‘cts’ means continuous.

![Relation between intuitionistic fuzzy π - generalized semi continuous mappings and other existing intuitionistic fuzzy mappings.](image)

In this diagram by “A \( \rightarrow \) B” we mean A implies B but not conversely and “A \( \rightarrow \) B” means A and B are independent of each other.

None of them is reversible.

Theorem 3.21: A mapping \( f : X \rightarrow Y \) is IF\( \pi \)G continuous then the inverse image of each IFOS in \( Y \) is an IF\( \pi \)GOS in \( X \).

Proof: Let \( A \) be an IFOS in \( Y \). This implies \( A^c \) is IFCS in \( Y \). Since \( f \) is IF\( \pi \)G continuous, \( f^{-1}(A^c) \) is IF\( \pi \)GCS in \( X \). Since \( f^{-1}(A^c) = f^{-1}(A^c) \), \( f^{-1}(A) \) is an IF\( \pi \)GOS in \( X \).

Theorem 3.22: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)G continuous mapping, then \( f \) is an IF\( \pi \)G continuous mapping if \( X \) is an IF\( \pi \)T\( \frac{1}{2} \) space.

Proof: Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IF\( \pi \)GSCS in \( X \), since \( f \) is an IF\( \pi \)G Continuous. Hence \( X \) is an IF\( \pi \)T\( \frac{1}{2} \) space, \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f \) is an IF\( \pi \)G continuous mapping.
Theorem 3.23: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\(\pi\)GS continuous function, then \( f \) is an IFG continuous mapping if \( X \) is an IF\(\pi\)g\(T_{1/2}\) space.

**Proof:** Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IF\(\pi\)GSCS in \( X \), by hypothesis. Since \( X \) is an IF\(\pi\)g\(T_{1/2}\) space, \( f^{-1}(A) \) is an IFGCS in \( X \). Hence \( f \) is an IFG continuous mapping.

Theorem 3.24: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\(\pi\)GS continuous mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) be IF continuous, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IF\(\pi\)GS continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Z \). Then \( g^{-1}(A) \) is an IFCS in \( Y \), by hypothesis. Since \( f \) is an IF\(\pi\)GS continuous mapping, \( f^{-1}(g^{-1}(A)) \) is an IF\(\pi\)GSCS in \( X \). Hence \( g \circ f \) is an IF\(\pi\)GS continuous mapping.

### 4. CONCLUSION

In this paper we have introduced intuitionistic fuzzy \(\pi\)-generalized semi continuous mappings and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy generalized continuous mappings and some of the intuitionistic fuzzy continuous mappings already exist.

### 5. REFERENCES


